Topic 27 — Interference - Various Phenomena

The interference effects we have looked at so far have involved splitting the wavefront up, sending part of the wavefront along one path and other parts along a different path, and recombining them to produce an interference pattern.

L27.1 Coated lenses - matching as an interference effect

There is an alternative way of splitting one signal into two or more – by division of amplitude. In fact we have already encountered interference of this kind in our treatment of the blooming of lenses, or quarter-wave matching. There by arranging that the path length through the matching layer was half a wavelength and with the same phase change of π at the reflection from the front surface and that at the back of the matching layer (because in each case the reflection was from an optically more dense medium) we could cancel the reflected signal.

We are going to look at several different instances of this effect.

L27.2 Interference fringes

thin film *FGT1036-1038, AF917-918*

The commonest situation in which we see interference effects is in oil on the surface of puddles, or in thin soap films or sheets of material such as mica[1],

[1]In some ways Newton set back the wave theory quite significantly. For example, he described the colours seen in thin films of mica as "fits of easy transmission and reflection" of his rays, and although he himself no doubt realised that this constituted no explanation it was enough to satisfy his less gifted successors for about a century. It was Thomas Young (1773-1829) who did much to force the wave theory back into favour. In a lecture to the Royal Society in 1801, entitled *On the Theory of Light and Colours*, he stated the principle of what he was later to call interference: "When two Undulations, from different Origins, coincide either perfectly or very nearly in Direction, their joint effects is a Combination of the Motions belonging to each." Unfortunately, Young was not a good communicator, and Hermann von Helmholtz wrote of him "He was one of the most acute men who ever lived, but had the misfortune o be too far in advance of his contemporaries. They looked on him with astonishment but could not follow his bold speculations, and thus a mass of his most important thoughts remained buried and forgotten in the *Transactions of the Royal Society*, until a later generation by slow degrees arrived at the rediscovery of his

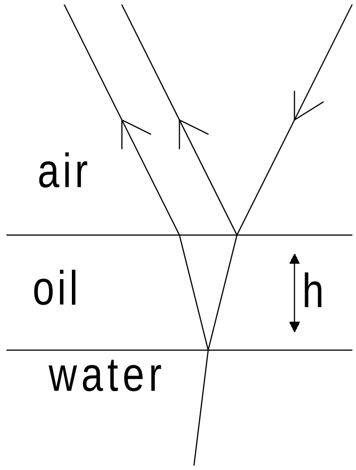

Figure L27.1: The geometry of fringe formation in an oil film.

as in figure L27.1. Let's consider the oil film first.

A typical oil has a refractive index of about 1.5, water 1.33. Thus the phase change on reflection in air from oil is π, that in oil from water is 0.

If we look vertically downwards on an oil film h thick, we will have

constructive interference between light reflected from the surface of the oil and that reflected from the oil-water interface if

$$2h = (p + \frac{1}{2})\lambda_{\text{oil}} = (p + \frac{1}{2})\frac{\lambda_{\text{air}}}{n_{\text{oil}}}$$

where p is an integer.

If we put in the numbers, the reflection coefficients at the interfaces are

$$r_{\text{air}-\text{oil}} = \frac{n_{\text{air}} - n_{\text{oil}}}{n_{\text{air}} + n_{\text{oil}}} = \frac{1.0 - 1.5}{1.0 + 1.5} = -0.20$$

and

$$r_{\text{oil}-\text{water}} = \frac{n_{\text{oil}} - n_{\text{water}}}{n_{\text{oil}} + n_{\text{water}}} = \frac{1.5 - 1.33}{1.5 + 1.33} = 0.06$$

so it is enough to consider one reflection at the interface.

discoveries, and came to appreciate the force of his arguments and the accuracy of his conclusions."

Oil films have thicknesses which are typically a fraction of a micron thick – say 0 to 1 micron. In fact oil films are usually thicker in the middle and thin at the edges. Visible light covers the range from about 0.4 to 0.7 microns in air, that is .27 to .47 microns in oil.

If the film is 0.5 microns thick in the centre, we can have values of

$$\frac{2h}{(p+\frac{1}{2})}$$

equal to 2, .667, .4, .28 microns, or maxima for air wavelengths of 3, 1, .6, .43 microns This will give a yellow fringe. On the other hand, a thickness of 0.525 microns will give

$$\frac{2h}{(p+\frac{1}{2})}$$

equal to 2.1, 0.7, 0.42, 0.3 microns, or air wavelengths of 3.15, 1.05, 0.63, 0.45, giving a mixture of yellow and blue.

