Topic 20 — Light: Some Basic Properties

The early history of light and vision makes for fascinating reading. I can firmly recommend David Park's *The Fire within the Eye* (Princeton University Press, 1997). Some of the early ideas were intriguing. According to Leucippus of Miletus (one of whose students, Democritus, became more famous than his mentor) light caused the surface of any visible object to peel off continuously in thin sheets. These sheets (eidola) keep their shape but alter their size as they fly off in all directions. How the eidolon manages to end up just the right size to fit through the pupil of our eye we will leave as a mystery.

Euclid produced (of course!) something more like a geometrical theory of light — but he still imagined, like Empedocles and Aristotle before him, that light started from the eye and bounced of the object being looked at. What is very significant is that Diocles, a century after Euclid, used geometric reasoning to show that a paraboloidal mirror has the right shape for focusing the sun's rays to a point (see later in these notes, where aberration of spherical reflecting surfaces is discussed).

The idea of the 'visual ray' emanating *from* the eye held sway at least up to the time of Abu Ali al-Hasan ibn al-Haitham (known as Alhazen) late in the tenth century CE. He studied the formation of images in detail: for example, in his *The Shape of the Eclipse* he explains the fact that the spots of light on the ground under a tree, normally circular, become crescent-shaped during an eclipse.

L20.1 The nature of light and radio waves *FGT941, AF782-787*

A full description of the electromagnetic field nature of light is best deferred until the differential forms of Maxwell's laws are encountered, in the Second Year. All we give here is a very basic summary, **but note that none of this electromagnetic material is required for the 1B24 examination**.

You have already encountered (in the First Year Electromagnetism course) the integral forms of Maxwell's equations in free space:

$$\oint_c \mathbf{E}.\mathbf{dl} = -\mu_0 \int\int_S \frac{\partial \mathbf{H}}{\partial t}.\mathrm{d}\mathbf{S}$$

$$\oint_c \mathbf{H}.\mathbf{dl} = \epsilon_0 \int\int_S \frac{\partial \mathbf{E}}{\partial t}.\mathrm{d}\mathbf{S}$$

$$\int\int_C \mathbf{H}.\mathrm{d}\mathbf{S} = 0$$
$$\int\int_C \mathbf{E}.\mathrm{d}\mathbf{S} = 0$$

where the subscripts c and C denote closed contours and surfaces respectively.

It is these equations which govern the behaviour of electromagnetic waves. The corresponding differential forms are

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathbf{H}}{\partial t}$$
$$\nabla \times \mathbf{H} = \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$$
$$\nabla . \mathbf{H} = 0$$
$$\nabla . \mathbf{E} = 0.$$

perpendicular E and H

These equations can be satisfied by an interdependent pair of electric and magnetic fields of the form

$$\mathbf{E} = \mathbf{E}_0 e^{i(\omega t - \mathbf{k}.\mathbf{x})}$$
$$\mathbf{H} = \mathbf{H}_0 e^{i(\omega t - \mathbf{k}.\mathbf{x})}$$

provided that the electric field amplitude $\mathbf{E}_0$, the magnetic field amplitude $\mathbf{H}_0$ and the wavevector $\mathbf{k}$ are mutually orthogonal (see figure L20.1). That is, electromagnetic waves are *transverse waves.*

Figure L20.1: An electromagnetic wave, made up of interdependent electric and magnetic fields.

The velocity at which the waves travel in a vacuum[1] is given by

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}},$$

and the ratio of the electric field to the magnetic field is

$$\frac{E_0}{H_0} = \sqrt{\frac{\mu_0}{\epsilon_0}} \equiv Z_0,$$

the impedance of free space.

In a medium, the velocity (phase velocity) is modified[2] to

$$c = \frac{1}{\sqrt{\mu_r \mu_0 \epsilon_0 \epsilon_r}} = \frac{c_0}{n},$$

which defines the refractive index n in terms of the relative permittivity (dielectric constant) ϵ_r and relative permeability (which is usually close to unity) μ_r. The impedance in a medium is modified to

$$Z = \frac{Z_0}{n}.$$

[1]The first person to publish a value for the speed of light was Christiaan Huygens, in his *Treatise on Light* (1690). His value relied on estimates of three quantities: a) the time taken for light to pass from the Earth to the sun, which Ole Römer (1644-1710) estimated, in about 1671, at 11 minutes. As the great Descartes had previously declared that light travelled from the Sun to the Earth "in an instant", Römers value was not universally accepted. b) The diameter of the earth, quite accurately determined in 1669 as 7920 miles. c) The ratio of the distance to the sun to the diameter of the earth, estimated by simultaneous measurements by Cassini in Paris and Jean Richer in Cayenne, Guyana, to be 11000. Römer noted that the time taken by Io to swing round Jupiter, measured by the interval between eclipses by Jupiter, varied by 22 minutes between closest and furthest Earth-Jupiter distances. Rø"mer was measuring the orbital period plus the time taken for the light to travel from Io.

