Tachyons and tachyonian systems. Generalisation of the special principle of relativity and an idea of six-dimensional space-time.

(Translation from unpublished notes in Swedish January 1971) Jan Pilotti B.Sc[*].

Introduction

In Alväger et al 1968 (1) the history and rational for tachyons are summarized. In short the special theory of relativity doesn't exclude particles "born" with the velocities greater than that of light even if the energy and rest-mass relation have a singularity for v=c and this precludes ordinary particles to reach v=c or v>c as well as precludes tachyon to have v<c or v=c.

In (1) they use $E = \dfrac{m_o c^2}{\sqrt{1-\dfrac{v^2}{c^2}}}$ even for v>c and rearrange it to $E = \dfrac{\mu c^2}{\sqrt{\dfrac{v^2}{c^2}-1}}$ where $m_0 = i\mu$ μ real and they argued that it is no problem that the tachyons (seems to) have imaginary rest-mass as they never can be at rest in our systems so it is not directly measurable. Seems possible and perhaps good enough for first attempt. But a little ad hoc? But actually it is not very clear how to handle tachyons theoretically or experimentally and I think this is due to the singularity at v=c. We are not entitled to use the old views on particles when trying to visualize tachyons.

Generalisation of the principle of relativity

There is yet another approach which seems to be more in line with the spirit of the special theory of relativity and therefore perhaps less ad hoc.

If tachyons exist I think it is conceivable that many tachyons, which have the same velocity **v** $|\mathbf{v}|>c$ relative some ordinary inertial system (OIS) S , would all be at rest relative each other and therefore it is conceivable to think of a tachyonian rigid system (TIS) ${}_tS_v$ with the velocity **v** relative S. And also think that there are tachyons with all the different characteristics as ordinary matter besides having $|\mathbf{v}|>c$ relative an OIS. So it will be conceivable that there is tachyon matter[1] and we can build tachyon clocks (eg a light clock where light is reflected between two mirrors).

In our world, "world 1" and for $|\mathbf{v}|<c$ we have the ordinary LT between S and S' in standard configuration

$$x' = \frac{x-vt}{\sqrt{1-\frac{v^2}{c^2}}} \qquad y'=y \qquad z'=z \qquad t' = \frac{t-\frac{vx}{c^2}}{\sqrt{1-\frac{v^2}{c^2}}}$$

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[1] (In (1) they assume that tachyons are created in pairs so it ought to exist both positive and negative charge for tachyons. They also argue that it is plausible that a charged tachyon can be captured by a charged nucleus or an electron. Is it not then also plausible that a positive tachyon could capture a negative tachyon so there could be tachyon atoms? Couldn't this give us the possibility to communicate definitions of length and time units.)

In a world which only consists of tachyons, world 2, we can as a first trial generalise the ordinary postulates and even for tachyon world **postulate:**

I. **the special principle of relativity** and

II. that the velocity of light is the same in all TIS $= c_t$

and the derivation of LT within the tachyon world, world 2 can be done exactly as in world 1

$$\xi' = \frac{\xi - v\tau}{\sqrt{1-\frac{v^2}{c_t^2}}} \qquad \eta' = \eta \qquad \varsigma' = \varsigma \qquad \tau' = \frac{\tau - \frac{v\xi}{c_t^2}}{\sqrt{1-\frac{v^2}{c_t^2}}}$$

Then I think it is conceivable that all the other physics in world 2 can be done as in our world[1].

Note that "humans" in world 2 will measure positive real rest-masses for their particles and will describe imaginary rest-masses for the particles in world 1 and that they never are at rest relative to IS in world 2.

A generalisation of the special principle of relativity ought to give "Tachionians" the right to view and handle there IS as we in world 1 view and handle our IS.

How are then world 1 and world 2 related? Can there be any meaningful transformation between them? I come back to that below.

How is the relation between c and c_t?

In world 1 c is a limit velocity and independent of $IS_1 \in$ World1

In world 2 c_t is a limit velocity and independent of $IS_2 \in$ World 2

So it seems reasonable, if not follows from the generalised special principle of relativity, to assume $c=c_t$. Of course we here also have the question about the units of length and time in world 1 and world 2 but we can here assume that these units can be chosen (is it a free choice?) so c and c_t have the same numerical value.

