6 Conceptual Problems of Quantum Mechanics

6.1 Introduction

- Extremely successful theory - matches experiment
- But has strange conceptual implications
- Philosophy or pointing to fundamental problem?
- Three broad areas of difficulty

6.2 Determinism

- Not a deterministic theory
- If wavefunction not an eigenfunction, cannot predict the result of a single measurement, only probabilities of different outcomes
- Example: Spin-1/2 particle - Stern-Gerlach (SG) measurements

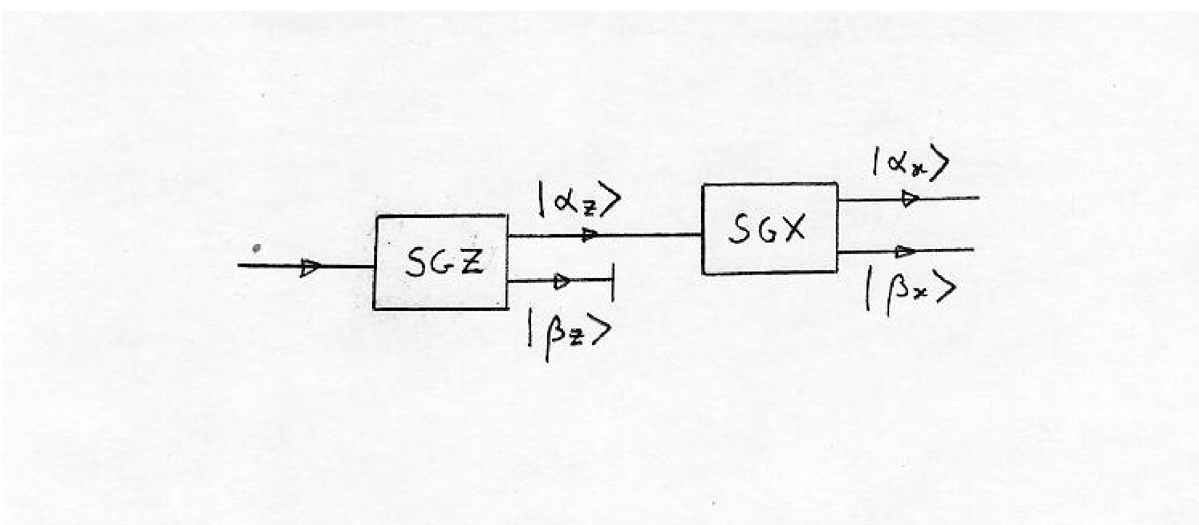


Figure 1:

- Select spin up in z-direction with SGZ. Subsequent SGX finds equal chance of spin up or down
- Cannot predict outcome for a single particle
- Compare to classical picture - all components of spin vector can be known in advance, so result predictable
- Hidden-variable theory would postulate that all components of spin are predetermined but incompleteness of QM makes it seem that we cannot know them

6.3 Locality

- Classically particles are point like or at least very small
- Classically motion determined by forces acting at that point through fields
- Three examples where particles seem not be local
- 1: The two-slit interference experiment - probability of detecting a particle at a point depends on whether one or both slits open
- 2: The correlated behaviour of two indistinguishable fermions - entangled
- The result of measuring the spin of one may depend on a the how a separate measurement of the other far away is carried out
- 3: Beam splitter experiment - exhibits same features as two-slit, using photons

6.3.1 Double slit experiment

- If light from a point source passes through a pair of slits, then you get a diffraction pattern (Young's slits experiment). Easily explained in terms of waves.
- It is possible to perform the experiment one photon at a time, producing one dot at a time on the recording screen. This is also particle like.
- If one slit is closed, a single slit diffraction pattern is produced
- If a slit changes from open to closed once the photon is in-flight, the pattern is still correct for the single slit.
- If we monitor the slits we can observe whether the photon goes to one or the other.

