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LECTURE (3)

PREGALACTIC STARS AND THE MICROWAVE BACKGROUND

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1. INTRODUCTION

Recent measurement of the microwave background spectrum (Woody & Richards 1979) suggest that the best fit black-body temperature in the submillimetre range is 3.0 K; this is significantly larger than the temperature measured in the Rayleigh-Jeans part of the spectrum (2.7 K). It is also claimed that there is a substantial deficit relative to the "best fit" black-body spectrum at wavelengths shortward of the peak (0.13 cm). Although this deficit remains to be confirmed, the discrepancy from the black-body prediction is estimated to be at the 5σ level. Such a deficit is not easy to explain within conventional cosmological models, in which all the background radiation goes back to the Big Bang, because the characteristic spectral distortions one would expect (e.g. from heating at early epochs) would exhibit an *excess* shortward of the peak due to Comptonization.

Rowan-Robinson *et al.* (1980) have suggested that both features of the Woody-Richards measurement (i.e. the general excess and the relative deficit) could be explained if the 3.0 K component is radiation generated by pregalactic stars and thermalized by amorphous silicate grains. The deficit is then explained by virtue of the fact that the absorption efficiency of such grains has a distinctive trough between 1 and 10 microns, which corresponds to a present wavelength of around 0.1 cm providing thermalization occurred at $z \sim 100$. On the other hand, the radiation cannot be thermalized at redshifts much larger than 100 since the heat from the stars would then evaporate the grains altogether. The metal abundance required in this scheme is low ($Z \sim 10^{-5}$) and could plausibly be generated by the same stars which produce the radiation. However, the energy requirements are severe: one needs the stars to have generated around 25% of the background radiation density, which in turn requires that they had a near-closure density. Nevertheless, if one believes the Woody-Richards effect, it may be difficult to envisage any other explanation. Thus, while many people have argued that one might *expect* pregalactic star formation, this could furnish the first direct evidence that it actually occurred.

The Rowan-Robinson *et al.* proposal has an interesting corollary: if pregalactic stars produce so much of the background radiation, it seems not implausible that they could produce *all* of it. This possibility is accentuated when one appreciates that one would expect pregalactic stars to span a range of masses and the stars which generate radiation at $z \sim 100$ may only correspond to a small part of that mass range. It is therefore natural to suggest that pregalactic stars in a higher mass range (which would complete their nuclear-burning phase more rapidly), or perhaps their black hole remnants, could generate an even larger contribution to the background radiation at redshifts exceeding 100. These more massive stars may perhaps produce the initial 75% component.

The possibility that pregalactic stars or black holes could have generated the entire background radiation has indeed already been suggested in the context of cold and tepid models for the early Universe (CARR 1977, CARR & REES 1977, REES 1978). In particular, REES has proposed that the background could all be starlight thermalized by the sorts of grains which Rowan-Robinson *et al.* invoke. However, the latter claim that Rees' model cannot work for two reasons. First, it requires a heavy element abundance larger than observed in extreme population II stars in order to thermalize the radiation effectively at millimetre wavelengths; and, second, grains fail to give sufficiently efficient thermalization at centimetre wavelengths. Even if it were possible to circumvent these objections by appealing to extra thermalizing agents (such as molecules), Rees' model could still not explain the 0.1 cm deficit. It would therefore seem more reasonable to suppose that the extra 75% of the background radiation was generated *before* the 25% component and that it was thermalized by a different mechanism. In this lecture I will consider the possibility that it was thermalized by free-free processes.

Another interesting feature of the Rowan-Robinson *et al.* proposal is that the redshift at which the stars responsible for the 25% component complete their nuclear-burning is roughly the same redshift ($z \sim 100$) at which the grains can first survive. Rather than regarding this as a coincidence, I suggest that the stars which generate the grain-thermalized radiation are merely the ones which are small enough to complete their nuclear burning phase after the grains can survive. This lends support to the suggestion that a significant amount of radiation may have been produced earlier by larger stars. The nuclear burning time is 10^7 y (corresponding to the age of the Universe when $z \sim 100$) for a star of mass $M_g \sim 20\, M_\odot$. If the mass above which stars or their remnants complete their radiation-production early enough for it to be thermalized by free-free processes is M_f, then the relative contributions to the microwave background from free-free-thermalized and grain-thermalized radiation would just depend on the relative number of stars larger than M_f and smaller than M_g. One needs the first contribution to be about three times as large as the second and in section II we examine whether this is possible. I will argue that it is, providing the value of M_f and the mass spectrum of the pregalactic stars satisfy certain restrictions.

