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LECTURE (2)

THE ORIGIN OF GALAXIES

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1. INTRODUCTION

The existence of galaxies implies that the early Universe must have contained initial density fluctuations. Overdense regions would then expand more slowly than the background and eventually — providing the fluctuations were not damped out first — they would stop expanding altogether and collapse to form bound objects. To understand how galaxies form we therefore need to know: (i) how the initial density fluctuations arise, (ii) under what circumstances they evolve into bound objects, and (iii) how the bound objects develop the observed characteristics of galaxies. The third question depends on details of gas dynamics and fragmentation [1—3] which are largely incidental to cosmology, so I will not discuss it here. My discussion of the second question will be largely review; my approach to the first question will be orientated to showing that the density fluctuations required to explain galaxies could arise spontaneously through statistical effects.

2. THE EVOLUTION OF DENSITY FLUCTUATIONS

Let us first ask what sort of fluctuations are required to produce galaxies. Observations indicate that the matter distribution in the Universe is clumpy, not just on the scale of galaxies, but on all scales up to 100 Mpc. It is therefore important to view galactic fluctuations in the context of a general spectrum of fluctuations which extend to much larger scales. A clue to the origin of the fluctuations may presumably be contained in the form of this spectrum. The form can be inferred from detailed analysis of the galaxy correlation function [4, 5]

$$\xi(r) \equiv \left\langle \frac{\delta\varrho}{\varrho}(\vec{x}) \frac{\delta\varrho}{\varrho}(\vec{x}+\vec{r}) \right\rangle \tag{1}$$

which is observed at present to be

$$\xi_0(r) \approx \left(\frac{r}{10\, h^{-1}\ \mathrm{Mpc}} \right)^{-1.8} \quad (100\ \mathrm{kpc} < r < 10\ \mathrm{Mpc}) \tag{2}$$

where h is the Hubble constant in units of $50\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$. The value of r for which $\xi_0 = 1$ corresponds to the scale 10 Mpc on which galaxies form bound clusters.

Because the effect of the 3K background on the matter is unimportant after decoupling (unless the Universe is reionized [6]), the matter behaves like a pressureless gas then. In these circumstances simple Newtonian arguments show that matter fluctuations just increase like z^{-1} until they grow to unity or until the "free expansion" epoch ($z_f \sim \Omega^{-1}$). One can infer that the density fluctuations which were present at decoupling must have had the form [4, 5]

$$\left(\frac{\delta\varrho}{\varrho} \right)_{\mathrm{dec}} \sim \left(\frac{M}{M_1} \right)^{-\alpha} \tag{3}$$

where α lies between 1/3 and 1/2, depending on the value of Ω. If $\Omega \sim 0.1$ the lower value pertains; but if $\Omega \sim 1$ one has $\alpha = \frac{1}{2}$ (corresponding to a "white noise" spectrum) and Peebles argues [4] that this fits the data best. The value of M_1 in equation (3) (i.e. the scale on which fluctuations at decoupling would be of order unity) also depends on Ω. For $\Omega \sim 0.1$, $M_1 \sim 10^8\, M_\odot$; for $\Omega \sim 1$, $M_1 \sim 10^6\, M_\odot$. While it must be stressed that there is no *direct* evidence that the decoupling fluctuations extended down to the scale M_1, because equation (3) is only inferred from observations on scales larger than galaxies, there is no obvious reason why they should be cut off below a galactic mass. It is therefore quite possible that some regions bound well before galaxies and perhaps at decoupling itself. (If the Universe was initially cold, regions may have bound even *before* decoupling [7].) Note that galactic scales must have $(\delta\varrho/\varrho)_{\mathrm{dec}} \sim 10^{-3}$.