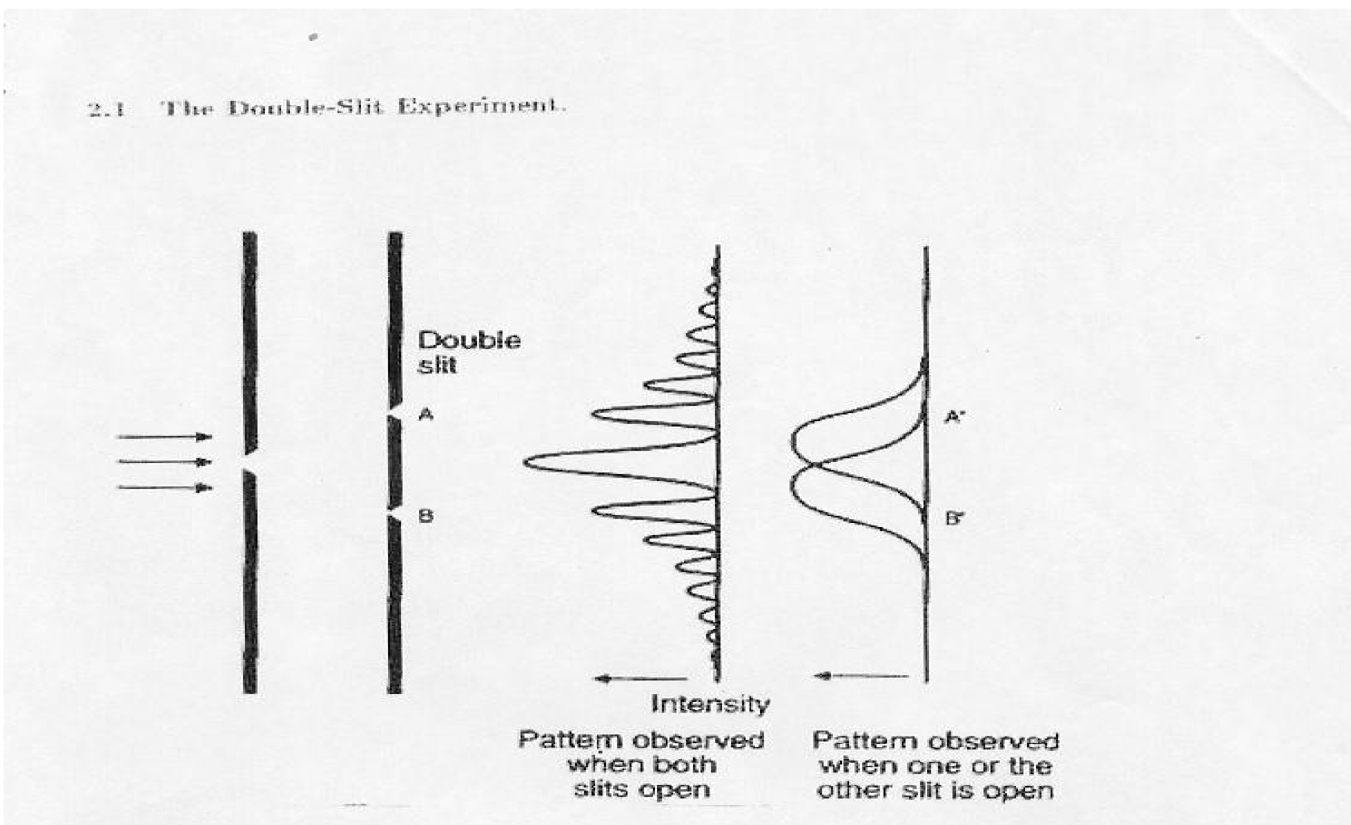


Figure 2:

6.3.2 Beam splitter experiment

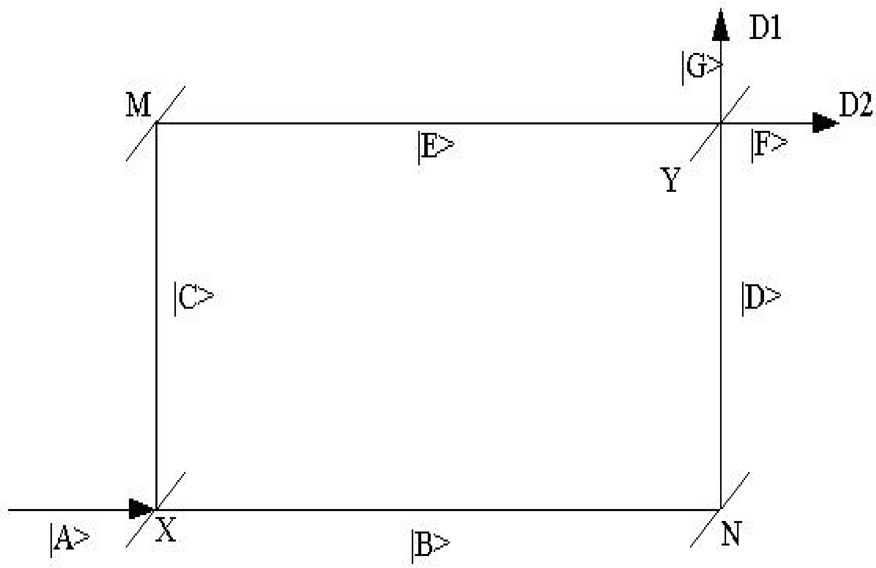


Figure 3:

- Photons incident on half-silvored mirror, X, half refelected, half transmitted
- Phase shift of $\pi/2$ between reflected and transmitted beams
- Mirrors M and N send beams to second splitter, Y which directs them to detectors D_1 and D_2
- The mirrors introduce a phase shift of π
- Wave picture -
- The phase shift in the path $XMYD_2$ is $3\pi/2$
- The phase shift in the path $XNYD_2$ is also $3\pi/2$
- The phase shift in thr path $XMYD_1$ is 2π
- The phase shift in the path $XNYD_1$ is π
- Destructive interference leads to zero intensity at D_1 and 100% at D_2.