In proposing that the 3K background is generated by pregalactic stars or their remnants, I am not necessarily assuming that there are *no* photons before the stars form. This would be unrealistic since many processes in the early Universe would inevitably produce *some* radiation (e.g. the dissipation of initial density fluctuations or primordial anisotropy). Furthermore a Universe which was "cold" at the neutron-proton freeze-out time (~ 1 s) would be unlikely to produce the observed 25% helium abundance through cosmological nucleosynthesis (although the helium might in principle be synthesized in pregalactic stars). A Universe with a photon-to-baryon ratio (S) larger than about 300, on the other hand, would still be radiation-dominated at 1 s and so would still produce roughly the amount of helium required. Therefore it is quite possible that the early Universe was "tepid", with a photon-to-baryon ratio much less than its present value ($\sim 10^8$). (The helium production is, in fact, weakly dependent on S (WAGONER 1973) and it would exceed 30% for $S < 10^5$.)

As discussed in my first lecture, a "tepid" scenario might find a natural setting in the context of the grand unified theories of strong, weak and electromagnetic interactions (GUTS). These theories permit the generation of a baryon asymmetry ($\Delta B/B$) in an initially symmetric Universe (YOSHIMURA 1978, DINOPOULOS & SUSSKIND 1978, ELLIS *et al.* 1979, TOUSSAINT *et al.* 1979, WEINBERG 1979) at the time when the temperature falls below the "unification" mass ($t \sim 10^{-35}$ s). When all the antibaryons annihilate at 10^{-5} s, the baryon excess will survive, leaving a photon-to-baryon ratio of order $(\Delta B/B)^{-1}$. The values of S predicted range from 10^4 to 10^{12} (NANOPOULOS & WEINBERG 1979), so although most GUT proponents are eager to predict the value 10^8 observed *today*, the picture could also produce a value of S much less than 10^8.

In my second lecture I argued that the tepid model is more attractive than the "standard" one from the point of view of galaxy formation. There were two reasons for this. First, it is well-known that *adiabatic* density fluctuations are erased by photon diffusion on scales less than $10^3 S^{5/4} M_\odot$. In the standard model this exceeds the typical mass of a galaxy and, since fluctuations *have* to be adiabatic in the GUTS scenario of baryon-production, this poses a problem. If $S \ll 10^8$, however, galaxy scale fluctuations survived and so the problem is averted. Second, in the standard model it is very difficult to explain how the density fluctuations originate in the first place (even if they *can* survive); they just have to be fed into the initial conditions of the Universe. On the other hand, the fluctuations required to produce galaxies (and indeed the stars which are supposed to generate the 3K background) can arise spontaneously in a tepid Universe through "statistical" effects (CARR & SILK 1980, HOGAN 1980). This possibility arises providing the Universe becomes "grainy" at some epoch as a result, for example, of undergoing a phase transition. Such effects cannot produce galaxies if $S \sim 10^8$ because the statistical fluctuations cannot then grow for a long enough period before decoupling, but they can if $S \ll 10^8$. This possibility was discussed in more detail in my second lecture. The tepid scenario may therefore explain the existence of structure in the Universe as well as the anomalous spectrum of the microwave background.

2. THE GENERATION OF THE MICROWAVE BACKGROUND

We first note that stars could not generate the microwave background through nuclear burning alone at redshifts exceeding ~ 100. If a fraction ε of the rest mass energy of stars is converted into radiation, then the earliest redshift at which the background can be generated is

$$z_{\min} \approx 70\left(\frac{\varepsilon}{0.007}\right)\left(\frac{T_0}{3\mathrm{K}}\right)^{-4} \Omega_* h^2$$

where Ω_* is the density of the stars in units of the critical density, T_0 is the background radiation temperature and h is the Hubble constant in units of 50 km/s/Mpc (at most 2). It seems unlikely that Ω_* could exceed 1 from measurements of the cosmological deceleration parameter and, even if all the mass in the stars was burnt from hydrogen into helium, ε could not exceed 0.007 (corresponding to a release of 7 MeV per baryon). Thus, while one could conceivably generate 25% of the background at $z \sim 100$, as suggested by ROWAN-ROBINSON *et al.*, one could not generate much more than that.

