STELLAR STRUCTURE AND EVOLUTION

Contents

Lecture 1 Introduction

The study of stellar structure and evolution plays a central role in modern astrophysics. For example, the study of distances and ages of stars, which are crucial to our understanding of the structure and history of our Galaxy, depends on stellar evolution calculations. Since the synthesis of almost all chemical elements is supposed to take place inside stars, an understanding of the chemical history of the Universe requires that one understands stellar evolution. Calculations of stellar structure and evolution depend on knowledge of physical properties of the matter in the stars; hence by testing the computed models against observations we are effectively testing the physics that was used to compute the models, often under conditions where it is impossible to carry out test in the laboratory.

This chapter provides an introduction to some of the terminology and concepts, which will be used later. It also gives a very rough sketch of the life history of a star, in an attempt to establish a framework for organizing the details, which follow in the subsequent chapters.

1.1. Stellar timescales

1.1.1. The dynamical timescale

Changes in a star may occur on a range of different timescales. The shortest relevant timescale is the *dynamical* timescale t_{dyn}. Consider a star of mass M and radius R. The gravitational acceleration at the surface of the star is

$$g_s = \frac{GM}{R^2} \tag{1.1}$$

$$g_s = \frac{GM}{R^2} g_s = \frac{GM}{R^2}$$

where G is the gravitational constant. Hence the time required for a particle to fall the distance a in the gravitational field of the star is

$$t = \left(\frac{2a}{g_s}\right)^{1/2} = \left(\frac{2aR^2}{GM}\right)^{1/2}.$$

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(1.2)

If we take a to be $R/2$ we obtain a timescale that is characteristic for motions over stellar scales in the gravitational field:

$$t_{dyn} = \left(\frac{R^3}{GM}\right)^{1/2}.$$

$$t_{dyn} = \left(\frac{R^3}{GM}\right)^{1/2} . t_{dyn} = \left(\frac{R^3}{GM}\right)^{1/2}.$$

(1.3)

It is obvious that there is a great deal of arbitrariness in this definition; after all, we could have chosen a distance of R, or $R/10$, instead of $R/2$. However, the point of arguments like this is not to obtain a *precise* values of the quantities that are being estimated; rather, the purpose is to get a feel for the magnitude of the quantity, and its dependence on basic stellar parameters. Hence we shall use equation (1.3) as a reasonable estimate for dynamical changes to a star. Using the solar values M $\odot\odot$ and R $\odot\odot$ for M and R, we may write the equation as

$$t_{dyn} = 30\ \mathrm{min} \left(\frac{R}{R_\odot}\right)^{3/2} \left(\frac{M}{M_\odot}\right)^{-1/2}.$$

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(1.4)

Stellar radii vary over a range from roughly $0.01R_{\odot}$ to roughly $1000R_{\odot}$, whereas the mass ranges from $0.1M_{\odot}$ to $100\ M_{\odot}$. Hence the dynamical timescale ranges from seconds to years. However, in most cases we see no evidence for motion with such timescales on the stars. This indicates that the forces on the star are very nearly balanced; we describe this situation by saying that the star is in *hydrostatic equilibrium*.

1.1.2. The timescale for release of gravitational energy (or the thermal timescale)

If the star has no internal sources of energy, it can still radiate energy by contracting. In this way it gets gravitationally more tightly bound: its gravitational potential energy decreases (i.e., becomes of larger negative magnitude), and the star has to get rid of the excess energy somehow. As will be discussed later using the virial theorem, half of the energy released goes to heat up the star, and another half is radiated away.

An estimate for the timescale of this process can be obtained by calculating the time a star could radiate at a given rate on the energy released through gravitational contraction to a given radius. Let L be the *luminosity* of the star, i.e., the amount of energy it radiates per unit time. The gravitational potential on the surface of the star is $-GM/R$, and so an estimate of the gravitational binding energy is

$$E_{grav} = -\frac{GM^2}{R},$$

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(1.5)

calculated as the gravitational potential energy of the stellar mass in the surface gravitational potential. Hence the relevant timescale, known as the *Kelvin-Helmholtz timescale*, is

$$t_{KH} = \frac{GM^2}{RL}.$$

(1.6)

$$t_{KH} = \frac{GM^2}{RL} t_{KH} = \frac{GM^2}{RL}.$$

In terms of solar values, the result is

$$t_{KH} = 30 \text{ million years} \left(\frac{M}{M_\odot}\right)^2 \left(\frac{R}{R_\odot}\right)^{-1} \left(\frac{L}{L_\odot}\right)^{-1}.$$

