Summary of Required Results from Other Courses in Part IA

Line Integrals

The following statements are equivalent:

(i) $\int_A^B \mathbf{F}\,.\,\mathrm{d}\mathbf{x}$ is independent of the path from A to B;

(ii) $\mathbf{F}\,.\,\mathrm{d}\mathbf{x}$ is an exact differential;

(iii) $\nabla \times \mathbf{F} = \mathbf{0}$;

(iv) $\mathbf{F} = \nabla\phi$ for some function $\phi(\mathbf{x})$.

In such a case, $\nabla\phi\,.\,\mathrm{d}\mathbf{x} = \mathrm{d}\phi$, so

$$\int_A^B \mathbf{F}\,.\,\mathrm{d}\mathbf{x} = \int_A^B \mathrm{d}\phi = \phi(B) - \phi(A).$$

Chain Rule in 3D

$$\frac{\mathrm{d}}{\mathrm{d}t}\,\phi(x,y,z) = \frac{\partial\phi}{\partial x}\frac{\mathrm{d}x}{\mathrm{d}t} + \frac{\partial\phi}{\partial y}\frac{\mathrm{d}y}{\mathrm{d}t} + \frac{\partial\phi}{\partial z}\frac{\mathrm{d}z}{\mathrm{d}t}$$

Radial Functions

$$\nabla f(r) = f'(r)\hat{\mathbf{e}}_r$$

where $r = |\mathbf{r}|$ and $\hat{\mathbf{e}}_r = \mathbf{r}/r$.

Jacobians for Polar Coordinates

In a 3D integral, the rule for changing variables from Cartesian coordinates (x,y,z) to spherical polar coordinates (r,θ,ϕ) is

$$\iiint \cdots\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \iiint \cdots\, r^2\sin\theta\,\mathrm{d}r\,\mathrm{d}\theta\,\mathrm{d}\phi$$

and for cylindrical polar coordinates (ρ,ϕ,z), it is

$$\iiint \cdots\,\mathrm{d}x\,\mathrm{d}y\,\mathrm{d}z = \iiint \cdots\, \rho\,\mathrm{d}\rho\,\mathrm{d}\phi\,\mathrm{d}z.$$

Taylor Series

For small $\delta\mathbf{x}$,

$$f(\mathbf{x}+\delta\mathbf{x}) = f(\mathbf{x}) + \delta\mathbf{x}\,.\,\nabla f(\mathbf{x}) + \cdots.$$