Spacetime and Gravity: Assignment 2

In what follows, unless otherwise stated, we will use units such that the speed of light, c=1.

1

In Kings I 7:23, in describing Solomon's temple there is a desription of a circular "sea of cast bronze". It is described as follows: *It was round in shape, the diameter from rim to rim being ten cubits and it took a line thirty cubits long to go around it.*

Assume the bible is correct and explain the seemly incorrect value of the ratio of the circumference to the diameter of a circle by postulating that it was built on the surface of a sphere of radius a.

Find the radius of this sphere a, in cubits to two decimel place. You may use:

$$c = 2\pi a sin(\frac{r}{a})\,. \tag{1}$$

Hint, expand the above formula for $r < a$.

2

Define,

$$z = x + \tau y \tag{2}$$

with τ a complex number given by, $\tau = \tau_1 + i\tau_2$ where τ_1 and τ_2 are real. What is the line element:

$$ds^2 = dz\bar{dz} \tag{3}$$

in terms of $dx, dy, \tau_1\tau_2$ (where $\bar{dz}$ is the complex conjugate of dz.)

3

Carry out the following coordinate transformations on the flat space line element,

$$ds^2 = -dt^2 + dx^2 + dy^2 + dz^2\,, \tag{4}$$

$$x = rsin(\theta)cos(\phi) \qquad y = rsin(\theta)sin(\phi) \qquad z = rcos(\theta)\,, \tag{5}$$

to get the line element in terms of the coordinates t, r, θ, ϕ. These are 3d spherical polars. Now, do anyother coordinate transformation to express the metric in terms of u, v, θ, ϕ where

$$u = t - r \qquad v = t + r\,, \tag{6}$$

u,v are called light cone coordinates. Why?