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This value gave rise to some controversy in the 19th century, at a time vhen the origin of the solar energy output was unknown. Gravitational contraction was considered as a valuable hypothesis, but this clearly limited the age of the Sun, and hence presumably of the Earth. On the other hand it was becoming clear from geological evidence, and from the time required for the evolution of the species, that the Earth had to be much older. As we now know, the resolution of the problem came with the realization that the solar energy derives from nuclear reactions in the solar core.

It will be shown later using the virial theorem that the gravitational binding energy and the total thermal energy of a star have the same magnitude. Hence t_{KH} also gives the time it would take for a star to radiate its thermal energy at a given luminosity, whence the name *thermal timescale*.

1.1.3. The nuclear timescale

During most of the life of a star, the energy it radiates comes from the fusion of hydrogen into helium. The total amount of energy that is available from this reaction may be estimated as $\Delta E = \Delta m\, c^2$, where Δm is the difference is mass between the original hydrogen and the resulting helium, and c is the speed of light. In the fusion of hydrogen to helium about 0.7 per cent of the mass is lost. The reaction occurs only in the inner about 10 per cent of the mass of the star. Hence the total amount of energy available is approximately $7\times10^{-4} M c^2$, and the corresponding timescale is

$$t_{nuc} = 7 \times 10^{-4} \frac{Mc^2}{L},$$

$$t_{nuc} = 7 \times 10^{-4} \frac{Mc^2}{L}, t_{nuc} = 7 \times 10^{-4} \frac{Mc^2}{L},$$

(1.8)

or

$$t_{nuc} = 10^{10} \text{ years} \left(\frac{M}{M_\odot}\right)\left(\frac{L}{L_\odot}\right)^{-1}.$$

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(1.9)

Since the star spends by far the largest part of its life in the hydrogen-helium burning phase, t_{nuc} provides a measure of the total lifetime of a star. As we will see later, the luminosity is a steeply increasing function of stellar mass. Hence, the dependence on L is dominant in the above equation, and the nuclear timescale decreases rapidly with increasing mass. For a $30M_\odot$ star the entire evolution, from birth to death, only lasts about 5 million years, whereas a star of $0.5M_\odot$ has barely had time to evolve over the age of the Universe.

1.2. The life of a star

The evolution of a star is largely a fight between gravity and nuclear reactions. The outcome of the fight depends critically on the mass of the star.

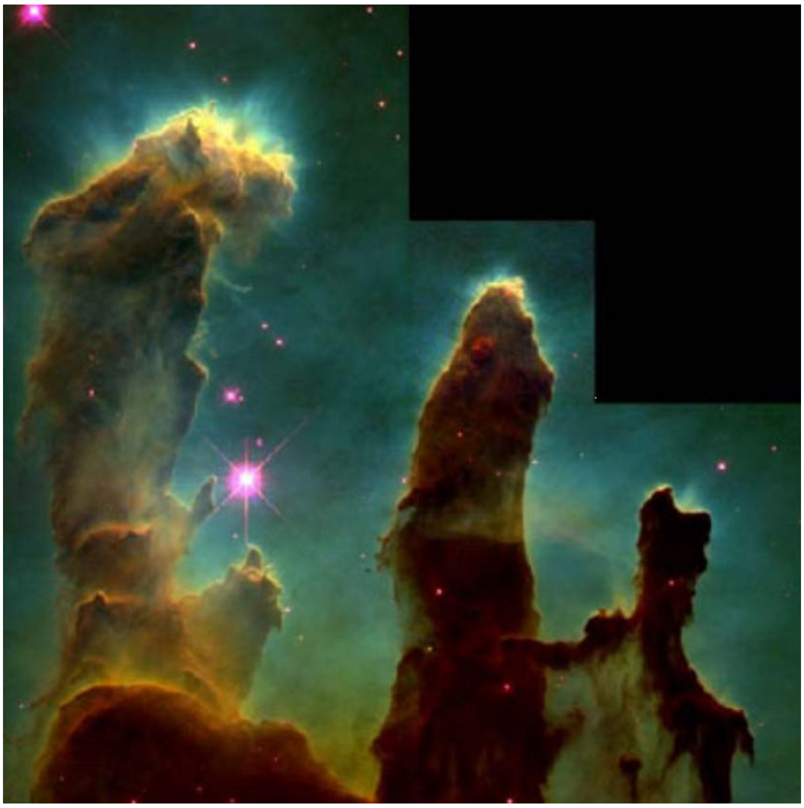

Figure 1.1. Nebula M16 in Serpens. M16 is thought to be associated with active processes of star formation.