However, there is an alternative and possibly more efficient source of radiation. Large enough stars may produce black hole remnants — either at the completion of the nuclear burning phase or, if they are larger than about $10^5\, M_\odot$, through direct gravitational collapse. Black hole accretion might then generate radiation with up to 30% efficiency (THORNE 1974). In this case ε, interpreted as the ratio of the radiation energy produced to the final mass of the holes, could jump by at least an order of magnitude and so

$$z_{\min} \approx 3000\left(\frac{\varepsilon}{0.3}\right)\left(\frac{T_0}{3\mathrm{K}}\right)^{-4} \Omega_{\mathrm{B}} h^2 \tag{2}$$

where Ω_{B} is the final black hole density. Since Ω_{B} cannot exceed ~ 1 and ε cannot exceed 0.3, we conclude that even black holes could not generate all of the 3K background at redshifts exceeding 3000, which corresponds to a time 10^{12} s (just before decoupling in the standard model).

In order to determine the relative contributions of the stars and the holes, one needs to know something about the pregalactic stellar mass spectrum. Let us assume

$$\frac{dN}{dM} \propto M^{-\alpha} \tag{3}$$

so that the density of stars with mass around M, $\Omega(M)$, goes like $M^{2-\alpha}$ (or $\log M$ if $\alpha = 2$). Present epoch stars, at least in a certain mass range, are described by the Salpeter mass function with $\alpha = 2.35$ (SALPETER 1955). On the other hand, TRURAN and CAMERON (1971) infer from the paucity of low-metallicity stars that the first generation of (possibly pregalactic) stars must have had a considerably shallower spectrum. In any case, for the model I am proposing to be self-consistent, α cannot be appreciably smaller than 2. Otherwise $\Omega(M)$ would be an increasing function of M and therefore, if the 20 $M_\odot$ stars had a near-critical density (which is required to explain the grain-thermalized component of the background radiation), larger stars would need to have *more* than the critical density, which as indicated above is precluded. We also need to know the mass $M_{\min}$ at which the mass spectrum begins. This depends on the very uncertain details of how bound regions in the early Universe fragment. However, since cooling will be less efficient at pregalactic epochs (because of the lack of heavy elements) one might expect $M_{\min}$ to be larger for pregalactic stars than for the ones forming in the present epoch; SILK (1977) has argued that one might expect $M_{\min}$ to be about 20 $M_\odot$ for the first generation of stars. As it happens, this is about the mass M_g of the stars which complete their nuclear burning when the grains can first survive. However, the proposed scenario would work for any value of $M_{\min}$ less than M_g. The mass at which the spectrum ends, $M_{\max}$, depends in part upon fragmentation details and in part upon the nature of the density fluctuations which initially produce the bound regions. We discuss this later.

The present radiation density (in units of the critical density) produced by stars of mass M and their remnants may be expressed as

$$\Omega_R(M) = \Omega(M)[\varepsilon_*(M)z_*(M)^{-1} + \varepsilon_B(M)z_B(M)^{-1}] \tag{4}$$

where $z_*(M)$ is the redshift at which the stars burn most of their nuclear fuel and $z_B(M)$ is the redshift at which they collapse to black holes (if this occurs at all). $\varepsilon_*(M)$ is the fraction of the star's rest mass which is converted into radiation by nuclear reactions and $\varepsilon_B(M)$ is the radiation energy produced by black holes whose progenitor stars have mass M divided by Mc^2. (This is not exactly the same as the quantity ε in equation (2) since ε is defined in terms of the *final* black hole mass.) Each of these four functions is, in principle, determined by stellar evolution theory.

