University College London

Department of Physics and Astronomy

2B21 Mathematical Methods in Physics & Astronomy

Suggested Solutions for Problem Sheet M1 (2003–2004)

1. Subtracting twice the first equation from the second shows that $x_1 = y_2 - 2y_1$. It then follows that $x_2 = \frac{1}{2}(3y_1 - y_2)$. **[2]**

   In matrix form,

$$\underline{y} = \underline{A}\,\underline{x} \quad \text{and} \quad \underline{x} = \underline{B}\,\underline{y}\,,$$

$$\underline{A} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \quad \text{and} \quad \underline{B} = \begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}. \qquad \textbf{[2]}$$

   Multiplying the two together,

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix} = \begin{pmatrix} -2+3 & 1-1 \\ -6+6 & 3-2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \qquad \textbf{[2]}$$

   Similarly

$$\begin{pmatrix} -2 & 1 \\ \frac{3}{2} & -\frac{1}{2} \end{pmatrix}\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} -2+3 & -4+4 \\ \frac{3}{2}-\frac{3}{2} & 3-2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}. \qquad \textbf{[2]}$$

   Thus $\underline{B}$ is the *inverse* matrix of $\underline{A}$.

2. Expanding by the first row,

$$\Delta = 2\begin{vmatrix} 0 & 3 & 2 \\ 3 & 2 & 1 \\ 2 & 1 & 0 \end{vmatrix} - \begin{vmatrix} 1 & 3 & 2 \\ 0 & 2 & 1 \\ 3 & 1 & 0 \end{vmatrix} - 3\begin{vmatrix} 1 & 0 & 3 \\ 0 & 3 & 2 \\ 3 & 2 & 1 \end{vmatrix}$$

$$= 2[-3(-2)+2(3-4)] - [-1+3(-1)] - 3[1(3-4)+3(-9)] = 8+4+84 = 96\,. \qquad \textbf{[5]}$$

   Alternatively, subtracting $2C_2$ from C_1 and $3C_2$ from C_4 gives

$$\Delta = \begin{vmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 3 & 2 \\ -6 & 3 & 2 & -8 \\ -1 & 2 & 1 & -6 \end{vmatrix} = -\begin{vmatrix} 1 & 3 & 2 \\ -6 & 2 & -8 \\ -1 & 1 & -6 \end{vmatrix}.$$

   Now add R_3 to R_1 and subtract $6R_3$ from R_2 to give

$$\Delta = -\begin{vmatrix} 0 & 4 & -4 \\ 0 & -4 & 28 \\ -1 & 1 & -6 \end{vmatrix} = 112 - 16 = 96\,. \qquad \textbf{[5]}$$

3.

$$\underline{C} = \begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \qquad \textbf{[2]}$$

$$\underline{D} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{pmatrix} = \begin{pmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{pmatrix} , \qquad \textbf{[2]}$$

Obviously $|\underline{C}| = 0$ and $|\underline{D}| = 0$ because the first and third rows are identical. [**1**]

On the other hand, $|\underline{A}| = 1(1) + 1(-2) + 1(-3+4) = 0$, whereas $|\underline{B}| = 0$ because two rows are again equal. [**1**]

Hence $|\underline{C}| = |\underline{D}| = 0 = |\underline{A}| \times |\underline{C}|$. [**1**]

The sums of the diagonal elements will be defined later in the course as the trace of the matrix.

$$Tr(\underline{C}) = 0 + 0 + 0 = 0 \, ; \quad Tr(\underline{D}) = -11 + 12 - 1 = 0 \, . \qquad \textbf{[2]}$$

The two numbers are identical, as suggested in the question.