Stars are born from a contracting cloud of interstellar matter. As the cloud contracts, gravitational potential energy is released. Part of this energy is used to heat up the gas; the cloud becomes hotter than its surrounding and starts to radiate energy away. As long as there are no other sources of energy in the cloud, the energy that is lost through radiation must be replaced by further release of gravitational energy, i.e. through further contraction. The rate of contraction is determined by the rate of energy loss. It is obvious that this phase occurs on something like the Kelvin-Helmholtz timescale discussed above.

The contraction continues up to the point where the temperature in the core of the star gets sufficiently high that nuclear reactions can take place, at a rate where the energy generated balances the radiation from the stellar surface. The temperature required is determined by the energy needed to penetrate the potential barrier established by the Coulomb repulsion between different nuclei. Hence the first nuclei to react are those with the lowest charge, i.e. hydrogen, starting when the temperature reaches a few million degrees. At this point a number of reactions sets in, the net effect of which is to fuse hydrogen into helium,

$$4\ {}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + 2e^{+} + 2\nu_{e}.$$

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$$4\ {}^{1}\mathrm{H} \rightarrow {}^{4}\mathrm{He} + 2e^{+} + 2\nu_{e}. \quad (1.10)$$

Because of charge conservation, two of the four protons on the left-hand side have to be converted into neutrons and positrons; the positrons are immediately annihilated by two electrons, so that the reaction can be thought as a reaction where four hydrogen *atoms* fuse into one helium atom (although at the temperature in the stellar core the atoms are fully ionized, i.e. separated into nuclei and free electrons). The reaction furthermore has to conserve the number of *leptons*, i.e. light elementary particles; since two anti-leptons (the positrons) are created, this must be balanced by the creation of two leptons, the neutrinos. Thus, regardless of the path the reactions take, the fusion of four hydrogen atoms into one helium atom leads to the production of two neutrinos.

Once hydrogen burning in the core has been established, the contraction of the stars stops. Stars in this phase of their evolution are said to be on their *main sequence*. It occurs on the nuclear timescale determined above; since this is the longest active phase of the life of the star, most of the “normal” stars that we observe within a given volume of space are main-sequence stars. During the main-sequence evolution, the structure of the star gradually changes as hydrogen is used up in the core. The result is a contraction of the core and an expansion of the outer layers, accompanied by an increase in the luminosity. For example, the luminosity of the Sun has increased by about 30 percent since it started the core hydrogen burning phase.

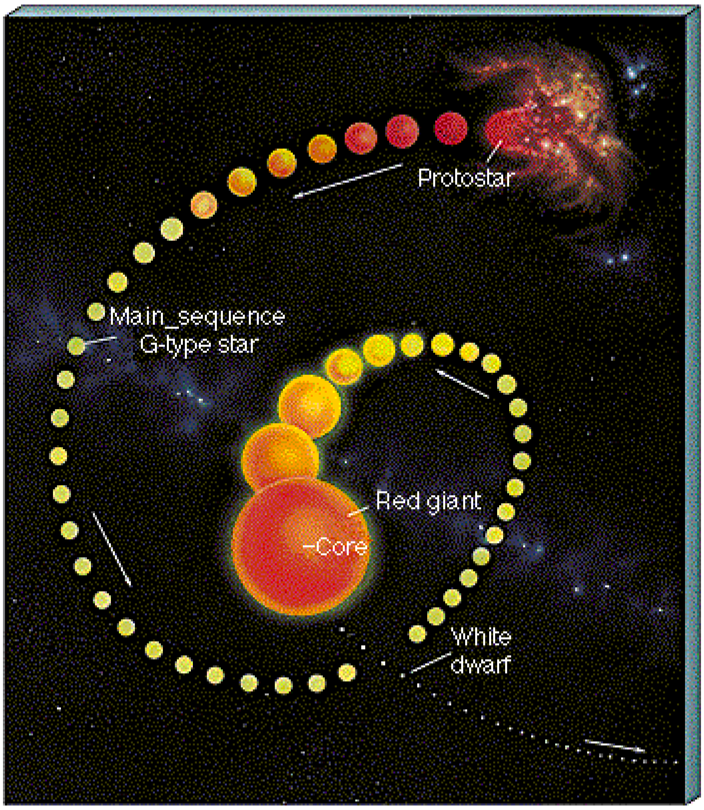

Figure 1.2. Schematic evolution of $1M_\odot$ star.