If we try to extrapolate the fluctuations back to still earlier times, we immediately face a problem because $(\delta\varrho/\varrho)$ no longer simply grows like z^{-1} before decoupling. There are two reasons for this. Firstly, the rate of growth of fluctuations depends on the equation of state of the Universe. If the equation of state is $p = \gamma\varrho$, then fluactuations grow like

$$\left(\frac{\delta\varrho}{\varrho} \right) \propto z^{-(1+3\gamma)} \propto t^{2(1+3\gamma)/3(1+\gamma)} \tag{4}$$

(using a suitable gauge). After decoupling, one just has dust so $\gamma = 0$ and $\delta \propto t^{2/3}$. But between 10^{-4} s (the end of the hadron era) and $t_{\mathrm{eq}} \sim 10^{10}\,\Omega^{-2}$ s (i.e. almost all the way up to decoupling) the density of the Universe is dominated by its radiation content in the standard "hot" model, so $\gamma = 1/3$ and $\delta \propto t$. Before 10^{-4} s the equation of state depends on details of the strong interaction. In the most natural situation (asymptotically free quarks) one would expect $\gamma = 1/3$ here also. However, it is possible that a strong nuclear repulsion produces a "stiff" equation of state during the hadron era in which case $\gamma = 1$ and $\delta \propto t^{4/3}$. It is also possible that the equation of state could go soft (as in Hagedorn's superbaryon picture [8]) and in this case $\gamma = 0$, so δ grows like $t^{2/3}$ as in the post-decoupling epoch. Thus the extent to which fluctuations can grow before decoupling is very dependent on details of particle physics.

