Topic 29 — The Fabry-Perot Interferometer *FGT1040*

In the Michelson interferometer, we observe interference between two beams, and the resulting fringes have a sin-squared profile. In a sense, this is the amplitude-division equivalent of Young's slits for wavelength division. In the diffraction grating, however, we found that many slits gave us much better resolution. The Fabry-Perot apparatus increases the resolution of an amplitude division system by allowing multiple beams to interfere.

The basic idea is to allow multiple reflections within a thin film (diagram). If the phase difference between successive reflections is δ, and if the reflection coefficients at the surfaces are r and r', transmission coefficients t and t', the successive transmitted fields will be (suppressing the factor of $e^{i\omega t}$),

$$\begin{aligned} E_{t1} &= E_0 t t' \\ E_{t2} &= E_0 t r' r t' e^{-i\delta} \\ E_{t3} &= E_0 t r' r r' r t' e^{-2i\delta}, \end{aligned}$$

where we have absorbed the change of phase from a single passage across the etalon into the factor E_0. When we add all these together we have a geometric series, first term

$$E_0 t t'$$

and common ratio

$$r' r e^{-i\delta}$$

which we may sum over all paths to give

$$E_t = E_0 t t' \frac{1}{1 - r' r e^{-i\delta}}.$$

To find the *intensity* we multiply E_t by its complex conjugate to obtain (assuming r, r', t and t' to be real)

$$\begin{aligned} I_t &= I_0 (t t')^2 \frac{1}{\left(1 - r' r e^{-i\delta}\right)\left(1 - r' r e^{i\delta}\right)} \\ &= I_0 \frac{(t t')^2}{1 + (r' r)^2 - 2 r' r \cos(\delta)} \\ &= I_0 \frac{(t t')^2}{1 + (r' r)^2 - 2 r' r + 2 r' r (1 - \cos(\delta))} \end{aligned}$$

$$
\begin{aligned}
&= I_0 \frac{(tt')^2}{1+(r'r)^2 - 2r'r + 4r'r\sin^2(\delta/2)} \\
&= I_0 \left(\frac{tt'}{1-r'r}\right)^2 \frac{1}{1+\left[\frac{4r'r}{(1-r'r)^2}\right]\sin^2(\delta/2)}
\end{aligned}
$$

It is convenient to define three factors, to simplify the final result:

$$
\begin{aligned}
R &= r\,r' \\
T &= t\,t' \\
F &= \frac{4R}{(1-R)^2},
\end{aligned}
$$

when we can write

$$
\boxed{I = I_0\left(\frac{T}{1-R}\right)^2 \frac{1}{1+F\sin^2(\delta/2)}}
$$

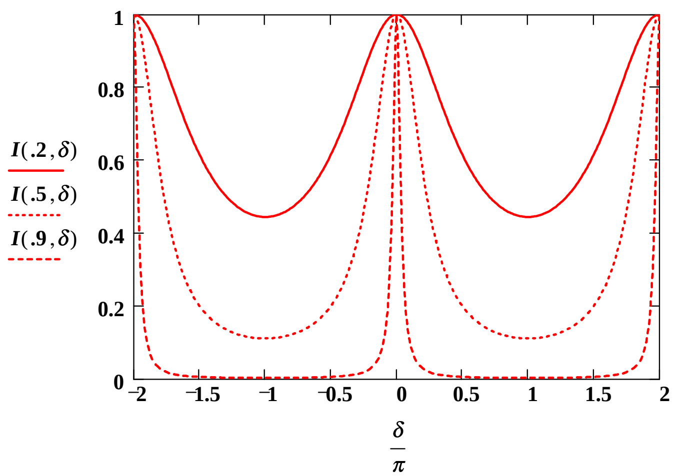

Figure L29.1: The variation of the Airy function with the reflectivity coefficient R, plotted as a function of the phase shift δ.

