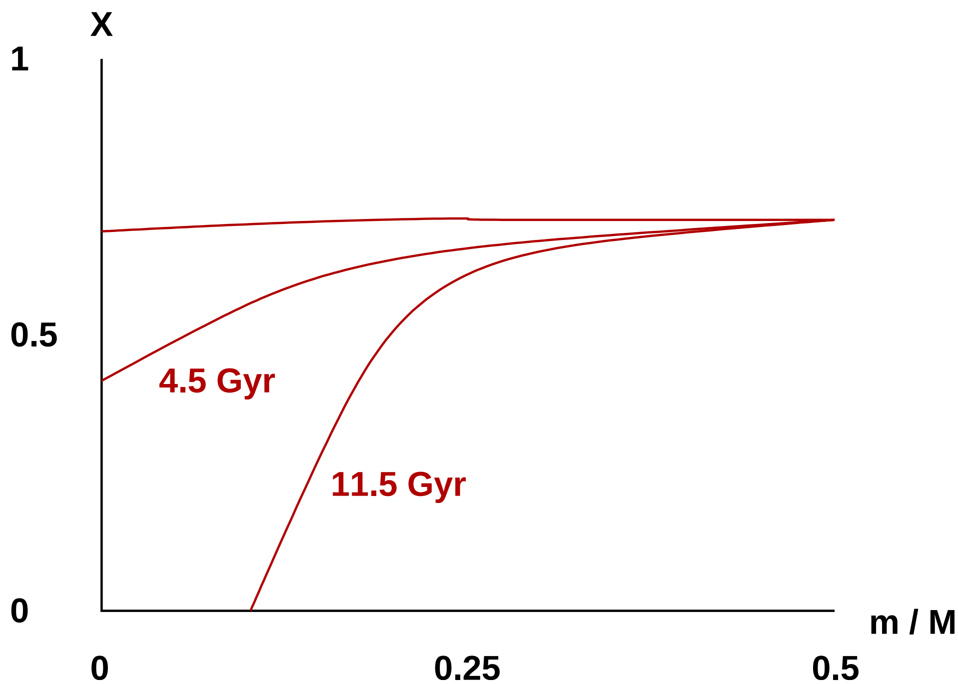
Evolution on the main sequence

On the main sequence, luminosity is provided by hydrogen burning into helium. Resulting change in the chemical properties of the star is concentrated towards the central region, since the nuclear reactions are strongly centrally peaked.

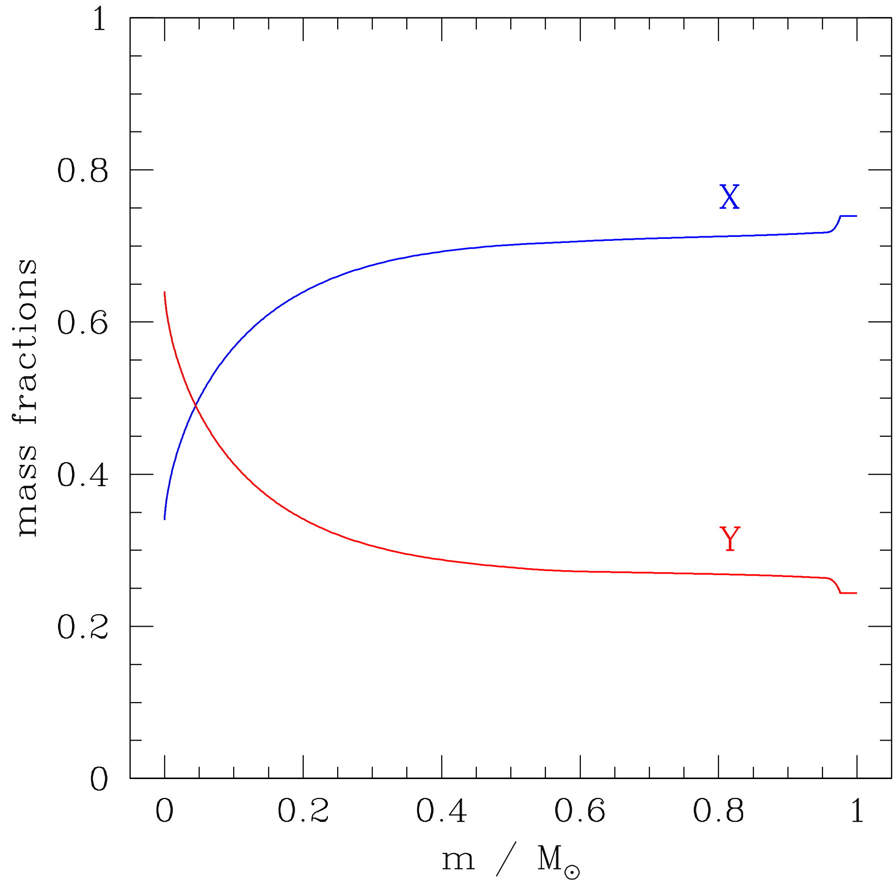
For low mass stars (roughly $M < M_\odot$) with radiative cores, situation is particularly simple. Absent mixing processes, at each radius,