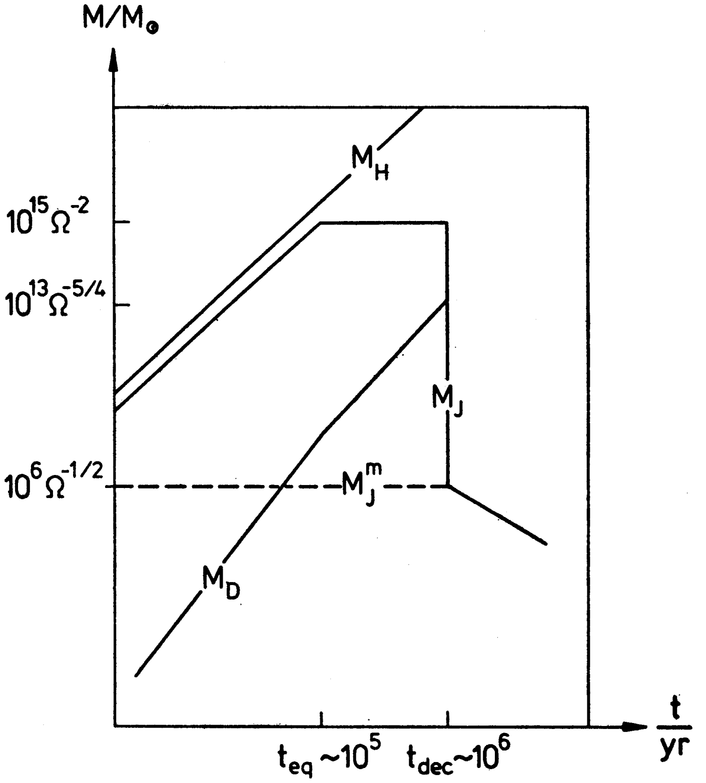

Fig. 1. This shows how various mass-scales evolve in the hot big bang model for the early Universe. These scales play a crucial role in determining the evolution of density fluctuations. M_H is the horizon mass (including the mass of the radiation content; this will dominate that of the matter content before t_{eq}). Until a fluctuation falls within the horizon it cannot be affected by any causal processes like viscous dissipation and photon drag. M_J is the total Jeans mass (including the mass of the radiation content). This specifies the scale below which pressure effects are important. The radiation pressure always exceeds the matter pressure until decoupling; this means that M_J is of order M_H until t_{eq} and that it flattens off between t_{eq} and t_{dec} ($t_{eq} < t_{dec}$ if $\Omega > 0.1$). When an adiabatic fluctuation falls within M_J, it will be converted into an acoustical wave. M_J^m is the matter Jeans mass; this specifies the scale below which matter pressure is important and it plays a similar role for isothermal fluctuations as M_J does for adiabatic ones. M_J^m is constant before t_{dec}; after t_{dec}, when the radiation decouples from the matter and no longer impedes the growth of matter fluctuations, the total Jeans mass drops to the value M_J^m. M_D is the mass below which adiabatic fluctuations are erased through photon diffusion. The value of M_D at decoupling, the "Silk" mass (M_S), specifies the scale above which adiabatic fluctuations can survive the radiation era

The second reason that the evolution of density fluctuations is more complicated before decoupling is associated with various effects of the 3K background. The radiation will tend to impede (and perhaps reverse) the growth of fluctuations on sufficiently small scales. It will do this in two ways, depending on the nature of the density fluctuations.

(i) If the fluctuations are *adiabatic* (in the sense that they maintain a constant photon-to-baryon ratio and are therefore fluctuations both in the matter and radiation densities), they will grow according to equation (4) only until they fall inside the Jeans length (which is of order the horizon size; see figure (1)). Thereafter they will turn into acoustical waves because of radiation pressure and the amplitude of these waves cannot grow again until the radiation becomes decoupled at 10^{13} s. Furthermore, below a critical scale, the amplitude of the waves will be reduced because of photon diffusion [9]. The critical scale is just the distance over which photons can random walk in a cosmological expansion time. The associated mass, M_D, is the geometric mean of the horizon mass, M_H, and the mass of optical depth unity, $M_{\tau=1}$:

$$M_D \sim \sqrt{M_H(t) \cdot M_{\tau=1}(t)}\,. \tag{5}$$

Since the dominant opacity before decoupling is electron-scattering, this gives

$$M_D \sim 10^{-18} \left(\frac{t}{s}\right)^{5/2} M_\odot \tag{6}$$

and by decoupling this has grown to $M_S \sim 10^{13} \Omega^{-5/4} M_\odot$ (the Silk mass). Any adiabatic fluctuations on scales smaller than this will be exponentially damped [9]:

$$\left(\frac{\delta\varrho}{\varrho}\right) \rightarrow \left(\frac{\delta\varrho}{\varrho}\right) \exp\left[-\left(\frac{M}{M_S}\right)^{2/3}\right]. \tag{7}$$

Another complication is that adiabatic fluctuations bigger than M_S but smaller than the pre-decoupling Jeans mass ($\sim 10^{15} \Omega^{-2} M_\odot$) may be boosted at decoupling by kinematic effects [10]. However, it seems likely that this boosting is just an artifact of assuming that decoupling occurs instantaneously [11]. In this case the fluctuations at decoupling would have the same amplitude for $M_S < M < M_J^{max}$ as they had on entering the horizon.

(ii) If the fluctuations are *isothermal* (in the sense that they affect only the matter density and not the radiation density), they will also stop growing once they have fallen within the horizon because of photon drag. However they will only be erased if they fall within the matter Jeans length. Otherwise they will merely be "frozen" with constant amplitude until decoupling. Since the matter Jeans mass has the constant value

$$M_J^m \sim 10^6 \Omega^{-\frac{1}{2}} M_\odot \tag{8}$$

before decoupling (after decoupling it falls; see figure (1), all isothermal fluctuations on scales larger than this will survive and begin the matter era with the amplitude they had when they first fell within the horizon. Note, however, that for isothermal fluctuations one must distinguish between the fluctuation in the total density, ϱ_T, and the fluctuation in the matter density, ϱ_M. Since

$$\left(\frac{\delta\varrho}{\varrho}\right)_T = \left(\frac{\delta\varrho}{\varrho}\right)_M \left(\frac{\varrho_M}{\varrho_T}\right) \approx \left(\frac{\delta\varrho}{\varrho}\right)_M \left(\frac{t}{t_{eq}}\right)^{\frac{1}{2}}, \tag{9}$$

