School of Mathematical Sciences
Mile End, London E1 4NS

MTH5112 Linear Algebra I
MID-TERM TEST

Date: 12 November 2010 *Time:* 11.00–11.40

FB328	Surnames A to C
Octagon	Surnames D to Z

Complete the following information:

Name	
Student Number (9 digit code)	

The duration of the test is **40 minutes**. Answer **all** questions **in the spaces provided**. Write the final answer clearly. Calculators are **not** allowed.

Total Marks	

Nothing on this page will be marked!

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1. Determine the solution set of the following system of linear equations:

$$\begin{array}{rcrcrcrl} x_1 & + & 2x_2 & - & 3x_3 & + & x_4 & = 1 \\ -x_1 & - & x_2 & + & 4x_3 & - & x_4 & = 6 \\ -2x_1 & - & 4x_2 & + & 7x_3 & - & x_4 & = 1 \end{array}$$

2. Find the inverse of the following matrix using Gauss-Jordan inversion:

$$A = \begin{pmatrix} 1 & 4 & 3 \\ -1 & -2 & 0 \\ 2 & 2 & 3 \end{pmatrix} .$$

3. Let A be an $n \times n$ matrix and let A_{ij} denote the (i, j)-minor of A for $i, j = 1, 2, \ldots, n$. Define the *adjugate*, adj A, of A. Given

$$A = \begin{pmatrix} 1 & 1 & 1 & 3 \\ 0 & 3 & 1 & 1 \\ 0 & 0 & 2 & 2 \\ -1 & -1 & -1 & 0 \end{pmatrix},$$

Compute the determinant $\det A$ and the product

$$A(\operatorname{adj} A).$$

4. Let V be a real vector space. Explain what is meant by a *subspace* of V.
   Let A and B be subspaces of V. Show that the sum

$$A + B = \{a + b : a \in A, b \in B\}$$

is a subspace of V.

Let $\mathbb{R}^{1\times 4} = \{(x, y, z, w) : x, y, z, w \in \mathbb{R}\}$ which is a real vector space.
Determine, **with a reason**, if the subset

$$W = \{(x, y, z, 1) : x, y, z \in \mathbb{R}\}$$

is a subspace of $\mathbb{R}^{1\times 4}$.

END