$$\frac{dX}{dt} \propto -\epsilon_H$$

where ϵ_H is the rate of energy production from hydrogen burning reactions. Evolution for a Solar mass star looks like:

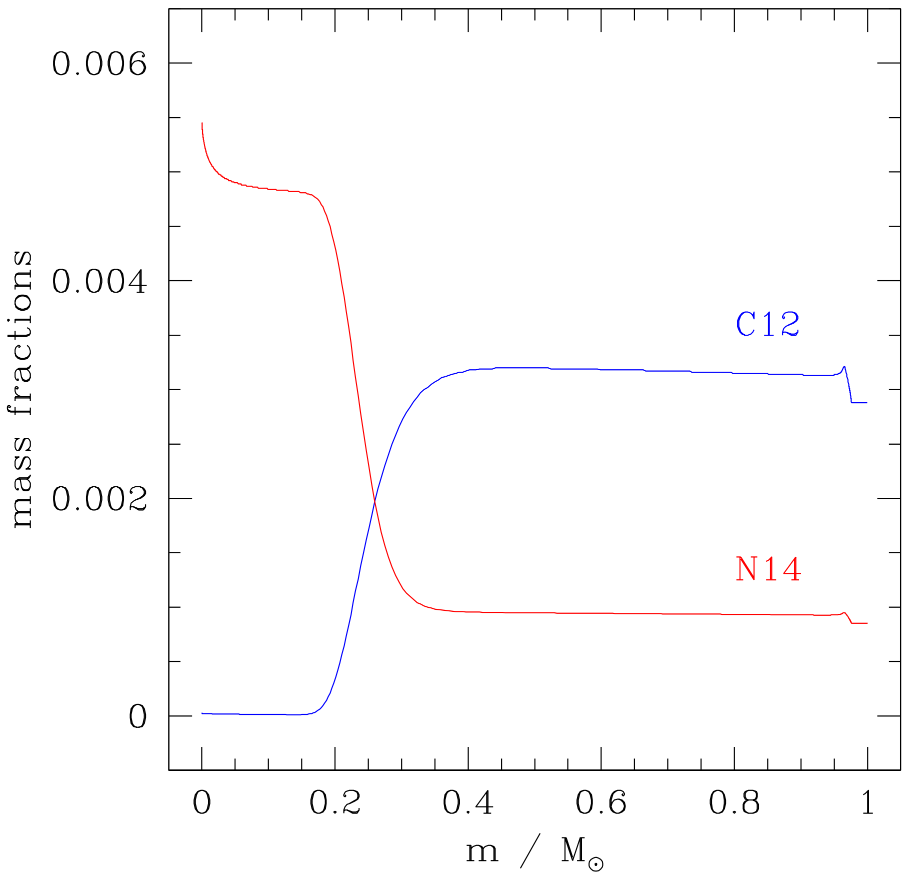


Hydrogen and helium mass fractions as a function of enclosed mass in the Sun,



Data from the standard Solar model of Bahcall, Pinsonneault & Basu, ApJ, 555, 990 (2001).