the fluctuations do not have the same form. In particular, since a matter mass M falls within the horizon at a time $t_H \propto M^{2/3}$ during the radiation era, the total horizon fluctuation is related to the matter fluctuation at decoupling by the equation [12]

$$\left(\frac{\delta\varrho}{\varrho}\right)_{T,H} \propto \left(\frac{\delta\varrho}{\varrho}\right)_{M,dec} M^{\frac{1}{3}} \propto M^{\frac{1}{3}-\alpha} \tag{10}$$

where α is defined by equation (3).

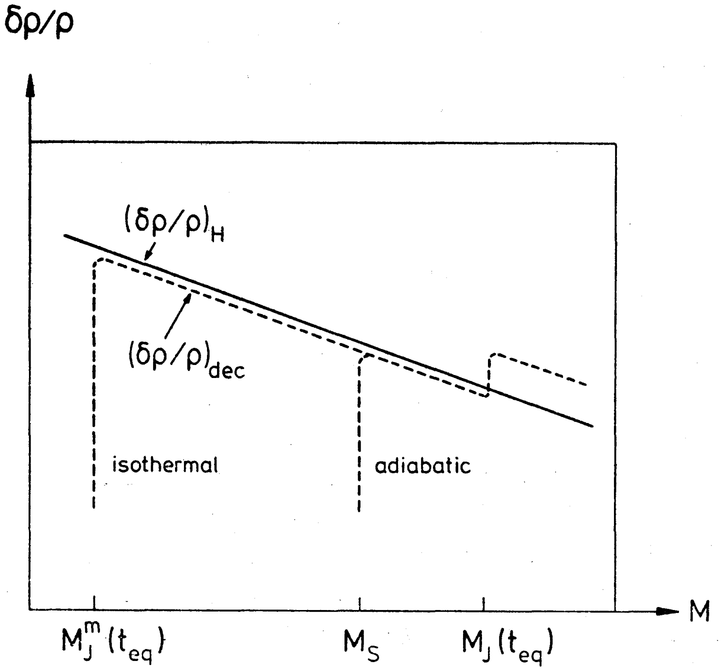

Fig. 2. This shows the relation between the density fluctuation when a mass-scale M falls within the horizon $(\delta\varrho/\varrho)_H$ (solid line) and the matter fluctuation it produces after decoupling $(\delta\varrho/\varrho)_{dec}$ (broken lines). Fluctuations in the radiation density (adiabatic fluctuations) are erased on scales below M_S by photon diffusion. Fluctuations in the matter density (isothermal fluctuations) survive on scales larger than the matter Jeans mass at decoupling. In both cases, for $M < M_H(t_{eq})$, the surviving scales have $(\delta\varrho/\varrho)_H \sim (\delta\varrho/\varrho)_{dec}$ providing one interprets $(\delta\varrho/\varrho)_H$ in the isothermal case as the fluctuation in the matter density alone. This is because the fluctuations stop growing between falling within the horizon and t_{eq} either because of radiation pressure in the adiabatic case or because of radiation drag in the isothermal case. For $M > M_H(t_{eq})$, both sorts of fluctuations will be boosted by a factor $(t_{dec}/t_{eq})^{2/3}$, providing $t_{eq} < t_{dec}$, on account of the flattening of the Jeans between t_{eq} and t_{dec}

The evolution of M_H, M_J, M_J^m and M_D is shown in figure (1) and the relation between the horizon-scale and decoupling fluctuations is illustrated in figure (2). In the conventional "hot" scenario, the fact that M_S is larger than a typical galaxy mass (and more comparable to the mass of a cluster of galaxies $\sim 10^{13}\, M_\odot$) suggests that, with adiabatic fluctuations, galaxies could only form by fragmentation: that is, the $10^{13}\, M_\odot$ regions have to bind first and then fragment into galaxies [13]. On the other hand, with isothermal fluctuations, the smallest surviving fluctuations would bind first and then ever larger units would form by hierarchical clustering. Exactly what singles out a galaxy as the smallest scale on which structure persists in this model is not clear — although various gas dynamical arguments are fairly plausible [3].