Problems about communication.

Can we communicate with world 2 and tell them about our results of measurement? Is this of any significance? Can we choose units for them?

The problem is that we never can have clocks from world 1 respectively world 2 at rest relative each other (other than for $v_1=c=v_2$ but then the time "stands still"?) The same with length measuring-rods. So to speak about sameness of measuring-rods and clocks doesn't seems so easy. But if we could communicate we could tell "them" how 1 second is a certain number of period of a special radiation from Caesium 133 and then define 1m as a distance light goes in a specified time and trust in the principle of relativity?[1]

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Can there be any meaningful transformation between our world 1 and tachyonian world 2?
If we can observe tachyons, that is giving real coordinates to them in our systems it must exist transformations with real numbers between world 1 and world 2 as the coordinates for tachyons in there own systems also are real. But for v>c the ordinary LT doesn't work.
So either we cannot observe tachyons or there must be other real transformation than the ordinary LT. Or (perhaps) more general; Can one and the same event be registered in both world 1 and world 2 and must this registration be expressed in real values? If so there must exist a transformation with only real numbers.
When we at first define IS we don't a priori know that there will be an upper limit for the speed between two IS, so it doesn't seem to bold to define all systems which move with uniform velocity relative to some IS also as an IS irrespectively of the velocity.
(In the way my teacher proved LT I think it works, but perhaps problem with "free particle" in Rindler' s derivation (2).)
It is then discovered that we must divide these IS in three classes

$$\left.\begin{array}{l}\text{Those which belongs to world 1 } |\mathbf{v}|<c \\ \text{Those which belongs to world 2 } |\mathbf{v}|>c \\ \text{The limit } |\mathbf{v}|=c\end{array}\right\rangle \text{ in relation to World 1}$$

$$\left.\begin{array}{l}\text{(Those which belongs to world 1 } |\mathbf{v}|<c \\ \text{Those which belongs to world 2 } |\mathbf{v}|>c \\ \text{The limit } |\mathbf{v}|=c\text{)}\end{array}\right\rangle \text{ in relation to World 2}$$

(Those in world 2 (according to us) will call their world world 1 but I (think I) belong to world 1 so I use our labels)

With this explicit "generalisation" (which basically is no generalisation) the special principle of relativity can be formulated exactly as before "Einsteins's principle":

Postulate I "All inertial frames are equivalent for the formulation of all physical laws." (3)

(Comment: This principle ought to be interpreted as: If one have exactly the same experimental situation, that is if all variables of causes have the same value of measurement in two different IS (in the generalised sense) the result will be the same, that is the measure of the effect variables).
That "tachyoninas" give our rest-masses imaginary numbers (and vice versa) *just shows* that rest-mass is not invariant, but its value in a drastically way depends on from which world it is described.(transformation $m_0 \rightarrow i\, m_0$?)

As done above this postulate at first ought to allowed to be used in each world by itself and we will have LT between intertial frames IS and IS' belonging to the same world (either wolrd1 or world2).

Generalised Lorents transformation
Now as stated above either we cannot observe tachyons or there must be a real transformation between an $IS_1 \in$ World 1 and an $IS_2 \in$ World2. So now we will examine if there is a possibility to find such real transformation between IS_1 and IS_2 with the relative velocity $|\mathbf{v}|>c$ and in "standard configuration" (this is not totally clear what it shall mean for $|\mathbf{v}|>c$)

We follow he derivation according to Rindler (2):

Two events P (a flash of light emitted) Q (some particle illuminated by that flash)

in IS_1 (x,y,z,t) $(x+dx,y+dy,z+dz,t+dt)$

in IS_2 $(\xi,\eta,\varsigma,\tau)$ $(\xi+d\xi,\eta+d\eta,\varsigma+d\varsigma,\tau+d\tau)$

Exactly as in Rindler (2) page 16-17 we get

$dx^2+dy^2+dz^2-c^2dt^2=0$ and $d\xi^2+d\eta^2+d\varsigma^2-c^2d\tau^2=0$ which with the same argument as in Rindler (e g differentiability) gives :