Mass fractions of ^{12}C and ^{14}N in the standard Solar model at the present epoch:



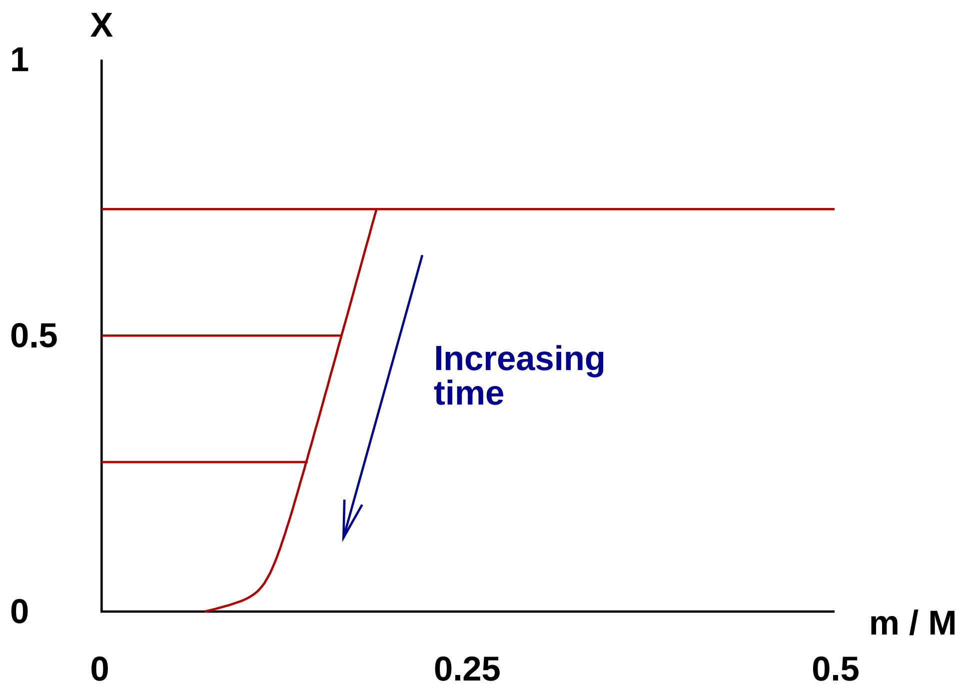
In more massive stars, helium production is *more* concentrated towards the center because the CNO cycle has a much higher temperature sensitivity than the PP chain. Convection, however, maintains a homogenous chemical composition within the core.

For central convection zones,

$$\frac{dX}{dt} \propto -\bar{\epsilon}_H$$

where $\bar{\epsilon}_H$ is the energy generation rate *averaged over the convective core.*

Allowing for a change (decrease) in the mass of the convective core with time, evolution of the hydrogen mass fraction for a 5 $M_\odot$ star looks like:



How do stars evolve in the HR diagram during hydrogen burning?