[2]The first publication of the law of refraction was by René Descartes (1596-1650)in his *Dioptrics* of 1637. The reasoning is false, relying on a treatment of light as a stream of particles (in his book, balls projected by a little man with a racket!) which travel faster in the optically more dense medium, but the result is correct. Descartes pushed his tennis ball analogy even further by claiming that the spin of the ball is responsible for colour. When he came to deal with lenses Descartes was more reliable, and he made good use of his exceptional skills in geometry, but he achieved no more than Ibn Sahl 650 years earlier.

polarization

Being a transverse wave, electromagnetic radiation exhibits *polarisation*: as for mechanical transverse waves, such polarisation may be plane, circular, or elliptical[3].

L20.2 Dipole radiator *FGT948-950, AF791*

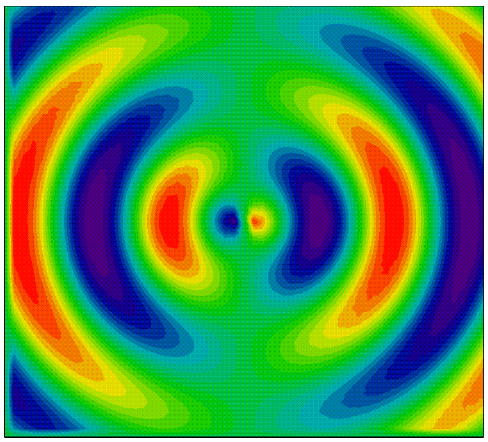

Figure L20.2: The radiation pattern from a dipole source: the dipole is pointing up the page, and does not radiate along that direction. In the diagram, red represents positive field, blue represents negative field, and green is zero.

The fundamental radiator of electromagnetic radiation is the *oscillating dipole*: basically an aerial the ends of which are made alternatively positive and negative. The radiation from a dipole is not isotropic (which is to be expected - we know that the static field of a dipole is not spherically symmetrical): figure L20.2 illustrates the radiation pattern.

[3]The problem of the polarisation of light took a long time to solve. Newton suggested, very vaguely, that rays of light had "sides". Thomas Young was close to the mark in an article in the 1817 *Encyclopedia Britannica* when he suggested that light, though he thought of it as a longitudinal motion like sound, might also have some sideways character similar to the motion of a shaken rope. Fresnel went the whole way, and suggested that waves of light are entirely transverse.

L20 Reflection and Refraction

So far we have concentrated on phenomena which depend on the wave-like nature of light. When we come to design optical instruments such as microscopes and telescopes, however, it is convenient to make the approximation that, unless reflected and refracted, light travels in straight lines. This leads us to draw *ray diagrams* to show the path of light through the system. These, and the basic properties of rays which are briefly mentioned here, are assumed to have been covered in school physics courses, and the diagrams which accompany this topic are intended merely as reminders.

A ray is best thought of as the normal to the local wave front, and in calculating the behaviour of each ray it is assumed that the local wavefront is part of an infinite plane wave front parallel to the local wave front. Thus we ignore diffraction and interference – or at best add them in at the end.

The phase of a wave changes with distance along the ray. Each wavelength along the ray corresponds to a phase change of 2π: but the relevant wavelength is the wavelength *in the medium.* The phase varies as

$$\text{phase} = kx = \frac{\omega}{c}x = \frac{\omega n}{c_0}x = \frac{\omega}{c_0}nx = k_0 nx,$$

that is, we can handle all the phase changes in terms of the wavevector in free space, k_0, provided that we scale the distance in each medium by the refractive index. This leads to the concept of

> optical path length: the sum of the geometrical path lengths with the geometric path in each medium multiplied by the refractive index of that medium.

L20.1 Reflection and refraction ***AF858-861***

In fact we can form a link between rays and waves through *Fermat's principle*[4], which states that

> a light ray passing from a point S to a point P must traverse an optical path length that is stationary with respect to variations of that path.