3. THE ORIGIN OF THE DENSITY FLUCTUATIONS

We have extrapolated the fluctuations required to make galaxies back until the time at which they first fall within the horizon ($t_H \sim 10^8$ s for $M \sim 10^{12}\, M_\odot$). Before this time the fluctuations are larger than the horizon and are therefore not subject to any of the causal dissipative effects described in the last section. They merely grow due to gravitational instability in the manner described by equation (4). The question now, therefore, is where do the horizon-scale fluctuations come from? Ideally one would like to be able to show that the fluctuations could arise spontaneously at some time and then grow to have the value required at t_H. Unfortunately, it has proved very difficult to explain how the required fluctuations could arise in this way and most cosmologists have therefore concluded that the fluctuations just have to be fed into the initial conditions of the Universe (i.e. at the Planck time). This is hardly a satisfactory attitude since it means that, ultimately, one has more or less given up hope of explaining the origin of galaxies.
However, the situation is not completely hopeless. One possibility is that the fluctuations were induced by quantum gravity effects at the Planck time [14]. Harrison has suggested that these will naturally endow the Universe with initial fluctuations of the form [15]

$$\left(\frac{\delta\varrho}{\varrho}\right)_\odot = \varepsilon_H \left(\frac{M}{M_0}\right)^{-\frac{2}{3}} \tag{11}$$

where ε_H is a constant ($0<\varepsilon_H<1$) and M_0 is the horizon mass at the Planck time $\sim 10^{-5}$ g. These fluctuations have the attractive feature that they have always grown to a scale invariant amplitude ε_H on first entering the horizon (independent of the equation of state). Various people [12, 16] have argued that such "constant curvature" fluctuations are just what is needed to produce galaxies and clusters of galaxies. In particular, ZEL'DOVICH has argued [16] that, if $\varepsilon_H \sim 10^{-4}$, one can produce both the photon to baryon ratio (via the dissipation of small scale fluctuations discussed in may first lecture) and the observed large scale structure of the Universe. On the other hand, the argument that quantum gravity effects produce density fluctuations of the form [11] is far from convincing, so one should also seek other natural ways of producing density fluctuations.

It has recently been suggested [17, 18] that the required fluctuations could arise in an initially homogeneous Universe through purely statistical effects if the Universe was ever "grainy". Of course, at some level, the Universe is bound to develop graininess since we know that (at sufficiently late epochs) the matter will be in discrete particles like protons. However, it is well known that grains of only 10^{-24} gm are far too small to produce galaxies through statistical effects. On the other hand, graininess might develop on much larger scales whenever the Universe undergoes a phase transition. Such a phase transition might occur, for example, at the GUT unification era [19], at the nuclear density epoch [20], when the superbaryons decay in Hagedorn's model [8], at the atomic density epoch in a cold Universe [21, 22], or when the quarks belatedly fuse into nucleons in Lasher's model [23]. In all these situations one expects grains of a specific mass M_* to form at a specific time t_*. In estimating the size of such statistical fluctuations, one must bear in mind that one is interested in fluctuations on scales initially much larger than the horizon (on scales containing N grains, say). It is quite unrealistic to appeal to the usual $\sqrt{N}$ effect on such scales since there has not been time for the grains to randomize their positions; and anyway one can show that such fluctuations are not really meaningful since they would close the Universe on a sufficiently large scale. If one assumes that grains can only randomize their positions on scales less than the horizon size, one would naively infer initial "surface" fluctuations of the form

$$\left(\frac{\delta\varrho}{\varrho}\right)_* \propto M^{-\frac{1}{3}}. \tag{12}$$