The term

$$
\frac{1}{1+F\sin^2(\delta/2)}
$$

is known as the *Airy function.* We can plot this function as a function of $R = r'r$, and this is done in figure L29.1. There are peaks as a function of δ which correspond to

$$\sin(\delta/2) = 0,$$

or

$$\delta = 2p\pi$$

for integer values of p.

Note that the peak which occurs at

$$\delta = 2p\pi$$

where p is an integer will fall to half its peak value at a value of δ, $\delta_{1/2}$ say, given by

$$\boxed{\delta_{1/2} = 2\sin^{-1}(1/\sqrt{F}) \approx 2/\sqrt{F}}$$

for large values of F

Thus the peaks get sharper as the reflection coefficients approach unity.

L29.1 Fabry-Perot interferometer *H368-372*

etalon

The arrangement of the Fabry-Perot interferometer (see figure L29.2) uses two glass plates to form the reflecting surfaces. The pair of parallel plates is called an etalon. For good performance we require a fairly large (few mm) plate separation, set accurately parallel, and partly silvered to increase the reflectivity. Often the outer surfaces of the plates are slightly wedged, to reduce intereference as a result of reflection from these surfaces.

silvered or unsilvered surfaces

With similar plates, $r = r'$, $t = t'$, and the phase difference δ is for light passing through the gap at an angle θ to the normal is

$$\delta = \frac{2\pi}{\lambda} 2d\cos(\theta) + 2\phi \qquad \text{(L29.1)}$$

where ϕ accounts for any phase change on reflection - if the mirrors are coated so as to increase the reflection coefficients ϕ may be between 0 and π, rather than being exactly π.

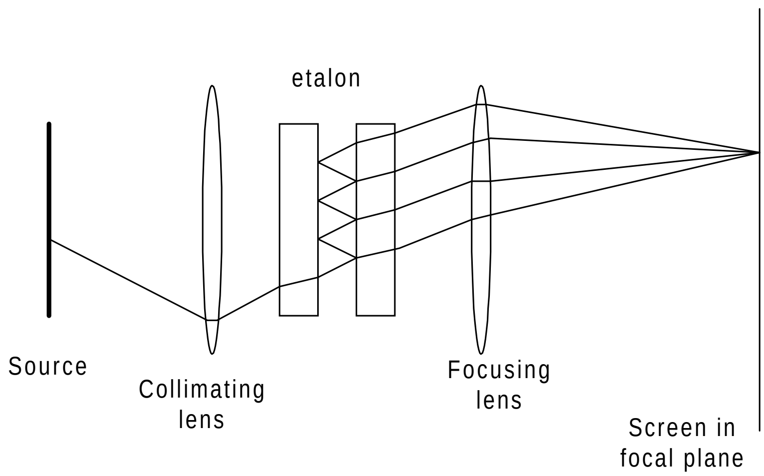

Figure L29.2: The Fabry-Perot interferometer.

As the phase shift depends on the angle θ, we see *circular fringes*, but because of the form of the Airy function they will be sharper than those in the Michelson interferometer.

resolving power

The resolving power may be increased by increasing the reflection coefficient. This can be done by silvering, or by appropriate dielectric layers - just as blooming a lens can reduce reflection, so appropriate coating can increase reflectivity.

The basic considerations which apply to the definition of the resolving power are the same as those that Rayleigh used in his definition of the criterion for the resolution of two slits, namely that the peaks have to be far enough apart compared with their widths that there is a discernible dip in intensity between them[1] This definition is not suitable for the present case, as

[1] In the case of two slits, the application of Rayleigh's criterion that the maximum of one peak should lie on or below the first minimum of the other gives a dip in intensity between the peaks to $8/\pi^2$ of the maximum. This follows because the separate intensity patterns vary as $(\sin(\beta)/\beta)^2$ where $\beta = (kd/2\sin(\theta))$, and if we set one peak at the point where $\beta = \pi$ for the other the dip in intensity corresponds to $\beta = \pi/2$ for each peak, giving a ratio of dip to peak intensity of $2(1/(\pi/2))^2 = 8/\pi^2$.

