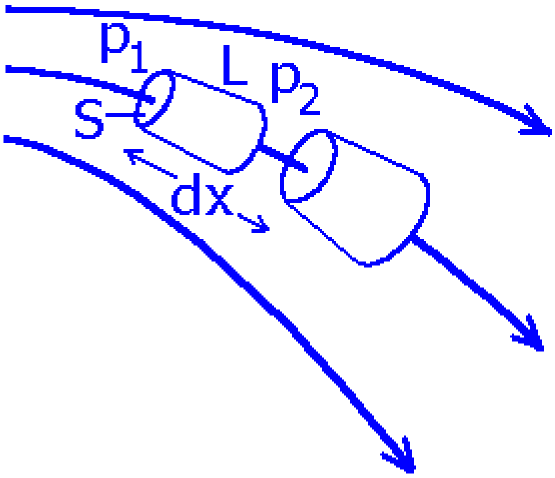
THE BERNOULLI'S THEOREM



Consider a small cylidrical fluid element with crossection S and length L, with mass m and volume V.

Choose the coordinate x along the velocity u of the fluid element.

Let dx be a small displacement along x during the motion.

The ammount of work done by the outer pressure forces is

$$dA = (p_1 - p_2)S\,dx = -\frac{dp}{dx} LS\,dx = -V\,dp$$

$$= -\frac{m}{\rho} dp = -m\,dh.$$

This work goes to the increase of the total mechanical (kinetic plus potential) energy of the fluid element,

$$-m\,dh = d\left(\frac{1}{2} mu^2\right) + d(m\psi).$$

We observe that $u^2/2 + \psi + h$ remains constant during the motion of the fluid element (along the streamline).

EXERCISE 3.2

From the mass conservation equation $\rho u A = const$ (3.16), we have

$$\frac{d\rho}{\rho} + \frac{du}{u} + \frac{dA}{A} = 0.$$

We now substract the equation (3.18) to eliminate $d\rho/\rho$, getting

$$\frac{du}{u} + \frac{dA}{A} = \mathcal{M}^2 \frac{du}{u},$$

which gives (3.19).

EXERCISE 3.3

Equations (3.27, 3.28) follow directly from (3.20, 3.21) when you substitute $r, u, \rho, \dot{M}$ measured in dimensionless quantities x, v, a, λ specified by (3.25, 3.26).

Equations (3.29, 3.30) is just the differential form of (3.27, 3.28).

When you substract (3.30) from (3.29) to eliminate da/a, you arrive to (3.31).