However, one can show that this is wrong (because equation (12) gives the fluctuation in the number of grains but not in the total energy) and that the "effective" density fluctuation is really [24, 25]

$$\left(\frac{\delta\varrho}{\varrho}\right)_* \sim N^{-7/6} \sim \left(\frac{M}{M_*}\right)^{-7/6}. \tag{13}$$

Fluctuations of this form are inevitable and although they are much smaller in amplitude than those described by equation (12), they can still be significant. For example, if the fluctuations on a matter scale M originate and fall within the horizon during the radiation era (when $p = \varrho/3$), one finds [17]

$$\left(\frac{\delta\varrho}{\varrho}\right)_{\rm dec} \sim 10^{-8}\left(\frac{M}{10^{12}\,M_\odot}\right)^{-1/2}\left(\frac{t_*}{10^{-4}\,{\rm s}}\right)^{3/4}\left(\frac{M_*}{M_{\rm H}(t_*)}\right)^{7/6}. \tag{14}$$

Note that, although the fluctuations go like $M^{-7/6}$ at t_*, they turn into $M^{-\frac{1}{2}}$ fluctuations at decoupling because of their growth before falling within the horizon at $t_{\rm H} \propto M^{2/3}$. It is interesting that such a "white noise" spectrum of fluctuations is just what Peebles argues is required to explain the galaxy correlation function[4]. Unfortunately, in the standard "hot" model, the statistical fluctuations described by equation (14) are not large enough to produce galaxies and clusters. For since causality requires the grains to be smaller than the horizon at t_*, and since galactic scales need $(\delta\varrho/\varrho)_{dec}$ to be about 10^{-3}, we require $t_* > 10^3$ s and it is hard to see how a phase transition could occur as late as this in a hot Universe [18]. However, if the photon-to-baryon ratio had an initial value S much less than its present value (i.e. if the early Universe was "tepid" or "cold" — a possibility discussed in my