The onset of nuclear burning puts a temporary halt on the tendency of gravity to make the star contract; but it is obvious that this is only effective until the time when hydrogen is exhausted in the core. At that point hydrogen burning stops in the core, although it continues in a shell around it. The core contracts, again releasing gravitational energy and heating up, while the outer parts of the star expand drastically and cool, until the star becomes a *red giant*, with a radius that may be as large as the distance between the Sun and the Earth. As in the case of the initial contraction, the contraction of the core may be halted when its temperature becomes high enough for helium to react, to produce carbon:

$$3\ {}^{4}He \rightarrow {}^{12}C,$$

$$3\ {}^{4}He \rightarrow {}^{12}C,$$

$$3\ {}^{4}He \rightarrow {}^{12}C, \qquad (1.11)$$

possibly followed by

$${}^{4}He + {}^{12}C \rightarrow {}^{16}O.$$

$${}^{4}He + {}^{12}C \rightarrow {}^{16}O. {}^{4}He + {}^{12}C \rightarrow {}^{16}O. \qquad (1.12)$$

The result of this is to revert the previous evolution: the core expands somewhat, the outer layers contract and heat up, and the star settles down on the *helium burning main sequence*, while still maintaining a hydrogen-burning shell.

When helium is exhausted in the core, the history to some extent repeats itself: gravity again gets the upper hand, and the core, which now consists mainly of carbon and oxygen, contracts and heats up, surrounded by a helium-burning shell and, further out, possibly still a hydrogen-burning shell. The subsequent evolution depends crucially on the mass of the star.

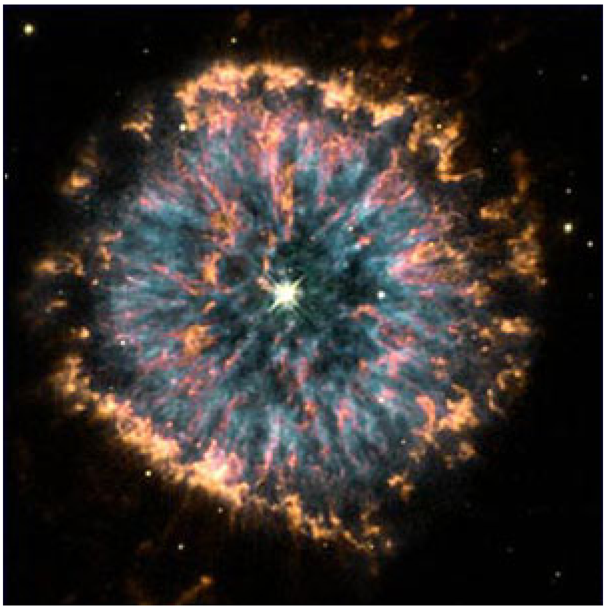

Figure 1.3. Planetary nebula NGC6751.

If the mass is less than about $10M_{\odot\odot}$, the core never becomes hot enough for the next nuclear reaction (between two carbon nuclei) to start. The core continues to contract and the outer layers expand until the star enters a second red giant phase. At this point an instability develops between the hydrogen- and helium-burning shells. It is thought that this instability leads to the loss of large amounts of mass from the star, undoubtedly aided by the large luminosity and radius of the star. The mass that is lost ends up as a *planetary nebula* and is later dispersed into the interstellar medium; this leaves behind the carbon-oxygen core which at that point has contracted to a radius comparable with the radius of the Earth, but is still extremely hot; such an object is observed as a *white dwarf*. It continues to radiate by loosing its internal thermal energy, a process that lasts forever.