Let us first consider the stars' contribution to $\Omega_R(M)$. The main-sequence lifetime of stars which are sufficiently massive ($>10\,M_\odot$) that their opacity is dominated by electron-scattering is approximately given by

$$t_{MS} \sim \begin{cases} 10^{10}\left(\dfrac{M}{M_\odot}\right)^{-2} \text{ y} & (M < 10^2\,M_\odot) \\ 10^6 \text{ y} & (M > 10^2\,M_\odot) \end{cases} \tag{5}$$

Only stars smaller than about 20 $M_\odot$ complete their main-sequence phase after the epoch ($z \sim 100$) at which grains can first survive. Stars more massive than $10^2\,M_\odot$ are radiation-dominated (WEINBERG 1972) and their main-sequence lifetime is independent of M and comparable to the time of decoupling (a coincidence which we exploit later). Assuming the stars form on a timescale shorter than t_{MS}, and assuming they do not collapse to holes or explode prematurely, equation (5) implies that the redshift z_* at which stars of mass M burn most of their nuclear fuel is

$$z_{MS} \sim \begin{cases} \left(\frac{M}{M_\odot}\right)^{4/3} & (M < 10^2\, M_\odot) \\ 10^3 & (M > 10^2\, M_\odot) \end{cases} \tag{6}$$

The form of the function $\varepsilon_*(M)$ is less clear, but if one assumes that the fraction of the star's mass burnt to helium is just the fraction of the mass in the convective core, given approximately as $f_c(M) \approx 0.1\,(M/M_\odot)^{\frac{1}{2}}$ for $10^2\, M_\odot > M > 20\, M_\odot$ (IBEN 1967), one has

$$\varepsilon_*(M) = 0.007 f_c(M) \sim 10^{-3} \left(\frac{M}{M_\odot}\right)^{1/2}. \tag{7}$$

As M goes from $20\, M_\odot$ to $10^2\, M_\odot$, $f_c(M)$ increases from 0.5 to 1; for $M > 10^2\, M_\odot$, $f_c(M) \sim 1$ (as expected for radiation-dominated stars) and so $\varepsilon_*(M) \approx 0.007$.

Let us now consider the black holes' contribution to $\Omega_R(M)$. The function $\varepsilon_B(M)$ depends on the fraction of the star's original mass which undergoes gravitational collapse, $f_B(M)$, and the efficiency ε with which the resultant hole generates radiation. Although $f_B(M)$ is probably small for $M < 10^2\, M_\odot$, stars larger than $10^2\, M_\odot$, being radiation-dominated, are unstable and it is possible that they collapse entirely to black holes ($f_B \sim 1$) even without burning their nuclear fuel. However, present calculations indicate that in the range immediately above $10^2\, M_\odot$ stars merely undergo nuclear-energized pulsations (STOTHERS & SIMON 1970), shedding material which is progressively enriched in helium (APPENZELLER 1970), until they form an explosive oxygen core (TALBOT & ARNETT 1971). In this case the mass above which complete collapse can occur must be considerably larger than $10^2\, M_\odot$. On the other hand, sufficiently large stars almost certainly collapse directly to black holes because nuclear reactions are still too slow to stop their collapse when they fall within their relativistic instability radius (CHANDRASEKHAR 1964). If only the PP cycle operates, this happens for $M > 10^5\, M_\odot$. If the CNO cycle were important, the stars might still be able to bounce after falling within their instability radius (FOWLER 1966) for $M < 10^6\, M_\odot$; however the first pregalactic objects presumably contain no heavy elements. It therefore seems clear that there is some mass M_B between $10^2\, M_\odot$ and $10^5\, M_\odot$ above which the collapse of stars, were they to form, would be complete ($f_B \sim 1$) — at least after accretion of the envelopes.

We now discuss the form of the function $z_B(M)$. If the black holes form only after their progenitors have burnt their nuclear fuel, $z_B(M)$ is just $z_*(M)$. But if they are bigger than M_B and can collapse *before* they have burnt their fuel, one might assume that they could form on a Kelvin-Helmholtz timescale. The dominant opacity is due to Thomson scattering for a large star, in which case this timescale increases as the star collapses. Thus the relevant Kelvin-Helmholtz timescale is that which pertains at the relativistic instability radius and this can be shown to be

$$t_{coll}(M) \sim 10^{18} \left(\frac{M}{M_\odot}\right)^{-1} \text{s}. \tag{8}$$