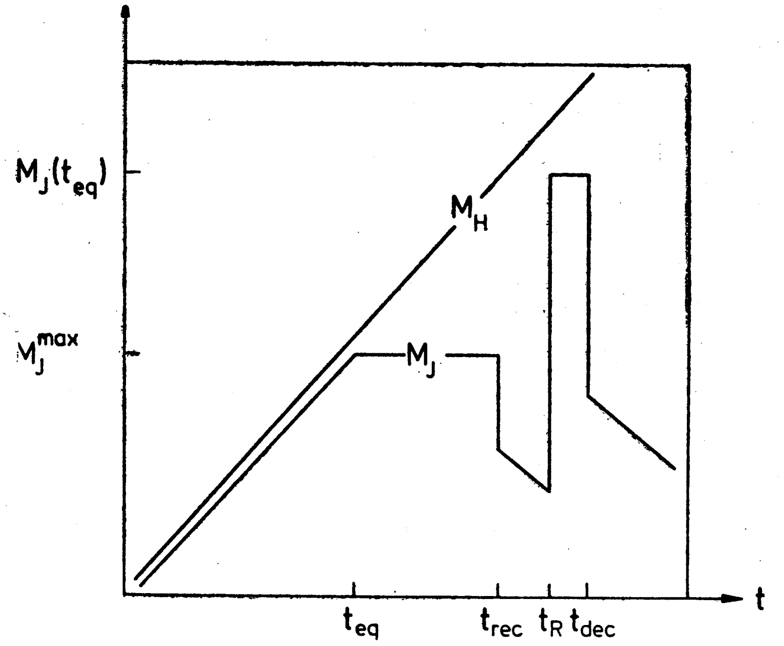

Fig. 3. This illustrates various evolutionary features of a Universe which starts off with a photon-to-baryon ratio S much less than 10^8, the extra photons being generated at some time t_R. The Jeans mass M_J is of order the horizon mass M_H until the matter and radiation densities become equal at $t_{eq} \sim 10^{-5}\, S^2$ s. Thereafter it remains constant at $10^{-1}\, S^2\, M_\odot$ until recombination at $t_{rec} \sim 10^8\, S^{\frac{1}{2}}$ s when it drops to $10^{-1}\, S^{\frac{1}{2}}\, M_\odot$. After t_R, M_J evolves as in the standard model. Because t_R cannot much precede decoupling ($t_{dec} \sim 10^{13}$ s) in the suggested scenario, the period in which M_J is flat is generally very long and this means that bound systems can form very easily then

next lecture), the statistical fluctuations could continue to grow even *after* falling within the horizon because the Jeans length would be very small (see figure (3)). In this situation $(\delta\varrho/\varrho)_{dec}$ could be much larger than indicated by equation (14). If the photon-to-baryon ratio is boosted to its present value at a time t_R one finds [17]

$$\left(\frac{\delta\varrho}{\varrho}\right)_{dec} \sim 10^{-3} \left(\frac{M}{10^{12}\, M_\odot}\right)^{-7/6} \left(\frac{t_*}{10^{-4}\ \mathrm{s}}\right)^{3/4} \left(\frac{M_*}{M_H(t_*)}\right)^{7/6} \left(\frac{S}{10^4}\right)^{-1/2} \left(\frac{t_R}{10^{13}\ \mathrm{s}}\right)^{2/3}. \quad (15)$$

S could probably only be boosted at *very* late times by black hole accretion and, in this context, we saw in my first lecture that t_R has to be around 10^{13} s in order to satisfy energetic and thermalization criteria. One also requires S to exceed 10^4 in order to avoid over-producing helium through cosmological nucleosynthesis [26].

One infers that galactic scale fluctuations could be as large as 10^{-3} providing $t_* \geqslant 10^{-4}$ s. In fact, in a tepid Universe, 10^{-4} s (the end of the hadron era) is really the last time one could expect a phase transition to occur. If such a transition *does* occur then, and if it produces horizon-size ($\sim 1\ M_\odot$) grains, one does indeed anticipate the sort of galactic scale fluctuations required providing S is not much larger than 10^4. The same statistical fluctuations (on a smaller scale) could produce the pregalactic black holes which are required to generate the extra radiation [17]. However, one no longer has $(\delta\varrho/\varrho)_{\rm dec} \propto M^{-1/2}$.

The possibility that the early Universe was tepid could be significant in the context of galaxy formation for another reason. This is because the two important mass-scales which arose in the discussion of section (2) the matter Jeans mass and the photon diffusion mass at decoupling, both depend on S:

$$M_{\rm J}^{\rm m} \sim 10 S^{1/2} M_\odot, \; M_{\rm S} \sim 10^3 S^{5/4} M_\odot \,. \tag{16}$$

If the mechanisms discussed in my first lecture produced an initial value for S less than 10^8, these characteristic masses could be much smaller than their conventional values. This would circumvent one of the prime problems in explaining galaxies. For if the present photon-to-baryon ratio was primordial or generated in one step, the Silk mass would exceed the mass of a galaxy and so, if the fluctuations were adiabatic (as they would have to be in the GUTS scenario), galaxies could only form via fragmentation. This is embarrassing since the fragmentation scenario faces severe theoretical difficulties [27]. However, if the present photon-to-baryon ratio was generated in two steps (e.g. first by GUTS and then by black hole accretion), this problem would be averted since adiabatic fluctuations could survive and become bound before the value of S was boosted to its present value.