- If one arm is blocked, at M, no interference and equal numbers of photons at D_1 and D_2
- Photon picture -
- If only one photon at a time passes through the apparatus, how does interference occur?
- **Either** photon passes down both arms at once, or
- It passes down only one arm but knows the other arm is there and how to react at Y

6.3.3 Delayed-choice beam splitter experiment

- Place photon detectors in each of the arms of the beam-splitter
- With detectors on, whole photons pass down one arm or other and are counted
- With detectors off we see interference
- Only turn on once the photon has passd through X
- Photon behaviour at X determined by a possible future measurement?

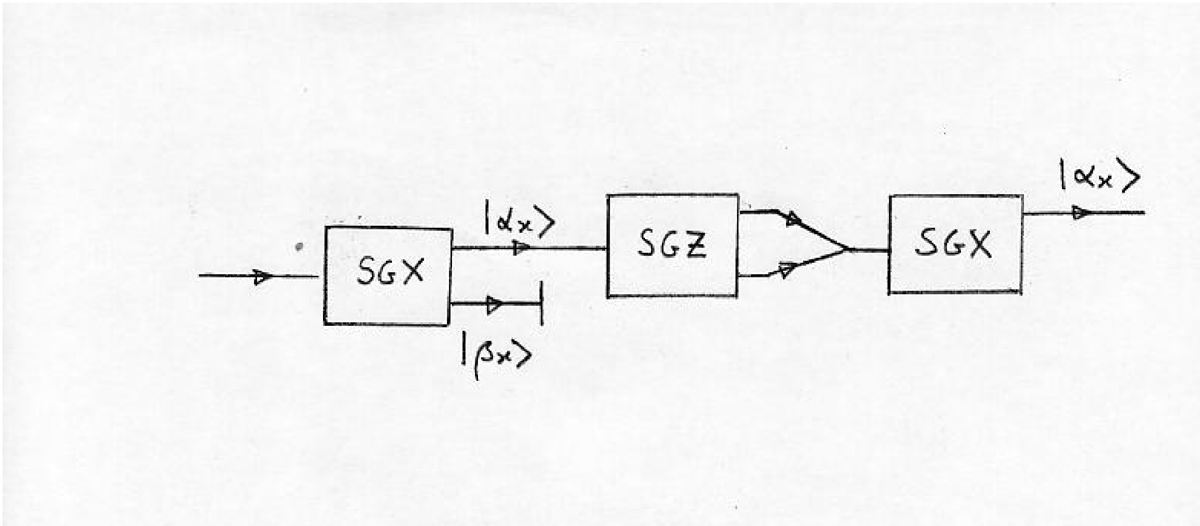


Figure 4:

6.4 Reduction of the wave function

- Postulate that measurement reduces wavefunction from prior form to an eigenfunction of the measurement operator
- In Section 3 thesequence of spin measurements in Figure 4 was discussed
- After first SGX select spin up particles
- Subsequent SGZ would find equal numbers up and down, but
- Beams recombined without counting photons before final SGX measurement
- Only spin up are found
- If reduction of the wave function had occurred at SGZ, creating $|\alpha_z>$ and $|\beta_z>$ states, the final SGX would have found equal numbers of up and down.
- Need to know which channel particles passed down after SGZ to cause reduction

- Difficulties arise in using QM to describe apparatus that records particles
- Consider measurement of S_z in Figure 4 above and
- suppose possible states of particle detector are $|0>$ if no paricle is detected, $|+1>$ if a particle is detected with z-component $+\hbar/2$, and $|-1>$ if particle is detected with $-\hbar/2$.
- The initial state of particle+detector is

$$|\alpha_x > |0> = \frac{1}{\sqrt{2}} \left(|\alpha_z > + |\beta_z > \right) |0>$$

After the particle has passed the state wil be

$$\frac{1}{\sqrt{2}} \left(|\alpha_z > |+1> + |\beta_z > |-1> \right)$$

- State

$$|\alpha_z > |0>$$

evolves to

$$|\alpha_z > |+1>$$

and the SE preserves the time evolution of the two parts of the wave function

- Hence the final state of the system does not specify a unique final state of the detector. It is also an entangled superpositon.
- A further measurement is required to reduce the wavefunction of the detector itself to either

$$|\alpha_z > |+1>$$

or

$$|\beta_z > |-1>$$

- This process could go on ad infinitum
- Possible solutions
- A conscious observer is required to “know” the result of the measurement (Is the Moon there when no one is looking?) or
- Laboratory apparatus does not obey QM. But then where does QM end and classical physics begin?
- Also, some find the reduction postulate unacceptable because it is not described by the Schroedinger equation, which is supposed to describe the time-evolution of the wave function.
- At the moment of measurement this concept is abandoned in favour of an ad hoc process of “reduction”