If we keep to this criterion, and note that in this case the maximum intensity is the we do not have a zero in the diffraction pattern. Instead, we use a definition due to Taylor, which states that the peaks are resolvable if their separate intensity curves intersect at their half-intensity positions. Near $\theta = 0$,

$$\delta = \frac{4\pi}{\lambda} d,$$

and differentiating and using the derivative as an approximation to the ratio of differences

$$\frac{\mathrm{d}\delta}{\mathrm{d}\lambda} = -\frac{4\pi d}{\lambda^2} \approx \frac{\Delta\delta}{\Delta\lambda}.$$

If the peaks are to be separated, the difference in δ between the peaks corresponding to the different wavelengths must be greater than double the

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central intensity of one peak plus the intensity of the other peak at the same point, for peaks to be resolved we require two values of δ, $\Delta\delta$ apart, which satisfy

$$\begin{aligned} \frac{8}{\pi^2} &\leq 1 + \frac{1}{1 + F\sin^2(\delta_1 + \Delta\delta/2)} \\ &= \frac{1}{1 + F\sin^2((\delta_1 + \Delta\delta/2)/2)} + \frac{1}{1 + F\sin^2((\delta_2 - \Delta\delta/2)/2)}. \end{aligned}$$

But we know that at the peaks δ_1 and δ_2 are each equal to a multiple of 2π, so we have

$$\frac{8}{\pi^2} 1 + \frac{1}{1 + F\sin^2(\Delta\delta/2)} = \frac{2}{1 + F\sin^2(\Delta\delta/4)},$$

giving, for small $\Delta\delta$ (expanding sins to first order, obtaining and solving a quadratic in $F\Delta\delta$),

$$\Delta\delta \approx \frac{4.2}{\sqrt{F}}.$$

Now, from Equation L29.1,

$$\Delta\delta = -\frac{2\pi\Delta\lambda}{\lambda^2} 2d\cos(\theta)$$

and δ itself is a large multiple, p, of λ, so making the reasonable approximation of neglecting the phase change ϕ compared with $2\pi p$,

$$\frac{\Delta\lambda}{\lambda} \approx \frac{\Delta\delta}{2\pi p},$$

which gives a resolving power

$$\frac{\lambda}{(\Delta\lambda)_{\min}} = \frac{4\pi d}{4.2\lambda}\sqrt{F},$$

since $p\lambda \approx 2d$.

half-width at half-height of each peak,

$$\frac{4}{\sqrt{F}} < \frac{4\pi d}{\lambda^2}\Delta\lambda.$$

This gives a resolving power[2]

$$\frac{\lambda}{(\Delta\lambda)_{\rm min}} = \frac{\pi d}{\lambda}\sqrt{F}.$$

A reasonable criterion for the resolving power is

$$\boxed{\frac{\lambda}{\Delta\lambda_{\rm min}} = \frac{\pi d}{\lambda}\sqrt{F}.}$$

For example, if $R = 0.9$, $d = 10$ mm, $\lambda = 500$ nm, we find that

$$F = \frac{4 \times 0.9}{(1-0.9)^2} = 360,$$

and then

$$\frac{\lambda}{\Delta\lambda_{\rm min}} = 1.2 \times 10^6$$

which is significantly larger than is achieved with most diffraction gratings (recall our previous result for a grating, which had a resolving power of 2000).

effect of extended source

Again, an extended source may be used because this system operates by amplitude division - the light from each region of the source is split up and thus each region contributes separately to the interference pattern. The patterns from each region are identical. Because we are dividing the *amplitude*, we do not require coherence across the source for the extended source to produce a stable pattern.

[2] This differs very little from the result deduced from our interpretation of the Rayleigh criterion.