For more massive stars, the carbon-oxygen core heats up sufficiently to start the next type of nuclear reactions. The star continues through a sequence of nuclear burning phases of gradually heavier elements, interspersed by phases of gravitational contraction. It is important that the later phases of chemical evolution proceed faster and faster: the final phases can be measured by years or even days, and hence the chances of observing a star while in these evolutionary phases are extremely small.

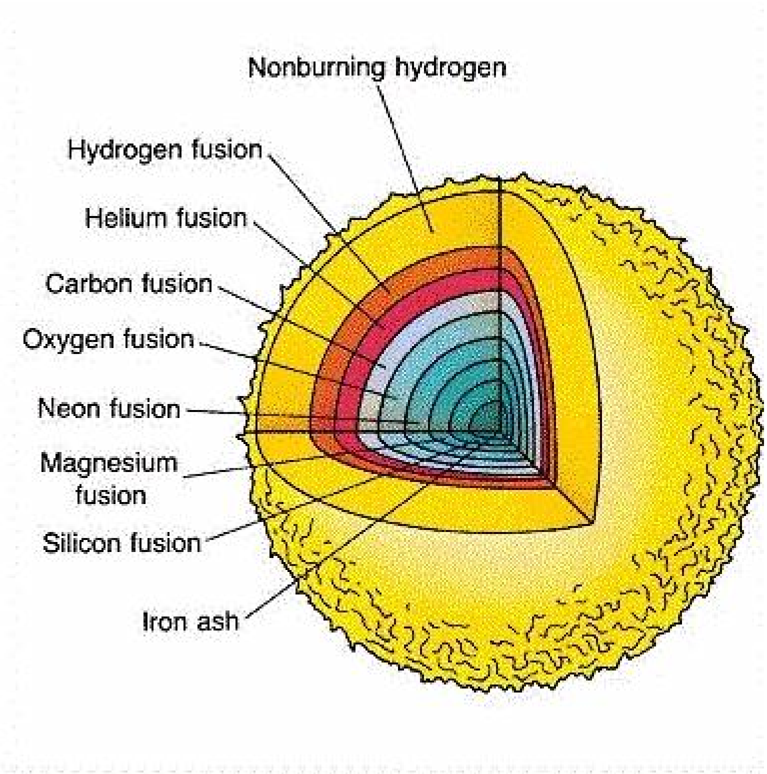

Figure 1.4. Schematic illustration (not to scale) of the “onion-shell” structure in the interior of a highly evolved massive star.

The burning of heavier and heavier elements has to end when the material of the core has been transformed into elements of the iron group; for these elements the nuclear binding energy is maximal, and hence fusion into even heavier elements *requires* energy instead of releasing it. At that point the gravity has won in the core; the core continues to contract and heat up, until the temperature gets so high that the iron nuclei are dissociated into protons and neutrons. The drastic increase in density forces the electrons and protons in the gas to recombine to newtrons, and the density gets so high that the neutrons essentially touch each other. At that point the core can contract no further; the result is a bounce which propagates out through the outer parts of the star as a shock wave, expelling them in a *supernova explosion*, in which the star for a few days becomes as luminous as all the stars in a normal galaxy combined. The energy derives from the gravitational energy released by the collapse of the core. In the reactions taking place during the explosion many neutron-reach nuclei are formed.

The fate of the core depends on its mass. If the core mass is less than about $2M_{\odot}$, a stable configuration is formed, consisting almost entirely of neutrons; this has a radius of only about 10 km. Observational evidence for such *neutron stars* has been found in the *pulsars* which are rapidly spinning neutron stars emitting radio pulses with a very precisely defined period. If the core mass is greater, even the pressure of neutrons cannot withstand gravity, and the core collapses into a black hole, where matter is essentially crushed out of existence. The ultimate victory of gravity!

Figure 1.5. Neutron star has radius of only about 10 km.

An excellent and somewhat more detailed description of stellar evolution, with special emphasis on supernova explosions, was given by Woosley and Weaver (1989).

An important consequence of these evolutionary scenarios is that interstellar matter is enriched by matter that has undergone nuclear burning, either in the mass loss in less massive stars, or in the supernova explosions of massive stars. This adds elements heavier than hydrogen and helium to the material out of which new stars are formed. It is believed that essentially all elements other than hydrogen and helium have been created and distributed in this way.

1.3. The physics of stellar interiors

The evolution sketched in the previous section is based on a large number of very complex numerical calculations. These, in turn, depend on knowledge and assumptions about the properties of stellar interiors.