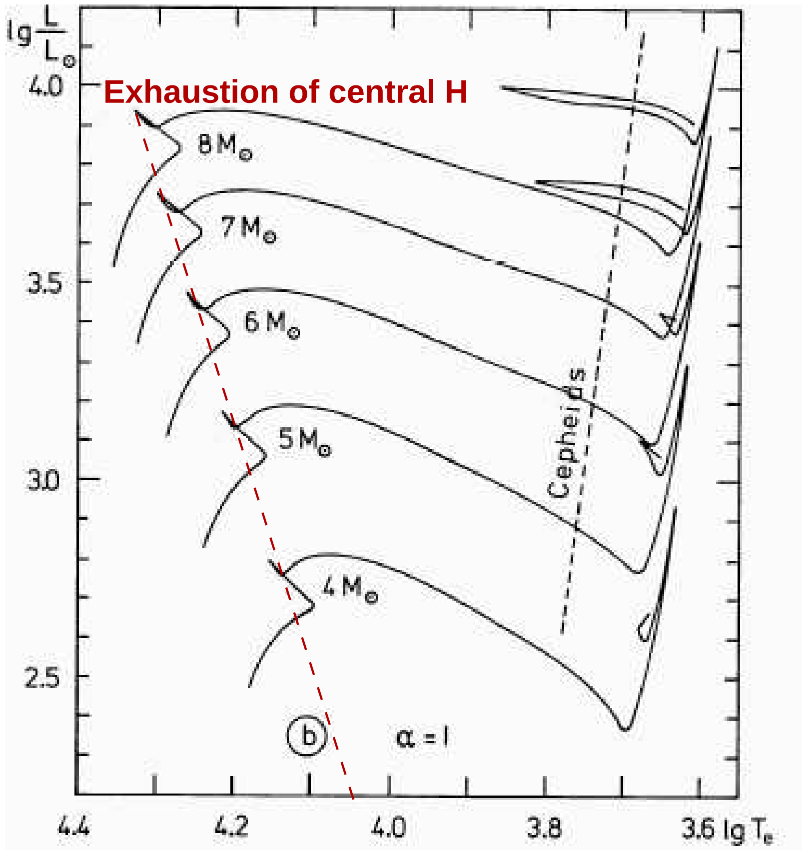


Fig. 4a and b. Evolutionary tracks through central He-burning; **a** without overshooting, **b** with overshooting ($\alpha = 1$) from the convective core during H-burning

(figure from Matraka et al. 1982)

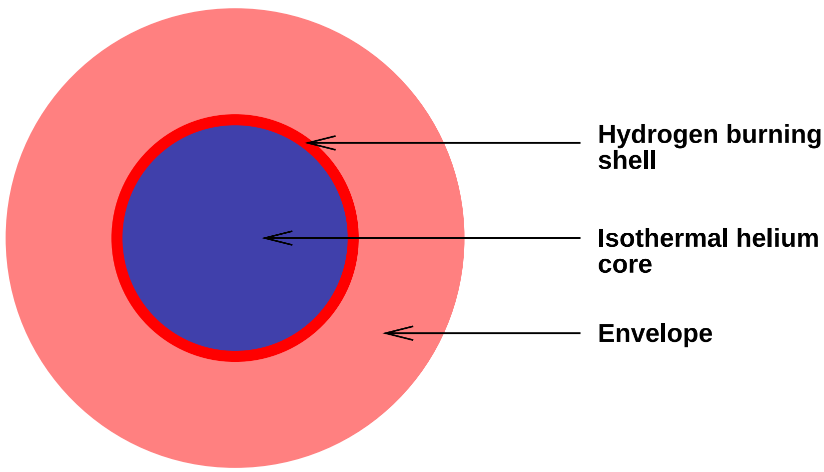
Slight evolution *above* the ZAMS during the phase of central hydrogen burning. Note that this is *not* the direction predicted for evolution of a chemically homogenous star.

Conditions for leaving the main sequence

As star evolves, form a helium core which grows in mass. Hydrogen burning continues in a shell around the core. Might imagine that this continues steadily until the entire hydrogen content of the star has been exhausted.

In reality, existence of a **maximum core mass** $\rightarrow$ abrupt departure from the main sequence.

Basic physical argument follows from the virial theorem and homology.



Structure:

- Isothermal helium core, mass M_c, temperature T_c.
- Hydrogen burning shell, maintained at a fixed temperature T_e independent of core properties by thermostatic nature of nuclear burning.
- Overlying envelope.

A consistent model requires that the temperature and pressure of the core and envelope match.