6.4.1 Schroedinger’s cat and Wigner’s friend

- Graphic examples of the measurement dilemma
- In the case of Schroedinger’s cat, a cat is in a sealed box, containing a radioactive source and a flask of hydrocyanic acid.
- A device is included so that if the source undergoes a decay, the flask is shattered and the cat is poisoned. A source is chosen that gives a 50% chance of a decay in a fixed time, say one hour. According to QM, before any measurement is made, the wave function is a linear superposition of the two possible states

$$|\psi>= C_l|\text{no decay; live cat}> +C_d|\text{decay; dead cat}>$$

- On opening the box, the wavefunction collapses into one or other states with probabilities $|C_l|^2$ or $|C_d|^2$.
- What does a superposition of live and dead states mean?
- When the observer opens the box does he/she then go into a superposition of

$$|\text{observer sees live cat; no decay; live cat} >$$

and

$$|\text{observer sees dead cat; decay; dead cat} >$$

- until another observer comes along etc etc
- Wigner replaced the cat by a friend and asked how a conscious being would expereince the state of superposition

6.4 The Copenhagen Interpretation

- The interpretation of QM given by Bohr and his collaborators, working in Copenhagen
- Accepted widely as the "standard" interpretation of QM
- In the CI, measurement is all that we can speak about. It is meaningless to seek deeper knowledge.
- An atom is not a collection of particles in motion but a total system whose energy levels (and other properties) can be predicted and measured.
- It is meaningless to seek any detailed picture of the individual "particles"
- Bohr also introduced the Principle of Complementarity

- This states that mutually exclusive descriptions (like wave/particle) can be applied to quantum systems but not simultaneously
- Wave and particle can be observed but not in the same experiment at the same time
- The CI comes close to denying the existence of physical reality in the absence of an observer

6.6 Hidden variables

- Einstein and others have suggested that the indeterminacy in QM is only apparent and due to some hidden structure or incompleteness of the theory
- Compare to kinetic theory of gases - the global properties of gases, pressure, temperature etc are statistical properties of many motions of individual molecules
- These motions are deterministic - perhaps QM has a deterministic substructure and the statistical nature of measurement is only apparent
- Called hidden variable theories

6.7 Non-locality and EPR

- In a famous paper Einstein, Podolsky and Rosen (EPR) laid down two criteria for an acceptable theory
  - Reality: Physical quantities should be "real" where in this context reality is defined by the criterion "If without in any way disturbing a system, we can predict with certainty the value of a physical quantity then there exists an element of physical reality corresponding to this physical quantity"

– Locality: The theory should be local with no action at a distance

- EPR described a thought experiment in which a single stationary particle explodes into two identical fragments
- From conservation of momentum a measurement of the momentum of one fragment can be used to deduce the momentum of the other
- Similarly for position since they must have both travelled an equal distance
- Position and momentum cannot be measured simultaneously to arbitrary precision
- So the type of measurement made on the first fragment determines the result of the measurement of the second even though they may be separated by a large distance
- David Bohm later developed this thought experiment to the case of the correlated spins of two spin-1/2 particles which we considered in Chapter 2

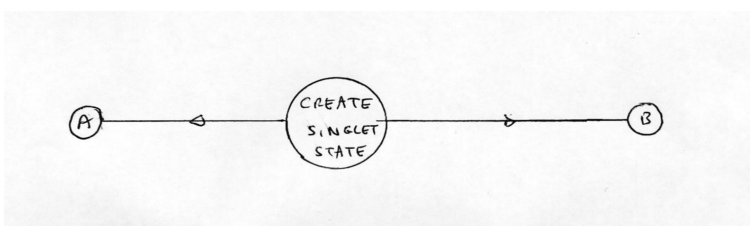


Figure 5:

- The wave function of the two particles in a singlet ($S = 0$) state is

$$|\chi(1,2)> = \frac{1}{\sqrt{2}}(|\alpha_1>|\beta_2> - |\beta_1>|\alpha_2>)$$

- Neither particle has a specific value for S_z until a measurement is made.

- The two particles are then allowed to separate spatially and measurements are made on their z-component of spin, first at A then at B
- Measurement of S_z for particle 1 at A finding spin up, say, collapses the wavefunction to $|\alpha_1 > |\beta_2 >$, so a subsequent measurement of S_z for particle 2 must reveal spin down.
- If A measures S_x, then the wavefunction collapses to $|\alpha_{1x} > |\beta_{2x} >$ and if B measures S_z on particle 2 she finds spin up or down with equal probability
- Thus the results of measurements at B are affected by the choice of measurement at A
- These long range correlations between measurements violate the EPR conditions for reality and locality .
- The choice of spin measurement direction can be delayed until the particles are far enough apart that no sub-light speed signal could pass between them but
- B cannot deduce the orientation of A's measurement from her measurements so no information can be passed in this way
- Non-local behaviour of this type can also be demonstrated with polarisation states of photons

6.7.1 Relativity and EPR

- It can be shown that, if relativity is included, the order of the measurements at A and B can appear reversed
- Depending on the velocity of the observer it can appear that either A or B made the first measurement collapsing the wave function of the composite system

