Topic 21 — Interference - Basic phenomena

We have already encountered the phenomenon of interference, in the context of the quarter-wave plate. The interference we met there was *interference by division of amplitude*, that is to say, the beams which interfered were formed from exactly the same original wave by reflecting part of it and transmitting another part. The two bits which later came together to interfere therefore had a well-defined phase relationship.

Young's slits ***FGT1027-1029, AF912-913***

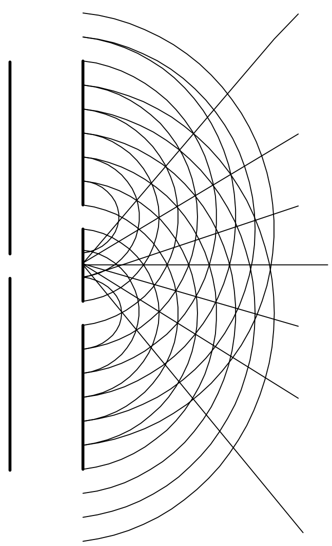

Figure L21.1: The pattern of peaks in waves from Young's slits, showing the directions in which they reinforce each other.

The first interference experiments we will look at, however, require *division of wavefront* to produce two separated sources. The prototype experiment was that of Thomas Young (1803), shown in figure L21.1.

If the light from the source slit arrives in phase at the two slits S_1 and S_2, the phase difference at the screen is given by the path length difference $d_2 - d_1$, as shown in figure L21.2. If the slits are h apart, and very narrow,

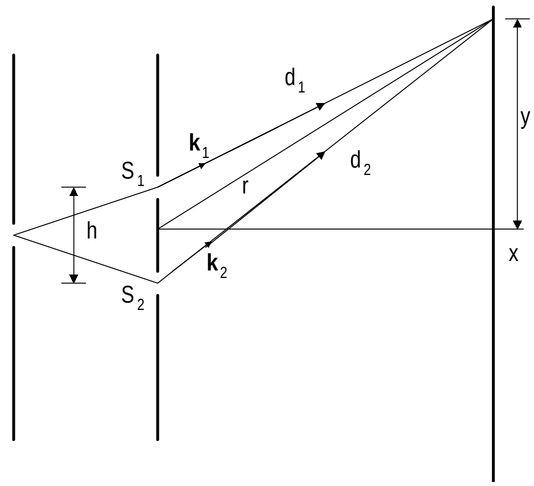

Figure L21.2: The geometry for calculating the path length difference in Young's experiment.

we have

$$d_2 - d_1 = \sqrt{x^2 + \left(y + \frac{h}{2}\right)^2} - \sqrt{x^2 + \left(y - \frac{h}{2}\right)^2}$$

and if x is large compared with y and h we may expand

$$\begin{aligned} d_2 - d_1 &= x\left[1 + \frac{1}{2}\left(\frac{y}{x} + \frac{h}{2x}\right)^2 - 1 - \frac{1}{2}\left(\frac{y}{x} - \frac{h}{2x}\right)^2\right] \\ &= 2x\frac{1}{2}2\frac{y}{x}\frac{h}{2x} \\ &= \frac{yh}{x}. \end{aligned}$$

This will give bright lines if the path lengths differ by an integer number m of wavelengths

$$\frac{yh}{x} = m\lambda,$$

so that the signals add in phase.

We can, of course, think of this in terms of phasors. The largest resultant we can obtain by adding two vectors occurs when the two vectors are pointing in the same direction: that is, the largest sum of two phasors occurs when the phasors are in phase.

We can make this rather more quantitative if we calculate the total electric field on the screen at (x, y). This will be the sum of the signals from the two slits, that is, the two cylindrical waves centred on the slits. If we incorporate in the amplitude factor A both the $1/\sqrt{kr}$ term which is appropriate to the decrease of amplitude with distance from the slit and the amplitude of the wave at the slit. We assume, as discussed before, that we can neglect the variation of amplitude with distance compared with the variation of phase with distance. Then

$$\begin{aligned} E(x,y,t) &= Ae^{i(\omega t - kd_1)} + Ae^{i(\omega t - kd_2)} \\ &= Ae^{i(\omega t - k(d_1+d_2)/2)}\left[e^{-ik(d_1-d_2)/2} + e^{ik(d_1-d_2)/2}\right] \\ &= 2Ae^{i(\omega t - k(d_1+d_2)/2)}\cos(k(d_2-d_1)/2)) \\ &= 2Ae^{i(\omega t - k(d_1+d_2)/2)}\cos(kyh/2x)) \end{aligned}$$

