UNIVERSITY COLLEGE LONDON
DEPARTMENT OF PHYSICS AND ASTRONOMY

2B21 MATHEMATICAL METHODS IN PHYSICS AND ASTRONOMY

Problem Sheet M8 (2003–2004)

Solutions to be handed in on Tuesday 2 December 2003

1. By using integration by parts twice, or otherwise, show that

$$\int \sin nx \sinh x \, dx = \frac{1}{1+n^2} \left[\cosh x \sin nx - n \cos nx \sinh x\right] + C \, .$$

[4 marks]

The function $f(x)$ is periodic with period 2π. In the interval $-\pi < x < +\pi$, it is given by

$$f(x) = \sinh x \, .$$

Is $f(x)$ even or odd? **[1 mark]**

If $f(x)$ is expanded as the Fourier series

$$f(x) = \tfrac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx \, ,$$

obtain the coefficients a_n and b_n and show that the Fourier series is

$$f(x) = \frac{2}{\pi} \sinh \pi \sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1} \sin nx \, .$$

[4 marks]

State Parseval's theorem and use it to evaluate

$$\sum_{n=1}^{\infty} \frac{n^2}{(n^2+1)^2} \, .$$

[6 marks]

2. Evaluate the Fourier transform

$$g(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x)\, e^{i\omega x} \, dx$$

of the function $(a > 0)$

$$f(x) = \begin{cases} e^{-ax} & x \geq 0 \, , \\ 0 & x < 0 \, . \end{cases}$$

[2 marks]

Verify Parseval's theorem for this example. **[4 marks]**