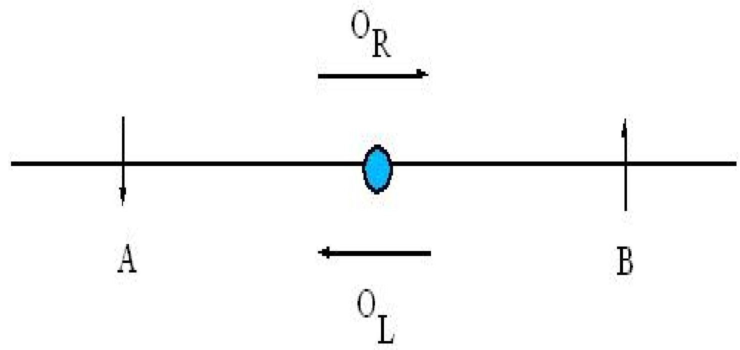


Figure 6:

- The sequence of cause and effect depends on the observers relative motion

6.8 Bell's Inequalities

- Bell derived inequalities that would have to be met by any real, local theory
- Consider the entangled singlet state above
- First the QM picture
- In QM a joint measurement of the spin-component of particle 1 in a direction described by the unit vector $\hat{n}$ and particle 2 in direction $\hat{n}'$ is given by the observable

$$\hat{K} = \sigma_n(1)\sigma_{n'}(2)$$

- We know that

$$< \chi|\hat{K}|\chi >= -\cos\gamma = -\hat{n}\cdot\hat{n}'$$

- Now the hidden-variable picture
- Suppose that after the two particles discussed above separate, system possesses some property that determines in advance the result of performing a spin measurement on either particle
- Suppose λ is a hidden variable that completely defines the state of the system
- Each spin-zero system has a particular value of λ
- Then the spin of particle 1 in the z-direction, S_{z1} is a function of λ, ie $S_{z1}(\lambda)$.
- Similarly for the second particle, whose spin is measured in some direction $\hat{n}$, $S_{n2}(\lambda)$

- Define a probability $p(\lambda)$ that a pair of particles is produced with spins $S_{z1}(\lambda)$ and $S_{n2}(\lambda)$
- Since $p(\lambda)$ is a probability

$$\int p(\lambda)\mathrm{d}\lambda = 1 \tag{6.1}$$

- Now suppose we measure S_{z1} and S_{n2} for a large number of pairs of particles and calculate the average value of the product $S_{z1}S_{n2}$, which we call $C(\hat{k}, \hat{n})$

$$C(\hat{k}, \hat{n}) = \int S_{z1}(\lambda)S_{n2}(\lambda)p(\lambda)\mathrm{d}\lambda \tag{6.2}$$

- Now suppose we carry out another series of measurements with the second axis in a direction $\hat{n}'$

$$C(\hat{k}, \hat{n}') = \int S_{z1}(\lambda)S_{n'2}(\lambda)p(\lambda)\mathrm{d}\lambda$$

therefore

$$C(\hat{k}, \hat{n}) - C(\hat{k}, \hat{n}') = \int \left[S_{z1}(\lambda)\ S_{n2}(\lambda) - S_{z1}(\lambda)\ S_{n'2}(\lambda)\right] p(\lambda)\mathrm{d}\lambda$$

- We know that a measurement of the spin of particle 2 in the same direction as particle 1 must imply an equal and opposite measurement for the spin of particle 1

$$S_{n1}(\lambda) = -S_{n2}(\lambda)$$
$$S_{n'2}(\lambda) = -S_{n'1}(\lambda)$$
$$\therefore C(\hat{k}, \hat{n}) - C(\hat{k}, \hat{n}') = -\int S_{z1}(\lambda)\left[S_{n1}(\lambda) - S_{n'1}(\lambda)\right] p(\lambda)\mathrm{d}\lambda$$

Now since $S_{n1} = \pm 1$,

$$S^2_{n1}(\lambda) = 1$$

$$\therefore C(\hat{k}, \hat{n}) - C(\hat{k}, n') = \int S_{z1}(\lambda)\ S_{n1}(\lambda)\,[1 - S_{n1}(\lambda)S_{n'1}(\lambda)]\,p(\lambda)\mathrm{d}\lambda$$

We now take the absolute value of both sides of this equation to get

$$|C(\hat{k}, \hat{n}) - C(\hat{k}, \hat{n}')| = |\int S_{z1}(\lambda)\ S_{n1}(\lambda)\,[1 - S_{n1}(\lambda)S_{n'1}(\lambda)]\,p(\lambda)\mathrm{d}\lambda|$$

$$\leq \int |S_{z1}(\lambda)\ S_{n1}(\lambda)\,[1 - S_{n1}(\lambda)S_{n'1}(\lambda)]\,p(\lambda)|\mathrm{d}\lambda$$

where we have used the fact that the absolute value of an integral is always less than or equal to the integral of the absolute value of the integrand. Now neither $p(\lambda)$ nor the term in the square brackets can ever be negative and also

$$|S_{z1}(\lambda)S_{n1}(\lambda)| = 1$$

because both these quantities are ± 1. Therefore

$$|C(\hat{k}, \hat{n}) - C(\hat{k}, \hat{n}')| \leq \int [1 - S_{n1}(\lambda)S_{n'1}(\lambda)]\,p(\lambda)\mathrm{d}\lambda$$

$$|C(\hat{k}, \hat{n}) - C(\hat{k}, \hat{n}')| \leq 1 + \int S_{n1}(\lambda)S_{n'2}(\lambda)p(\lambda)\mathrm{d}\lambda \tag{6.3}$$