First consider the core. Appropriate form of the virial theorem (allowing for the existence of a surface pressure is),

$$3(\gamma - 1)U + \Omega = 3P_s V.$$

Substituting,

$$\begin{aligned} \Omega &\propto -\frac{GM_c^2}{R_c} \\ U &\propto M_c T_c \\ V &= \frac{4}{3}\pi R_c^3 \end{aligned}$$

obtain an expression for the surface pressure,

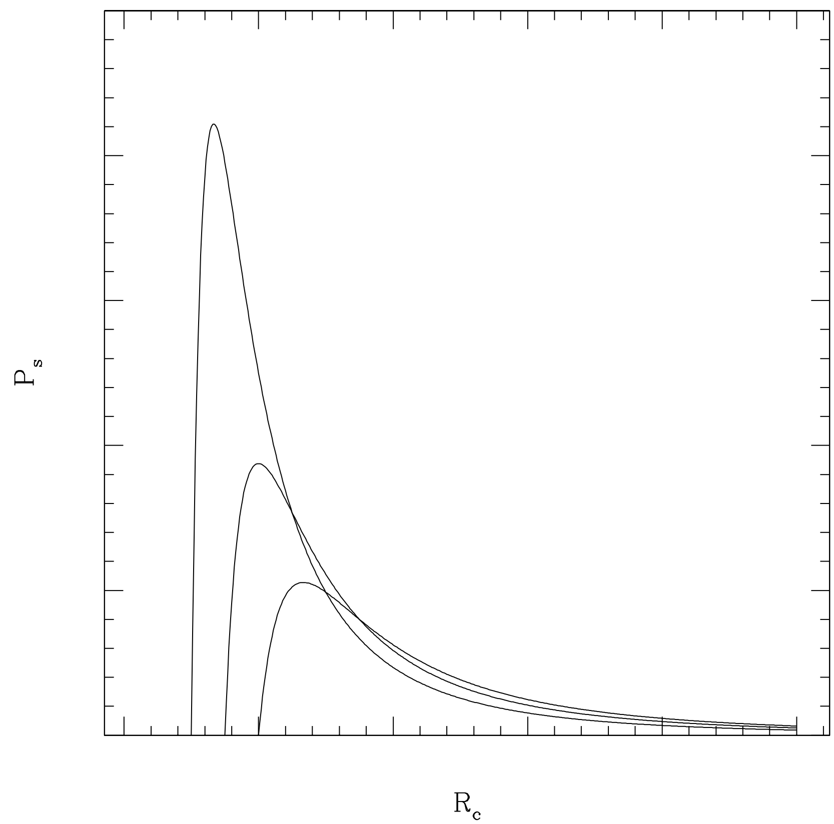
$$P_s = \frac{C_1 M_c T_c}{R_c^3} - \frac{C_2 M_c^2}{R_c^4}$$

where C_1, C_2 are positive constants.

If there is no energy generation within the core, expect that it will be isothermal with temperature $T_c = T_e$, where T_e is the temperature in the envelope immediately exterior to the core.

Assume that T_e is fixed by hydrogen burning requirement. Then T_c is a constant independent of R_c.

The function $P_s(R_c)$ has the form,



ie there is a maximum whose height *decreases* with increasing core mass M_c.

At constant core mass,

$$\frac{\partial P_s}{\partial R_c} = -3\frac{C_1 M_c T_c}{R_c^4} + 4\frac{C_2 M_c^2}{R_c^5}.$$

Setting this to zero to find the maximum, obtain,

$$R_c = \frac{4C_2 M_c}{3C_1 T_c}$$

which implies,

$$P_{s,max} \propto \frac{T_c^4}{M_c^2}.$$

The maximum surface pressure of the core decreases strongly with increasing mass M_c

Next consider conditions in the envelope. For simplicity, assume that all envelopes are homologous to each other. Then,

