EXERCISE 4.1

If $\Omega = \Omega(r_c)$, a scalar function $f = f(r_c)$ exists such that $r_c\Omega^2 = df / dr_c$. We then have $\hat{\mathbf{r}}_c r_c\Omega^2 = \nabla f$.

If there exists a scalar function $f(\mathbf{r})$ such that $\nabla f = \hat{\mathbf{r}}_c r_c\Omega^2$, this function depends on r_c only (it is constant on cylinders), because its gradient is directed along r_c (gradient is always orthogonal to surfaces of $f = \text{const}$). But then, the absolute value of ∇f also depends on r_c only. This absolute value is $r_c\Omega^2$, hence Ω depends on r_c only.