If the temperature of the collapsing star gets high enough for electron-positron pairs to be created ($T > 10^9$ K) before it falls within the instability radius, the collapse timescale may actually be shorter than this. However, this only applies for stars smaller than $10^5\ M_\odot$ and such stars probably cannot collapse until they have burnt their nuclear fuel. In general the value of $z_B(M)$ which applies in equation (4) is that associated with the time

$$t_B(M) = \begin{cases} \max[t_{form}(M),\ t_{coll}(M)] & (M > 10^5\ M_\odot) \\ \max[t_{form}(M),\ t_{MS}(M)] & (M < 10^5\ M_\odot) \end{cases} \tag{9}$$

where $t_{form}(M)$ is the time at which stars of mass M form. This depends upon the nature of the density fluctuations which produce the stars.

These considerations suggest that a sensible approximation to equation (4) is

$$\Omega_R(M) \sim \begin{cases} \Omega(M)\,\varepsilon_*(M)\,z_{MS}(M)^{-1} & (M < M_B) \\ \Omega(M)\,\varepsilon_B(M)\,z_B(M)^{-1} & (M > M_B) \end{cases} \tag{10}$$

i.e. we assume that most of the radiation produced by stars smaller than M_B comes from their nuclear burning phase, with z_{MS} given by equation (6), and that most of that produced by stars larger than M_B comes from their black hole accretion phase. We will not attempt to specify M_B more precisely at this stage, except to point out that two possible values are obviously $10^2\ M_\odot$ and $10^5\ M_\odot$. Equations (6), (7) and (10) imply that $\Omega_R(M)$ is a decreasing function of M for $M < 10^2\ M_\odot$ providing the spectral index α exceeds 7/6 (which is expected); it is also a decreasing function of M for $10^2\ M_\odot < M < M_B$ providing $\alpha > 2$, and we have seen that this condition is probably necessary anyway for the self-consistency of the model. We infer that the largest stellar contribution to $\Omega_R(M)$ comes from the lowest mass stars. The radiation from all stars with z_{MS} less than 100 could be thermalized *à la* Rowan-Robinson *et al.*, but if there were also stars with z_{MS} much less than 100, the redshifted trough in the spectrum of the background radiation would be smeared out over a wide range of wavelengths. It is not clear, in fact, from the observations how far the trough *does* extend, but if M_{min} is comparable to the mass of the stars which have $z_{MS} \sim 100$ (as claimed by Silk), one would not expect it to extend to wavelengths much less than 0.1 cm.

3. THE FORMATION OF THE BLACK HOLES

In determining the black hole contribution to Ω_R, we must make some assumption about the form of the function $z_B(M)$. This is problematic since it depends upon the function $t_{form}(M)$ which appears in equation (9), and this in turn depends on the details of cosmological evolution before the stars form. Let us assume that the photon-to-baryon ratio before star formation is S ($\ll 10^8$). As argued in the Introduction, any value of S in the range above 10^4 could plausibly be generated by GUTS at the "unification" epoch and such a value would lead to roughly 25% cosmological helium production. The crucial feature of the tepid scenario (CARR & REES 1977) is that the Jeans mass, instead of rising like the horizon mass at early times, flattens off when the radiation density falls below the matter density at a time $t_{eq} \sim 10^{-5} S^2$ s. At this point it has a value

$$M_J^{max} \sim 10^{-1} S^2 M_\odot . \tag{11}$$

Thereafter the Jeans mass stays constant until "first" recombination occurs when T falls below ~ 4000 K at $t_{rec} \sim 10^8 S^{1/2}$ s. After this, the Jeans mass drops even further and it does not rise above M_J^{max} until the rest of the radiation is generated. These features are illustrated in figure (3) of my second lecture.