which shows that the *intensity* has a $\cos^2$ variation as shown in figure L21.3, with a bright line at $y = 0$, and the next bright line at

$$\frac{2\pi yh}{2\lambda x} = \pi$$

or

$$\frac{yh}{x} = \lambda.$$

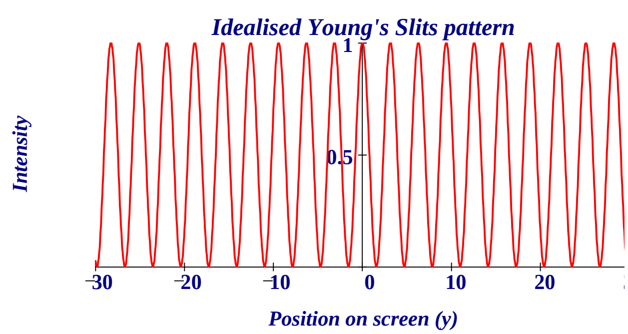

Figure L21.3: The (ideal) pattern of intensities in Young's experiment.

In a typical experiment, we might have $\lambda = 500$ nm, $h = 0.2$ mm, $x = 0.5$ m. Then the separation of the fringes will be

$$\Delta y = \frac{x\lambda}{h} = \frac{.5 \times 500 \times 10^{-9}}{0.2 \times 10^{-3}} = 1.25 \text{ mm}.$$

Visibility of fringes

We may define the *visibility* of fringes, the contrast between the maxima and the minima, by

$$\boxed{V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}.}$$

In the case we have just discussed, in which the two slits gave rise to signals of equal amplitude, the visibility is 1, the maximum possible.

Suppose, though, the two slits let through light with *different* amplitudes, A and B say, so that

$$\begin{aligned} E(x,y,t) &= e^{i(\omega t - k(d_1+d_2)/2)}\left[Ae^{-ik(d_1-d_2)/2} + Be^{ik(d_1-d_2)/2}\right] \\ &= e^{i(\omega t - k(d_1+d_2)/2)}\left[(A+B)\cos(k(d_1-d_2)/2) - i(A-B)\sin(k(d_1-d_2)/2)\right] \end{aligned}$$

Now the intensity is proportional to $|E|^2$, and

$$\begin{aligned} |E|^2 &= (A+B)^2[\cos(k(d_1-d_2)/2)]^2 + (A-B)^2[\sin(k(d_1-d_2)/2)]^2 \\ &= A^2 + B^2 + 2AB\{[\cos(k(d_1-d_2)/2)]^2 - [\sin(k(d_1-d_2)/2)]^2\} \\ &= A^2 + B^2 + 2AB\cos(k(d_1-d_2)). \end{aligned}$$

The maximum value of this clearly occurs when the cosine equals 1, the minimum when the cosine equals -1, and so

$$\begin{aligned} V &= \frac{(A+B)^2 - (A-B)^2}{(A+B)^2 + (A-B)^2} \\ &= \frac{4AB}{2(A^2+B^2)} \\ &= \frac{4A/B}{2(1+A^2/B^2)}. \end{aligned}$$

This expression is 1 when $A = B$, as we expect from our previous result, and the visibility of the fringes will be less than 1 if A and B are different.

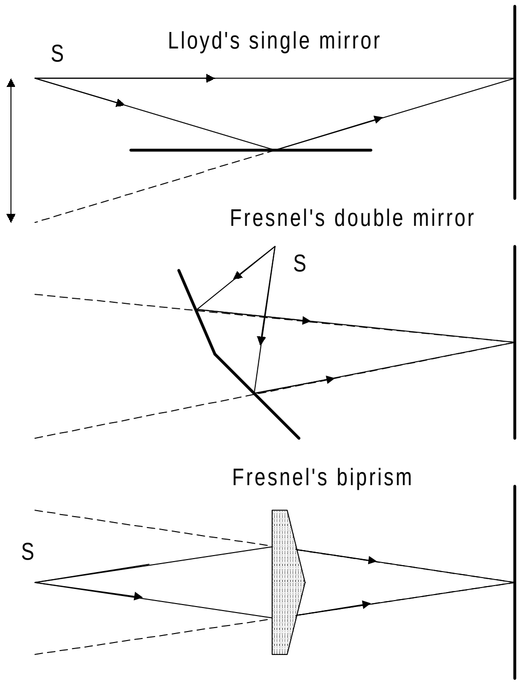

Figure L21.4: Alternative methods of dividing the wave-front for interference experiments.