Once the film becomes very thin, there is no phase shift as a result of the thickness, leaving only the shift of π between the front and back reflection, giving a minimum of reflectivity - the film appears dark in reflection.

The same phenomena occur in a soap film. Here the film's thickness varies as the water drains out. Again the difference in the phase changes is π on reflection from the front and back surfaces, so the film is dark by reflection but bright in transmission when the film becomes very thin.

Thin film - non-normal incidence

Of course, we do not always observe a film at normal incidence. As well as oil films, the geometry of the situation here is important in the descriptions of the Michelson and Fabry-Perot interferometers, which we shall encounter shortly. If the light is incident at an angle θ_1 in a medium of refractive index n_1 on a slab of material of thickness d and with a refractive index n_2, then the *optical* path length difference between paths AD and ABC (see figure L27.2) is

$$\Delta = n_2(\bar{AB} + \bar{BC}) - n_1\bar{AD}.$$

Now

$$\bar{AB} = \bar{BC} = d/\cos(\theta_2)$$

and, using Snel's law that

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2),$$

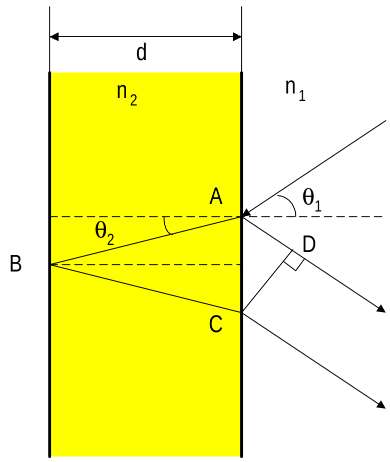

Figure L27.2: The geometry of fringe formation by a layer at non-normal incidence.

we have

$$\bar{AD} = \bar{AC}\sin(\theta_1) = \bar{AC}\frac{n_2}{n_1}\sin(\theta_2).$$

Finally

$$\bar{AC} = 2d\tan(\theta_2),$$

and putting all these together we have

$$\begin{aligned}\Delta &= 2n_2\frac{d}{\cos(\theta_2)} - n_1 2d\frac{n_2}{n_1}\sin(\theta_2)\tan(\theta_2)\\ &= \frac{2dn_2}{\cos(\theta_2)}(1-\sin^2(\theta_2))\\ &= 2n_2 d\cos(\theta_2).\end{aligned}$$

Again, the interference conditions depend on the phases changes on reflection from the two interfaces.

Newton's rings *FGT1035-1036, AF918-919*

This phenomenon was first discovered by Robert Hooke, and investigated both by him and by Newton.

For a lens with lower surface radius of curvature R resting on an optical flat (that is, a glass plate whose upper surface is smooth on the scale of the

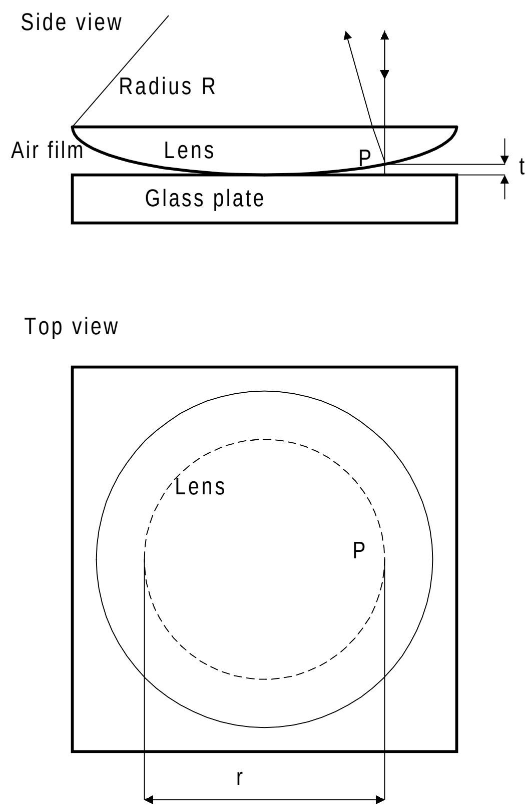

Figure L27.3: The set-up of a Newton's rings experiment.