Let us assume that the early universe has small density fluctuations and that these have a dimensionless amplitude δ on the scale M_J^{max} when that scale first falls inside the particle horizon. One can show that, whether the fluctuations are adiabatic or isothermal, the first overdense regions to condense out have a mass M_J^{max} and do so at a time

$$t_{cond}(M_J^{max}) \sim 10^{-6} S^2 \delta^{-3/2} \text{ s} . \tag{12}$$

(If S is less than 10^4, fluctuations somewhat larger than M_J^{max} are erased through photon diffusion, so the first regions to bind are larger than M_J^{max} and consequently bind later than indicated by equation (12); however, in the context of the GUTS scenario, we can assume $S > 10^4$). Thereafter ever larger regions will bind and at a time $t_{cond}(M)$ which, for scale-independent δ, is just proportional to M. Bound regions larger than $10^5 M_\odot$ (for which nuclear energy release is unimportant) will collapse directly to black holes on the Kelvin-Helmholtz timescale given by equation (8) providing they do not fragment first. Fragmentation will occur only if pressure gradients are important in impeding the collapse. The relevant condition is that the dynamical timescale exceed the Kelvin-Helmholtz timescale at binding, and this applies only for objects larger than

$$M_{frag} \sim 10^{15} \delta^{5/3} M_\odot . \tag{13}$$

If this exceeds M_J^{max} (i.e. if $S < 10^8 \delta^{5/6}$), the first bound regions collapse directly on the timescale $\max[t_{cond}(M), t_{coll}(M)]$. Otherwise black holes can only derive from the fragments of bound objects larger than M_{frag} and, in this case, they form on a timescale $\max[t_{cond}(M), t_{MS}(M')]$ where M is the mass of the bound object, M' is the mass of the fragment, and $t_{MS}(M')$ is specified by equation (5). These considerations imply that the *first* holes form at a time

$$t_B \sim \begin{cases} 10^{-6} S^2 \delta^{-3/2} \text{ s} & (10^8 \delta^{5/6} > S > 10^6 \delta^{3/8} \text{ or } S > 10^9 \delta^{3/4}) & (14.1) \\ 10^7 \delta^{-3/4} \text{ s} & (S < 10^6 \delta^{3/8}) & (14.2) \\ 10^{13} \text{ s} & (10^9 \delta^{3/4} > S > 10^8 \delta^{5/6}) . & (14.3) \end{cases}$$

This equation specific three regimes in the (S, δ) plane, as illustrated in figure (1). In regime (1) the first objects collapse at the time given by equation (12). (For $S < 10^8 \delta^{5/6}$, the first objects collapse directly without fragmenting because

$M_J^{max} < M_{frag}$; for $S > 10^9 \delta^{3/4}$, the first objects fragment but the fragments still collapse on the timescale (12) because it exceeds t_{MS}.) In regime (2) the first objects to collapse are the ones for which $t_{coll}(M) = t_{cond}(M)$ and these will be larger than M_J^{max}; the boundary $S = 10^6 \delta^{3/8}$ corresponds to the line $t_{coll}(M_J^{max}) = t_{cond}(M_J^{max})$. In regime (3) the first objects to collapse are the first stellar fragments to complete their nuclear burning; we assume that some of the fragments are larger than $10^2\ M_\odot$. Contours of constant t_B are shown in the figure. We note that there is a wide band of values for S and δ (shown shaded) in which the first holes form at 10^{13} s.

