2B28 Statistical Thermodynamics Problem Sheet 3
Answers should be handed in by *Monday 7 March 2005*.

1. Derive Planck's law for the energy density of radiation in a black-body cavity at a temperature T. Explain any assumptions or approximations that you use.

[11 marks]

Sketch the radiation spectra emitted by stars with surface temperatures of 6000 K and 8000 K, marking the visible and ultraviolet regions on your graphs. What is the frequency of the maximum in the spectrum, in each case? What does the area under the curve represent, and how does it vary with temperature?

[6 marks]

Measurements of the cosmic microwave background suggest that the Universe has an effective temperature of about 3 K. Calculate the wavelength of the maximum in the radiation spectrum in this case, and the energy (in meV) of the corresponding photons.

[3 marks]

2. For an ideal gas comprised of N identical non-interacting *fermions*, explain the meaning of the Fermi energy, ε_F, and Fermi temperature T_F.
Draw the Fermi-Dirac distribution for the average occupation number of single-particle states as a function of energy, at T=0 K. Mark ε_F on your diagram.

[6 marks]

Consider N free electrons in a volume V as an ideal Fermi-Dirac gas. Show that the Fermi energy is given by $\varepsilon_F = (h^2 /2m)(3N /8\pi V)^{2/3}$.

[8 marks]

Calculate the Fermi temperature and the average Fermi-Dirac pressure for:

(a) ^{3}He atoms in liquid ^{3}He, where the number density is 2.2 x 10^{28} m^{-3},

(b) conduction electrons in copper, where the number density is 8.45 x 10^{28} m^{-3},

(c) neutrons in a neutron star, for which the *mass* density is 10^{15} kg m^{-3}.

[6 marks]