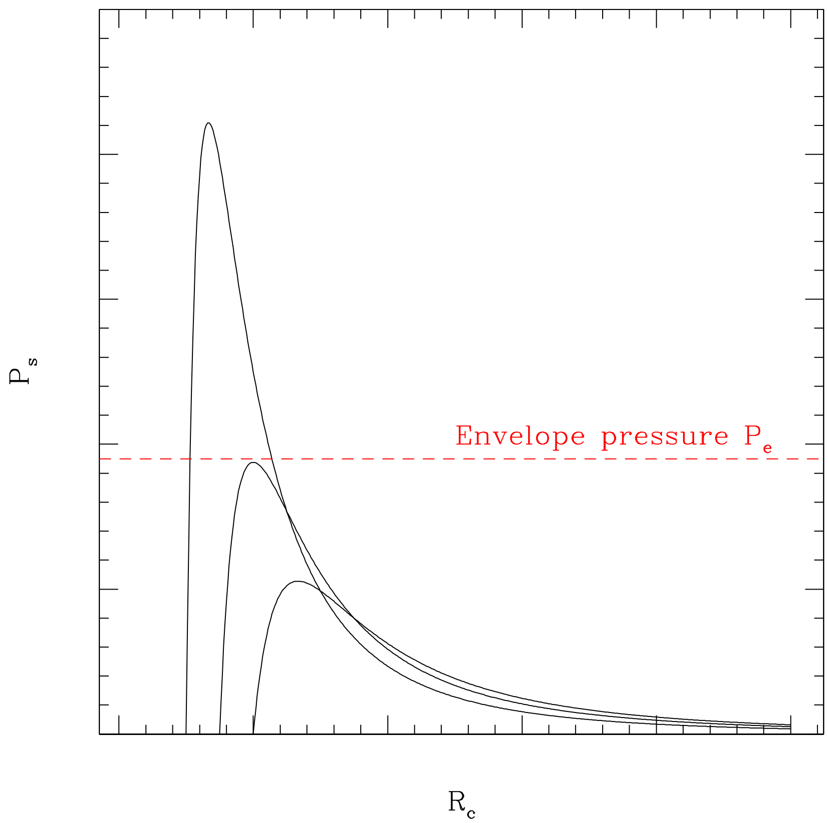
$$\begin{aligned} P_e &\propto \frac{M^2}{R^4} \\ T_e &\propto \frac{M}{R}. \end{aligned}$$

Making use of $T_e = T_c =$ constant, find,

$$P_e = C_3 \frac{T_c^4}{M^2}$$

ie P_e has the same scaling with T_c as $P_{s,max}$, but is independent of R_c.

In a plot of P against R_c, the envelope pressure is a straight horizontal line. Allowed solutions for the core + envelope structure are intersections of this line with the $P_s(R_c)$ curve,



Two regimes,

- For sufficiently small core masses,

$$q \equiv \frac{M_c}{M} \leq q_{SC},$$

  expect two solutions.

- For core masses above some critical mass, no solution.

(1) $q < q_{SC}$

Solution with the smaller core radius is thermally unstable, solution with the larger core radius is stable.

For the larger value of R_c, imagine a perturbation that slightly expands the core.

- Core surface pressure *decreases*.
- $P_e > P_s$.
- Excess pressure in envelope tends to compress the core $\rightarrow$ stability.

(2) $q > q_{SC}$

Once the core mass becomes too large, no solutions are possible. Signals some breakdown of the basic assumptions (e.g. equilibrium, ideal gas etc).

Critical core mass ratio q_{SC} is called the Schönberg-Chandrasekhar limit. Numerically,

$$q_{SC} \equiv \frac{M_c}{M} \simeq 0.37 \left(\frac{\mu_{env}}{\mu_{core}}\right)^2 ,$$

depending upon the ratio of the molecular weights in the envelope and the core. Taking $\mu_{core} = 4/3$ and $\mu_{env} = 0.62$, find,

$$q_{SC} \simeq 0.08$$

i.e. main sequence evolution comes to an end with only a fraction of the hydrogen exhausted.