Confine discussion to the case where the z-axis, $\hat{n}$ and $\hat{n}'$ are all in the same plane. Comparing the integral in this final equation to eqn 6.2 we see that

$$\int S_{n1}(\lambda)S_{n'2}(\lambda)p(\lambda)\mathrm{d}\lambda = C(\hat{n}, \hat{n'}) \tag{6.4}$$

So finally,

$$|C(\hat{k}, \hat{n}) - C(\hat{k}, \hat{n}')| \leq 1 + C(\hat{n}, \hat{n'}) \tag{6.5}$$

Now consider a specific example where $\hat{n}$ is at an angle θ to the z-axis and $\hat{n}'$ is at an angle 2θ to the z-axis. Equation 6.4 describes the correlation of measurements separated by an angle θ.

We know that the average value of such a correlated measurement in QM is

$$-\cos\theta$$

so equation 6.3 becomes

$$|-\cos\theta + \cos 2\theta| \leq 1 + (-\cos\theta)$$

or

$$|\cos 2\theta - \cos\theta| + \cos\theta \leq 1 \tag{6.6}$$

Now consider $\theta = \pi/4$ when $\cos\theta = \frac{1}{\sqrt{2}}$, so

$$\sqrt{2} \leq 1$$

- In fact the inequality is violated for all $0 < \theta < \pi/2$ and at other values of θ.
- So the inequality 6.5, which must be satisfied by a local theory is apparently violated by QM
- Argument still continues as to the exact conclusions to be drawn

6.9 The Aspect Experiments

- A. Aspect and co-workers carried out experiments to test Bell's inequalities
- They used photons rather than spin-1/2 particles
- Photons are spin-1 particles and carry intrinsic angular momentum parallel to their direction of motion of $\pm\hbar$
- We would expect $M_S = -1, 0, +1$ but $M_S = 0$ is not allowed due to relativity
- The states $M_S = \pm 1$ correspond to the polarization states of the photon, which as with a spin-1/2 particle is a two-state system, which we call $|h>$ and $|v>$.
- These refer to horizontal and vertical polarization
- The Figure 7 shows the Aspect experiment
- The $4p^2\ {}^1S_0$ state of Ca is pumped by lasers
- The state decays via two electric dipole transitions back to the $4s^2\ {}^1S_0$ ground state
- Overall there is no change of momentum, so the the two photons, emitted at 551.3 nm and 422.7 nm must carry a net angular momentum of zero
- The state of the two photons is described by

$$|\chi_{00}>=\frac{1}{\sqrt{2}}(|h_1>|h_2>-|v_1>|v_2>)$$

- where $|h_1>$ signifies photon 1 in state $|h>$ etc.
- The photon polarisations are correlated as were the spins in a singlet state