To make the computations even possible, drastic simplifications are required, relative to the complex phenomena that might occur in real stars. The stars are assumed to be spherically symmetric; thus effects of rotation, which probably take place in all stars at some level and which must lead to departures from spherical symmetry, are neglected. The same is true for large-scale magnetic fields, which could also have an effect on the structure of the stars. Convective motions (to be discussed below), which probably take place in almost all stars, are treated very crudely. Other instabilities which may develop in the star and which could cause mixing between the core, where nuclear burning is taking place, and the outer parts of the star, are generally ignored. Mass loss from the star is normally either ignored or treated very approximately.

These complicating effects have been studied under other simplifying assumptions, but in most calculations of the stellar structure, including those described in these notes, they are ignored. One reason for this is that to include them all would make the computations completely intractable. A more fundamental difficulty is that we simply do not know how to handle them consistently; these problems are still very much at the frontier of current work on stellar evolution. And finally, it is probably wise to try to understand simplified stellar evolution theory, and to test it against observations, before trying to incorporate the complications.

Given these simplifications, the main features which determine the structure and evolution of stars are the *microscopic properties* of stellar matter, more specifically its equation of state, the transport of radiation through it, and the nuclear reactions. The equation of state determines the relations between the various thermodynamic properties, such as the temperature, density and pressure, of the gas that stars are made of. At the most elementary level (which is adequate for much of these notes) this is very simple: due to the high temperature, the gas is fully ionized and behaves essentially as an ideal gas. To carry out realistic calculations of stellar models, however, complicating effects have to be included. Near the stellar surface the gas is only partially ionized, and hence its properties depend on the degree of ionization, which in turn is determined by the interaction between the various components in the gas. At even lower temperature, in the atmospheres of cool stars, the formation of molecules also affects the equation of state. On the other hand, in the cores of massive stars in advanced stages of evolution the temperature may get so high that the formation of electron-positron pairs has to be taken into account, as well as processes involving the production of, and energy loss through, neutrinos. Also, at high densities quantum-mechanical effects set in, leading to the properties of the gas being dominated by *degenerate electrons*.

The energy transport is carried out by radiation under many circumstances, and hence is determined by the interaction between radiation and matter, as specified by the *absorption coefficient* or *opacity* of the matter. This depends on the detailed distribution of the atoms in the gas on energy levels, and hence on the equation of state of the gas, on the cross-section for absorption in each level in the atoms, and on the interaction between the atoms. Thus the calculation of opacities is a major undertaking. As an example it may be mentioned that for some years a large number of scientists in several countries have been engaged in collecting the atomic data and recomputing the equation of state with the goal of setting up new tables of opacities; even so, the resulting tables are restricted to relatively low densities where the interactions between the atoms can be ignored.

When the opacity or the amount of energy to be transported gets too high, energy transport by radiation can no longer be achieved in a stable manner. It is replaced by transport through motions in the gas, the so-called *convection*, which is quite similar to the motions in a pot of water being heated. Even convection in a pot of water gives rise to complex hydrodynamical phenomena which are far from being understood; hence it is not surprising that convection in stars is still an area of considerable uncertainty in studies of stellar structure. Besides its effect on energy transport, convection also affects the evolution of a star by mixing material, thereby for example homogenizing the composition in regions of nuclear burning.

The rates of nuclear reactions are determined by the speed with which the nuclei move relative to each other, which in turn depends on the temperature, and by the cross-sections for the reactions, which again is a function of the relative energy of the nuclei. The cross-sections can in principle be measured experimentally; a problem is, however, that reactions under stellar conditions often occur at such low energies that the corresponding rate of reactions under laboratory conditions is almost immeasurably small. Hence a considerable amount of theoretical extrapolation is required to determine the stellar rates. Furthermore, the reactions are also affected by the presence of other particles in the gas, which may partly shield the charges of the nuclei and hence increase the reaction rates; this again depends on the thermodynamic state of the gas.

It is obvious that the description of the physical state of stellar interiors gives rise to a number of difficult problems, which are still being investigated. Fortunately, it is possible to obtain a basic understanding of the structure and evolution of stars without going into such detail. Thus, in the following we shall consider only the simplest possible physics, while occasionally hinting at some of the complications.

LITERATURE

Woosley, S., Weaver, T. 1989, Scientific American 261, 24