$d\xi^2+d\eta^2+d\varsigma^2-c^2d\tau^2 = \mathrm{K}\,(dx^2+dy^2+dz^2-c^2dt^2)$ (I)

and again with the same argument as in Rindler (the orientation can be arbitrary and IS are isotropic so the relation between IS_1 and IS_2 are completely symmetric)

$dx^2+dy^2+dz^2-c^2dt^2 = \mathrm{K}(d\xi^2+d\eta^2+d\varsigma^2-c^2d\tau^2)$
So it follows that K= ±1 (up to here Rindler had said nothing about the relative velocity v)

As K=1 gives ordinary LT which does not work for $|\mathbf{v}|>c$ we have to choose K=−1 even if it is against the law of inertia for quadratic forms if we not allow complex numbers[2].

$d\xi^2+d\eta^2+d\varsigma^2-c^2d\tau^2 = -\,(dx^2+dy^2+dz^2-c^2dt^2)$ (II)
this (exactly as in Rindler (2) exercise IV 1 page 74) proves linearity. Thus (II) is valid for finite differentials and if we assume we have standard configuration (a question is if simultaneity in a point is an invariant between world 1 and world 2?) and again as in Rindler For $\eta = Ay$ $\varsigma = Cz$ (II) which gives* A= ± 1 C=± 1 which gives

$\xi^2+\eta^2+\varsigma^2-c^2\tau^2 = c^2t^2-x^2-y^2-z^2$ (III) $\Rightarrow$ $\xi^2-c^2\tau^2 = c^2t^2-x^2-2y^2-2z^2$ (III')

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[2] This was pointed out by my teacher Dr Lars Söderholm at a lecture in november 1970
* 2018 march 6: This is not correct for the minus sign which instead give A=±i C=i there for (III') is not solvable Fortunately I didnt stopp here but did (what Parker unknown to me had done 1968) start with $x'^2 - ct'^2 = \pm(dx^2 - c^2t^2)$ where both + and – are possible but as Parker say cant be used in 4D.But can be seen as a heuristic argument and led to the idea of 6D

$\xi = 0$ must give x=vt $\Rightarrow$ $\xi = B(x - vt)$

If we for simplification just take one space dimension x resp ξ we get

$$\xi^2 - c^2\tau^2 = \pm (dx^2 - c^2t^2)$$

Rindlers argumet to discards $-$ is that (I) must be valid for v $\rightarrow$ 0. But this argument is invalid when we are looking for LT for $|\mathbf{v}|$>c. And now we can freely choose sign (because $+ -$ and $- +$ is the same signature in the law of inertia for quadratic forms).

Now from

$$\xi^2 - c^2\tau^2 = - (dx^2 - c^2t^2)$$

a simple calculation (as in Rindler using $\xi = B(x - vt)$ which gives $\tau = Cx + Dt$) we get

$$\xi = \frac{x - vt}{\sqrt{\frac{v^2}{c^2} - 1}} \qquad \tau = \frac{t - \frac{vx}{c^2}}{\sqrt{\frac{v^2}{c^2} - 1}} \qquad |\mathbf{v}|>c$$

To be able to use $-$ so $d\xi^2 + d\eta^2 + d\varsigma^2 - c^2d\tau^2 = - (dx^2 + dy^2 + dz^2 - c^2dt^2)$ (IV)without imaginary numbers (which as we argued seems to prevent tachyons from being observed) we according to the law of inertia for quadratic forms must have the same signature on both sides. As it stands the signature is $+ + + -$ in left member of (IV) but $- (+ + + -)$ that is $- - - +$ in the right member. A mathematical possible way to have the same signature is to add two dimension so the signature in left member is $+++ - - -$ and in the right member $-(+++ - - -)$ that is $- - - +++$ which is the same signature in the sense of quadratic forms as only the number of $+$ and $-$ counts. Can there exist two more parameters?

As augured above even if tachyons exist for us to be able to observe them there must exist more dimensions so we have symmetry in positive, space-dimensions and negative "time"-dimensions eg a six-dimensional space-time. Of course it is not straightforward how to interpret these extra dimensions.

References:

(1) Alväger ,T., Kreisler, M. Physical Review **171,**1357, (1968)
(2) Rindler, W. Special Relativity, Oliver &Boyd Second Edition 1969, pp 13-21 See Appendix A
(3) See (2) p8