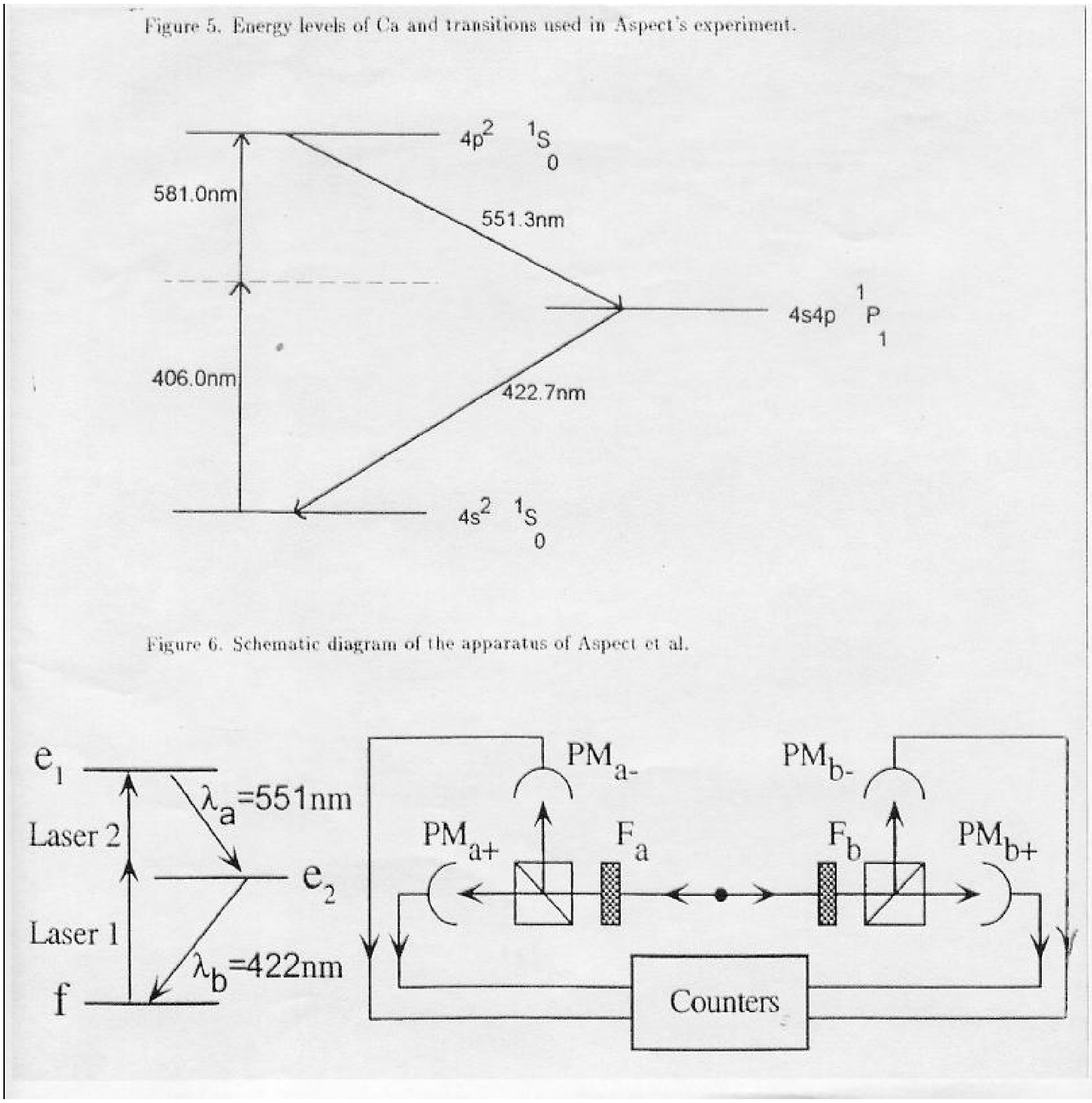


Figure 7:

- The photons enter polarisers that transmit photons with polarisation $|v>$ so that they enter detectors PM_{a+} or PM_{b+}
- The deflect photons with polarisations $|h>$ to detectors PM_{a-} and PM_{b-}
- Aspect used four different orientations of the two polarisers $\hat{a}, \hat{a}', \hat{b}, \hat{b}'$
- A correlation coefficient based on hidden variable theory is defined

$$C(\hat{a}, \hat{b}) = \int S_{a1}(\lambda) S_{b2}(\lambda) p(\lambda) \mathrm{d}\lambda$$

- The quantity

$$X = C(\hat{a},\hat{b}) + C(\hat{a},\hat{b}') - C(\hat{a}',\hat{b}) + C(\hat{a}',\hat{b}')$$

- satisfies the "Bell" inequality

$$-2 \le X \le 2$$

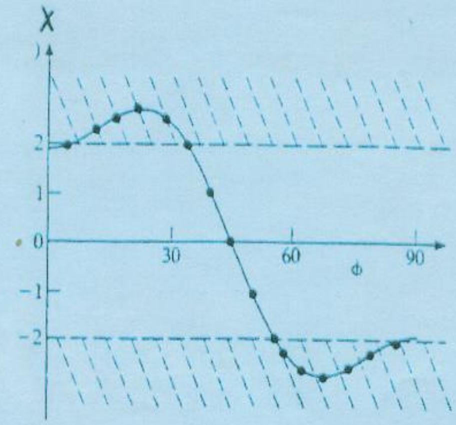
- Aspect et al restricted the four directions according to

$$\hat{a}\cdot\hat{b} = \hat{b}\cdot\hat{a}' = \hat{a}'\cdot\hat{b}' = \cos\phi$$

  and

$$\hat{a}\cdot\hat{b}' = \cos 3\phi$$

- and then measured X as a function of ϕ
- Results are shown in Figure 8
- The Bell inequality assumes locality but not QM
- Results violate the Bell inequality for a range of ϕ and therefore are inconsistent with this local hidden variable theory
- It appears that any successful theory must be non-local
- Note that this coclusion is valid even if QM is not the correct theory!
- A non-local hidden-variable theory is not ruled out



Experimental data obtained for the correlation quantity χ by A. Aspect, P. Grangier and G. Roger. The solid curve represents the prediction of quantum mechanics and the shaded areas are those in which Bell's inequality is violated. (*By courtesy of A. Aspect.*)

Figure 8:

6.10 Alternative Interpretations

- The Copenhagen interpretation held sway for about 50 years after the development of QM
- The work of Bell and Aspect particularly has re-awakened interest in alernative theories and interpretations
- Theories seek to provide falsifiable alternatives to QM eg hidden variable theories
- Interpretations provide alternative ways of understanding the results of QM