Other ways of splitting the wavefront

Slits are not the only way in which two closely-spaced sources can be formed. The same thing can be done using three other arrangements (see figure L21.4). In all cases, though, different spatial regions of the original wavefront are sent by different paths and allowed to interfere.

Modifications of the Young's slits arrangement

If the space between the slits and the screen is filled with, instead of air, a material with refractive index n, the wavelength will be reduced to λ/n, and the spacing of the fringes will thus be *reduced* to

$$\Delta y = \frac{x\lambda}{hn}.$$

If a slip of transparent material of thickness t, refractive index n, is placed over one slit, that will increase the optical path length in that path by $(n-1)t$. As a result the fringes will be shifted, the fringe which would have appeared at y now appearing (if the slip is placed over the lower slit) at $y-(n-1)tx/h$.

L21.1 Coherence *FGT409, FGT414, AF911*

interference of two coherent sources *FGT415-416, AF909- 911*

If the sources in Young's experiment were not slits illuminated by light, but dipole aerials driven by sinusoidal electric signals, there would be no question about what the fields would look like. For each dipole it would have a spatial distribution characteristic of the dipole and a time variation consisting of on continuous single-frequency wave. For light, however, from say a gas discharge tube each atom will send out a pulse, and another atom will fire at a time that is not correlated (unless we arrange this specially, as in a laser).

wave fronts *AF937*

The spatial distribution of atoms will mean that there is also an spatial distribution of phase - a wavefront is only approximately flat.

The degree to which a wave approximates to the ideal of a plane infinite wave train is described by its *coherence.*

Coherence is measured as

- *temporal coherence*, measuring the time for which the waves passing a particular point maintain a clear phase relationship. This may also be quoted as a *longitudinal coherence length*, the product of coherence time with wave speed.
- *spatial coherence*, measuring the distance across the wavefront which one can go without substantial drift of phase, also quoted as *transverse* or *lateral coherence length.*

Typical longitudinal coherence lengths

- white light – a few wavelengths
- discharge tubes (such as street lamps) – few millimetres
- low-pressure discharge tubes, e.g. ^{86}Kr lamp – few metres

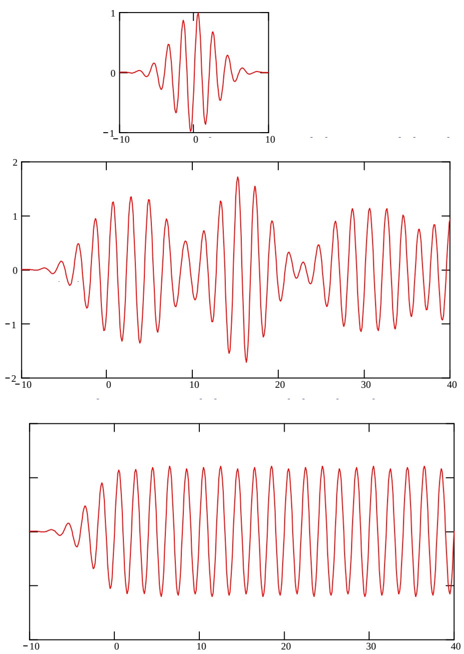

Figure L21.5: In light from, for example, a sodium flame, each atom sends out a short pulse of light (top). If all the atoms radiated 'in step' the result would be as shown in the middle graph. In reality, the atoms 'fire' at random, giving the irregular wave shown at the bottom.

- laser – kilometres readily achievable.

practical coherent sources

As a result of temporal coherence limits, if we look at large path differences we begin to lose the interference. In Young's experiment, this gives a reduction in fringe visibility as we move away from the straight-through direction.

If we use large sources, the spatial coherence limits the level of interference.

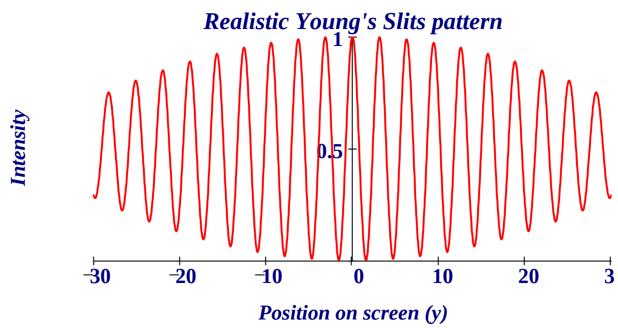

Figure L21.6: The fading of the fringe pattern at larer angles in Young's experiment, as a result of the finite coherence length of the light.