2B28 Problem Sheet 2
Answers to be handed in by Monday 14 February 2005

1. Derive the Boltzmann distribution for a system in equilibrium with a heat bath at temperature T. Explain the symbols used, and the conditions under which the distribution is valid.

A certain atom has 3 possible energy levels. These energies are $E_1 = 1.3 \times 10^{-22}$ J, $E_2 = 2.3 \times 10^{-22}$ J, and $E_3 = 3.2 \times 10^{-22}$ J. The first level is non-degenerate (g_1=1), whereas the other levels are degenerate : g_2=3 and g_3=5. If the atom is in equilibrium at (i) T=5 K, and (ii) T=10 K, calculate:

(a) the partition function Z(1,V,T);
(b) the probability that the atom has energy E_2;
(c) the mean energy of the atom.

2. A paramagnetic material has non-interacting magnetic dipoles with spin $S = {}^1/_2$, and magnetic moment μ. It is in contact with a heat bath at temperature T, and a magnetic field **B** is applied.

Show that the partition function of a single magnetic dipole Z(1,V,T) is 2 *cosh* x, where x = μB/kT.

Show that the average energy of a dipole is E = - μB *tanh* x, and sketch E as a function of x.

By considering the possible states of a system of *two* dipoles, find an expression for Z(2,V,T).

If the magnetic moment $\mu = 0.93 \times 10^{-23}$ Am^2, T = 4.2 K, and the applied field B = 5 T, calculate Z(1,V,T) and Z(2,V,T). Deduce the temperature at which the probability of the moment being aligned parallel to the applied magnetic field is 0.8.

3.State the general definition of entropy in terms of the probabilities P_j of the accessible microstates. From this definition, show that the entropy of a system in contact with a heat bath at temperature T can be written as

S(N,V,T) = k *ln* Z(N,V,T) + E/T.

Use this expression to obtain a definition of thermal equilibrium for a system in contact with a heat bath at temperature T.

A particular system can exist in two energy levels. The ground level has energy 3.0×10^{-19} J, and has a degeneracy of 2. The excited level has energy 7.0×10^{-19} J, and a degeneracy of 4. If the system is in equilibrium at T = 300 K, calculate (a) the entropy, and (b) the Helmholtz free energy. Give the units in each case.

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