6.10.1 The many-worlds interpretation/theory

- In this model, due originally to Everett in 1956, each time a measurement is made, the universe splits into a number of copies, in each one of which each of the possible outcomes is realised

- The wave function has an observer-independent existence and obeys the standard wave equation at all times
- There is no collapse of the wave function at measurement so the observer plays no special role
- A split occurs when a thermodynamically reversible process occurs
- So for Schroedinger's cat, the split occurs when the radioactive decay happens
- In one world the cat lives and is observed to be alive
- In the other, the cat dies and another version of the observer finds it dead
- Since the information about the cat's demise could be transmitted to the whole universe two different and permanently separate universes are created each time such a measurement occurs
- Two universes can recombine if no irreversible process has taken place
- In Figure 3, for example, when the beams are recombined the universes merge
- Advantages - economical on assumptions; avoids unexplained collapse of the wavefunction; proponents claim it provides a local theory; may make testable predictions
- Disadvantages - a seriously large number of universes are created; criterion for splitting of universes may have similar problems to criterion for a measurement in the CI

6.10.2 Pure and mixed states

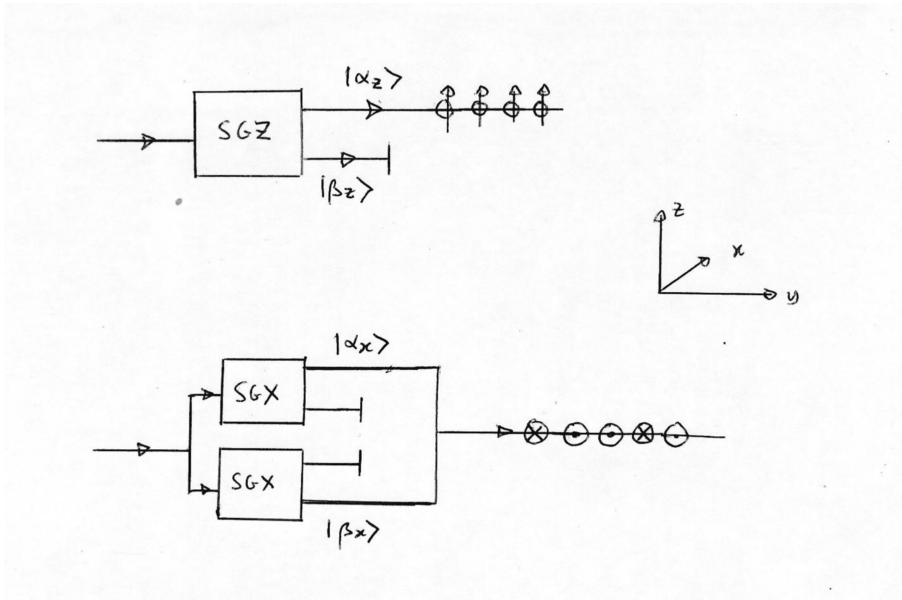


Figure 9:

- A beam of electrons with spin-up in the z-direction consists of an ensemble of identical fermions, all in the same quantum state $|\alpha>$
- Such an ensemble is said to be in a pure state
- It need not be an eigenstate but could be a superposition such as

$$|\chi>= a|\alpha> + b|\beta>$$

Such a state might be formed by the SGZ measurement in Figure 9

$$|\alpha_z>= \frac{1}{\sqrt{2}}|\alpha_x> + \frac{1}{\sqrt{2}}|\beta_x>$$

- All the particles are in a state which is a linear combination of the states $|\alpha_x>$ and $|\beta_x>$.
- Because such quantum states exhibit interference, they are called coherent states

- A measurment of S_z on this state naturally finds only spin up
- A mixed state on the other hand is a mixture of particles in different states.
- In a two state system, a particle is either in state $|\alpha_x>$ or state $|\beta_x>$.
- Such a mixed state could be created as shown in Figure 9
- A measurement of S_z on state $|\alpha_x>$ yields spin up or down with equal probability as does a measurement of S_z on $|\beta_x>$

6.10.3 Decoherence

- Decoherence is the process whereby a pure state is converted to a mixed state
- Any real system will consist of an entanglement of very many states not just involving a single spin-1/2 particle, for example but also states of the surrounding environment
- In an SG experiment we have an unusual situation where the particles are isolated from their environment
- In a more realistic situation where there are many entanglements, the interference terms due to all the quantum states cancel each other out leaving mixed states and the "classical" probability picture
- In this picture Schroedinger's cat's wave function would decohere almost immediately that the radioactive decay took place
- Decoherence may therefore help with the problem of the quantum state of macroscopic systems but does not help with the issue of collapse of the wave function

- If this picture is correct, a macroscopic system should exhibit quantum behaviour if it can be prevented from decohering
- In practice kept isolated from its environment
- Experiments are underway to test this
- In the field of quantum information processing, decoherence is an obstacle to be overcome since coherent states must be maintained for this to succeed