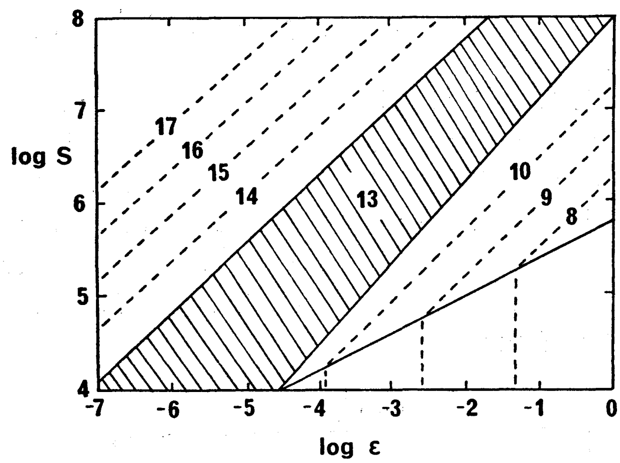

Fig. 1. This shows how the time (in seconds) at which the first holes form in a tepid universe depends on the pre-enhanced photon-to-baryon ratio (S) and the amplitude of the density fluctuations on a scale M_J^{max} when that scale first enters the horizon (δ). Throughout the shaded region the first holes form (via fragmentation into stars) at 10^{13} s (the nuclear-burning timescale of a massive star), which is just what is required to boost S to a value of 10^8 through black hole accretion. To the right of the shaded region the first holes form through direct collapse at the time indicated by the dotted lines. S would also be boosted here, although its final value would be more arbitrary and probably less than 10^8. (Above the line $S \sim 10^6\ \varepsilon^{3/8}$ the first holes form when objects of mass M_J^{max} bind; below this line the first holes to form are larger than M_J^{max} and they do so on a Kelvin-Helmholtz timescale.) To the left of the shaded region, bound regions also fragment into stars but the first regions bind on a timescale longer than t_{MS} and too late for any radiation generated to be thermalized

The holes, once formed, could accrete very rapidly because of the large background density and thereby generate radiation (Carr 1979). This radiation could be thermalized by free-free processes at wavelengths exceeding (Rees 1978)

$$\lambda_0 \sim 200(1+z)^{-\frac{1}{4}} \Omega_g^{-3/4} h^{-3/2} \left(\frac{n}{n_e}\right) \left(\frac{\langle n_e \rangle^2}{\langle n_e^2 \rangle}\right)^{\frac{1}{4}} \left(\frac{T_e}{3(1+z)}\right)^{3/4} \text{cm} \tag{15}$$

where n_e and T_e are the electron number density and temperature, and Ω_g is the density of the gas which has not gone into stars. Thus even if ionization is complete, one can thermalize down to wavelengths ~ 0.1 cm (as required) only at redshifts

$$z > 4 \times 10^4 (\Omega_g^{\frac{1}{2}} h)^{-3} \left(\frac{\langle n_e \rangle^2}{\langle n_e^2 \rangle}\right). \tag{16}$$

This puts an upper limit on the time at which black holes can produce the 75% component of the thermalized background. With a sufficiently large "clumpiness" factor, $\langle n_e^2\rangle/\langle n_e\rangle^2 \sim 40$, this could in principle be as late as decoupling $\sim 10^{13}$ s, the time until which the Universe is ionized in the conventional picture. (Such clumpiness is very plausible since most of the Universe will be in bound clumps at decoupling with density contrast $(t_{dec}/t_{cond}(M_J^{max}))^2$.) On the other hand, we have seen that black hole accretion cannot generate the 75% component before the redshift z_{min} specified by equation (2) and this corresponds to a time at least as late as 10^{12} s. For this model to work, therefore, the time at which the bulk of the radiation is generated can be specified very precisely (10^{12} s $< t_R < 10^{13}$ s). One also needs Ω_B to exceed ~ 0.1.

What is remarkable is that, for a wide range of values for S and δ, the first holes do indeed form at the time required. The scenario is plausible therefore providing the holes can produce radiation on the timescale with which they form. One infers immediately that the holes have to produce radiation at a rate exceeding the Eddington limit (since the Eddington timescale is $\sim 10^{16}$ s). One can envisage various ways in which this could happen. For example, the holes might accrete by capturing and tidally disrupting stars (HILLS 1975).