The evolutionary features of a tepid Universe are shown in figure (3). The crucial point is that the Jeans mass no longer goes like the horizon mass all the way until decoupling but flattens off after the radiation density falls below the matter density (cf. figure (1)). This means that bound systems can form very easily then [28]. These bound systems will either collapse directly to black holes or fragment into stars which may themselves collapse to black holes after the nuclear burning timescale $t_{\rm MS}$. For massive stars $t_{\rm MS} \sim 10^{13}$ s, so the resultant holes form just at the time required to boost S to 10^8. One would expect these holes to cluster because of the statistical density fluctuations discussed above. If galaxies form out of the gas which sinks to the centres of the cluster potential wells, they would therefore be born with a *halo* of black holes; and in rich clusters of galaxies one would expect the individual members to be tidally stripped of their holes, leaving a collective black hole halo [29]. The black holes which boost the value of S in the tepid scenario would therefore be natural candidates for the "missing" mass in galactic halos and clusters.

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REFERENCES

[1] JONES, B. J. T., 1976. Rev. Mod. Phys., **48**, 107.
[2] GOTT, J. R., 1977. Ann. Rev. Astr. Astrophys., **15**, 235.
[3] REES, M. J. & OSTRIKER, J. P., 1977. Mon. Not. R. astr. Soc., **179**, 541.
[4] PEEBLES, P. J. E., 1974. Astrophys. J. Lett., **189**, L51.
[5] FALL, S. M., 1979. Rev. Mod. Phys., **51**, 21.
[6] HOGAN, C., 1979. Mon. Not. R. astr. Soc., **188**, 781.
[7] CARR, B. J., 1977. Mon. Not. R. astr. Soc., **181**, 293.
[8] HAGEDORN, R., 1970. Astr. Astrophys., **5**, 184.
[9] SILK, J., 1968. Astrophys. J., **151**, 459.
[10] SUNYAEV, R. A. & ZEL'DOVICH, YA. B., 1970. Astrophys. Space Sci., **7**, 1.
[11] PEEBLES, P. J. E. & YU, J. T., 1970. Astrophys. J., **162**, 815.
[12] GOTT, J. R. & REES, M. J., 1975. Astr. Astrophys., **45**, 465.
[13] DOROSHKEVICH, A. G., SUNYAEV, R. A. & ZEL'DOVICH, YA. B., 1974. *Confrontation of cosmological theory and observational data*. Ed. M. S. Longair, Dordrecht: D. Reidel.
[14] WHEELER, J. A., 1957. Ann. Phys., **2**, 604.
[15] HARRISON, E. R., 1970. Phys. Rev. D., **1**, 2726.
[16] ZEL'DOVICH, YA. B., 1970. Sov. Phys. JETP, **12**, 307.
[17] CARR, B. J. & SILK, J., 1980. Preprint.
[18] HOGAN, C., 1980. Nature, **286**, 360.
[19] PRESS, W. H., 1980. *The Universe at Large Redshifts*. Ed. J. Kalckar O. Ulfbeck & N. R. Nilsson, Phys. Scripta, **21**.
[20] IRVINE, M., 1978. *Neutron Stars*, Clarendon Press, Oxford.
[21] ZEL'DOVICH, YA. B., 1963. Sov. Phys. JETP, **16**, 1395.
[22] LAYZER, D., 1976. *Stars and Stellar Systems* (Vol. 9). Ed. A. Sandage, M. Sandage & J. Kristian, University of Chicago Press.
[23] LASHER, G., 1979. Phys. Rev. Lett., **42**, 1646.
[24] ZEL'DOVICH, YA. B., 1965. Adv. Astr. Astrophys., **3**, 241.
[25] PEEBLES, P. J. E., 1974. Astr. Astrophys., **32**, 391.
[26] WAGONER, R. V., 1973. Astrophys. J., **179**, 343.
[27] PRESS, W. H. & VISHNIAC, E. T., 1980. Astrophys. J., **236**, 323.
[28] CARR, B. J. & REES, M. J., 1977. Astr. Astrophys., **61**, 705.
[29] WHITE, S. D. M. & REES, M. J., 1978. Mon. Not. R. astr. Soc., **183**, 341.