wavelength of light λ) the thickness t of the gap at a distance r from the point of contact is given by (see figure L27.4)

$$r^2 = R^2 - (R-t)^2 = 2Rt - t^2$$

or, as we will only observe interference near the point of contact where $t << R$ (typical R is about 1m, typical t a few λ)

$$r^2 \approx 2Rt.$$

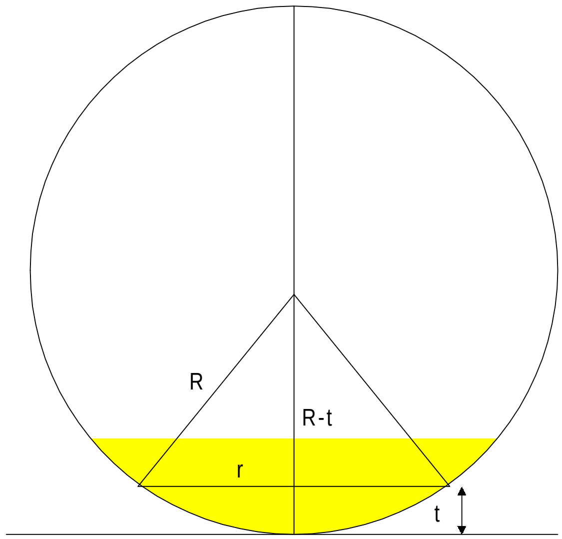

Figure L27.4: The geometry of the Newton's rings experiment.

For Newton's rings with air in the gap, light reflected from the flat experiences a phase shift of π, that from the bottom of the lens experiences no phase shift, so for constructive interference

$$2t = (p + \frac{1}{2})\lambda.$$

This will occur all round the radius r, giving a pattern of rings, so

$$r^2 = (p + \frac{1}{2})\lambda R.$$

Note that one can fill the gap with a fluid (oil, water...) in which case it is the wavelength *in that medium* that must be used in this formula.

The centre of the pattern (if the surfaces are in good contact) will be dark.

For example, if R is 1m, λ is 500nm, the first ring (with $p = 0$) has a radius

$$r = \sqrt{\frac{1}{2}500 \times 10^{-9} \times 1} = 0.5 \text{ mm}$$

and for the tenth (with $p = 9$)

$$r = 2.18 \text{ mm}.$$

wedge ***FGT1032-1034***

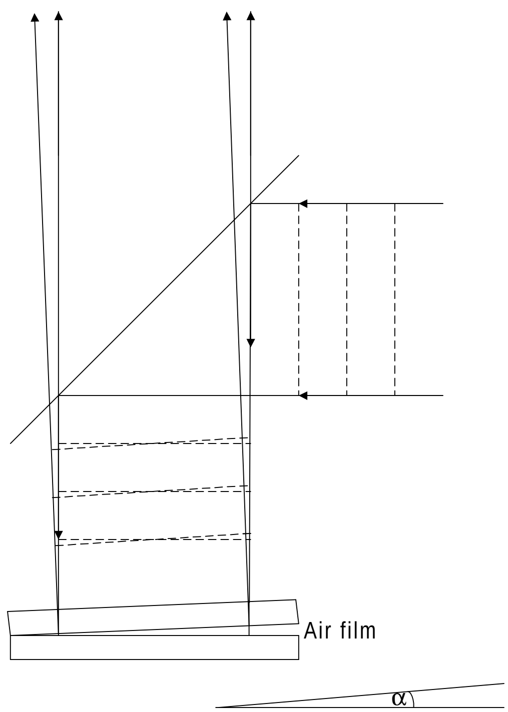

Figure L27.5: The formation of fringes in a wedge.

A similar but slightly simpler geometry uses two plates at a very slight angle α: the gap thickness a distance x from the edge is $x\alpha$, so we see bright fringes where

$$(p + \frac{1}{2})\lambda = 2\alpha x$$

or

$$x = \left(\frac{p + \frac{1}{2}}{2\alpha} \right) \lambda.$$

For example, suppose a slip of tissue paper of thickness 0.01mm is placed between two glass slides so as to form a wedge with a base of 20mm, so the angle is 5×10^{-4}. With light of wavelength, straight fringes will be seen with a spacing of

$$\frac{500 \times 10^{-9}}{2 \times 5 \times 10^{-4}} = 0.5 \text{ mm}$$

The set-up for a practical Newton's rings or wedge experiment uses a glass plate as a beam-splitter (see figure L27.5) and a travelling microscope to measure the fringes.

applications

More generally, interference effects between glass plates can be used to assess their flatness. A local bump on one plate, when pressed against another flat plate, will give locally fringe patterns similar to Newton's rings.