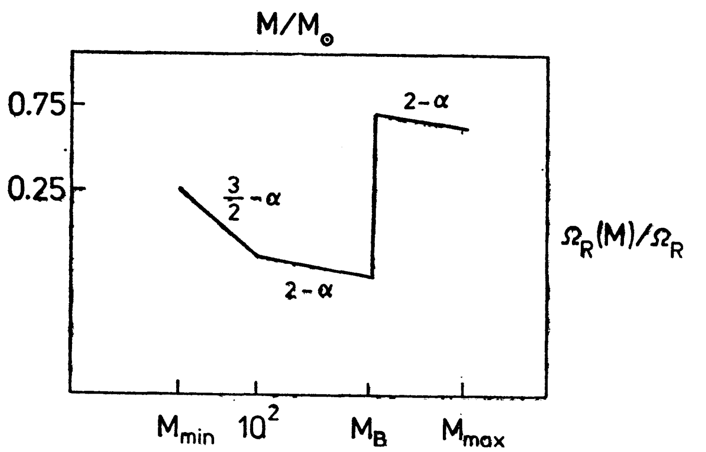

Fig. 2. This shows the contribution of pregalactic stars to the background radiation density. $\Omega_R(M)$ is the radiation density generated by stars with mass around M in units of the critical density, the total density being $\Omega_R \sim 10^{-4}$. Most of the radiation from stars smaller than M_B derives from their nuclear burning phase. Stars larger than M_B collapse to black holes and most of their radiation is generated by accretion. M_B probably lies between $10^3\, M_\odot$ and $10^5\, M_\odot$. The slope of the lines depends on the mass spectrum of the stars ($dN/dM \propto M^{-\alpha}$) and one expects $\alpha > 2$. The important feature of this diagram is that $\Omega_R(M)$ is a decreasing function of M except for a discontinuous jump at M_B due to the greater efficiency with which accretion generates radiation. Thus most of the grain-thermalized radiation comes from stars of mass M_{min} and most of the free-free-thermalized radiation comes from holes of mass M_B. One requires these contributions to be in the ratio 1 : 3 and this fixes a relationship between α and M_B.

Even though black holes might in principle generate the initial 75% of the 3 K background, the ratio of the free-free-thermalized and grain-thermalized contributions will be as required only if the function $\Omega(M)$ in equation (10) satisfies certain conditions. If one is in the shaded region of figure (1) or to the left of it, the mass spectrum of the holes will be the same as that of the stars smaller than M_B (assuming $f_B \sim 1$) and so should be described by the same value of α. If one is to the right of the shaded region, the spectrum of the first holes could be different, since they form directly from the density fluctuations and not via fragmentation, and CARR (1977) has argued that it should have $\alpha = 1$. However, the most straightforward situation arises if M_J^{max} exceeds M_{frag} because in this case *no* holes form except via the fragmentation mode. This is anyway the most plausible assumption since the holes tend to form too early in the domain to the right of the shaded region.

Assuming the holes do only form via fragmentation, equation (10) with $z_B(M) \sim 10^3$ and $\alpha \geqslant 2$ implies that holes of mass M_B make the largest contribution to Ω_R, larger holes making a contribution which falls off as $M^{2-\alpha}$. The form of the function $\Omega_R(M)$, allowing for both stellar and black hole contributions, is therefore as shown in figure (2). Since the growth of fluctuations will be temporarily suppressed when the radiation is generated at t_R, a natural value for M_{max} is the mass of the regions which are just binding at t_R. The important feature of figure (2) is that $\Omega_R(M)$ is a decreasing value of M except for a discontinuous jump at the value $M = M_B$. The ratio of the black hole and stellar contributions is

$$\frac{\Omega_R^B}{\Omega_R^*} \sim \left(\frac{M_{min}}{100\, M_\odot}\right)^{\alpha - 7/6} \left(\frac{M_B}{100\, M_\odot}\right)^{2-\alpha} \left(\frac{\varepsilon_B(M_B)}{\varepsilon_*(M_{min})}\right)$$

$$\sim 5^{3.3-\alpha} \left(\frac{M_{min}}{20\, M_\odot}\right)^{\alpha - 7/6} \left(\frac{M_B}{100\, M_\odot}\right)^{2-\alpha} \left(\frac{\varepsilon}{0.1}\right). \quad (17)$$

Therefore, if we normalize the curve so that Ω_R^* is $0.25\,\Omega_R$ (as required in the Rowan-Robinson *et al.* model), the value Ω_R^B is uniquely determined in terms of the parameters α, ε, M_{min} and M_B. Evidently the ratio given by equation (17) can be about 3 (as required) only for certain combinations of these parameters. One possible set of values, for example, would be $M_{min} \approx 20\, M_\odot$, $\varepsilon \approx 0.1$, $\alpha \approx 2.6$ and $M_B \approx 10^2\, M_\odot$ (corresponding to the assumption that *all* radiation-dominated stars undergo complete collapse). Another set of possible values would be $M_{min} \approx 20\, M_\odot$, $\varepsilon \approx 0.1$, $\alpha \approx 2.1$ and $M_B \approx 10^5\, M_\odot$. Lacking any model which predicts the values of α (the most uncertain parameter), one can at least infer that — for the present scenario to work $-\alpha$ must lie between 2 and 3.

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