Due March 13, 2008

MAS423/AST001 Solar System
Coursework #2

1. Consider two unbound bodies with masses m_1 and m_2 (e.g., stars, planets, planetesimals, ... etc.) that encounter and gravitationally scatter each other. Their motion is hyperbolic. Long before the encounter at time $t = -\infty$, the stars' separation r is infinite, and body m_2 (the scattered body) has an intial speed v_∞ relative to m_1 (the scatering body) and impact parameter b: (see the diagram on the back)

   (a) show that m_2's orbit has a semimajor axis a specific angular momentum h and eccentricity e that obey

$$a = -\frac{\mu}{v_\infty^2}$$
$$h = bv_\infty$$
$$e^2 = 1 + \frac{b^2 v_\infty^4}{\mu^2}$$

   (b) Let $f_{max} = m_2$'s maximum true anomaly, which is achieved at times $t = \pm\infty$. Also let $\theta_s =$ angle through which m_2 is scattered. Show that

$$\cos f_{max} = -\frac{1}{e}$$
$$\theta_s = 2\cos^{-1}\left(-\frac{1}{e}\right) - \pi$$

2. (Adapted from DePater & Lissauer Problem 2.9) This problem concerns the stability of orbits in the planar restriced three-body problem. The ratio of the star's mass to that of the planet is 333. Choose a rotating coordinate system with the center of mass at the origin and the planet located at x=1,y=0.

   (a) Calculate (approximately) the location of the equilibrium points.

   (b) Which, if any of these points are stable?

(c) In what region about the planet may moons have stable orbits? What regions must asteroid avoid in order to have stable orbits about the star?

(d) Go to the "Binary Star Integrator" (from Prof. Doug Hamilton's website), http://janus.astro.umd.edu/
"Orbital Integrators" and use the "Three-body Integrator". Start with "2D Orbits" as they're easiest to work with. Check your answers to part b) and show examples of tadpole and horseshoe orbits if they exist. View these in both the rotating and inertial systems to help you understand what is going on. Check your answer to part c) by starting the third particle on a circular orbit 60 degrees ahead of Jupiter and varying its semimajor axis. Make a table showing which values of the semimajor axis allow the particle to cross Jupiter's orbit and which do not.

[You can also use the 'Orbital Elements' link to plot and refresh the geometric definitions of the orbital elements.]

[Don't wait to the last minute to try this.]