[4]Pierre Fermat (1601-1665) corresponded with Descartes about the contents of the latter's *Dioptrics.* His comments were not always appreciated, and in a private letter to a friend Descartes described one of Fermat's criticisms as *stercus.* He first wrote down his famous principle in about 1662.

In other words, the first derivative of the path length with respect to the parameters defining the path is a minimum (Fermat's original statement) or a maximum[5]. Why does this work? If the path is an extremum, then the ray path and neighbouring paths all have, to a first approximation, the same length – thus the signals all add in phase. This tells us that the ray path corresponds to a maximum amplitude in the context of the Huygens-Fresnel principle (see lecture 24). The link that is made here with the Huygens-Fresnel principle is significant — from that principle we have the picture that light can travel by almost *any* route from one point to another, because each point on a primary wavefront makes its own contribution to every point on the secondary wavefront. Here we see that it is the *stationary* paths that are most important. This has its parallel in quantum mechanics, in Feynman's path integral formulation of the subject.

This immediately gives us the result that in a medium in which the refractive index is constant the paths will be straight lines.

laws of reflection ***AF858***

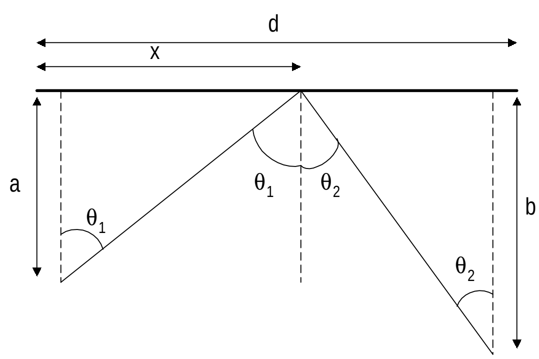

Figure L20.3: The application of Fermat's principle to reflection from a plane surface.

[5]Actually, Fermat's principle was known in the first century CE, by Hero of Alexandria. His statement is based on the false premise that light starts from the eye, and his deduction is not sound, but it is worth quoting: "Thus it follows that the ray is emitted with infinite speed. And therefore it moves without interruption, it makes no detours, but travels along the shortest way, which is a straight line."

Let's use this to look at reflection and refraction. For reflection as in figure L20.3 from a plane mirror, source a in front, detector b in front, a distance d apart parallel to the mirror, with the ray reflected from a point x, the path length is

$$\begin{aligned} l &= \sqrt{a^2 + x^2} + \sqrt{b^2 + (d-x)^2} \\ \frac{\mathrm{d}l}{\mathrm{d}x} &= \frac{x}{\sqrt{a^2+x^2}} - \frac{d-x}{\sqrt{b^2+(d-x)^2}} \\ &= \sin(\theta_1) - \sin(\theta_2) \end{aligned}$$

which is zero if $\theta_1 = \theta_2$ – that is,

the angle of incidence is equal to the angle of reflection.

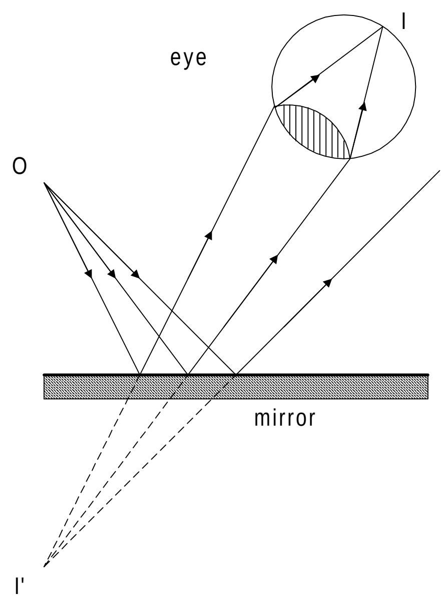

Figure L20.4: The image in a plane mirror is a *virtual* image, as the rays do not pass through the image.

Note that the image formed in a mirror is a *virtual* image: that is, the rays themselves do not pass through the image, only the extensions of the rays. This is shown in figure L20.4.

Laws, of course, tempt people to break them. Artists, in particular, enjoy taking liberties with the laws of reflection. Magritte, in *La Reproduction Interdite*, shows a blatantly impossible reflection. Manet's *Bar at the Folies Bergeres* is more subtle: the reflections of the back of the barmaid's head and the face of the artist are displaced to one side: in their correct positions they would have been invisible. Van Eyck's *Arnolfini Marriage*, on the other hand, uses the laws of reflection to great advantage, to show a complete image not just of the other side of the scene in the main painting but of two onlookers not otherwise seen.

Snel's law *AF859*

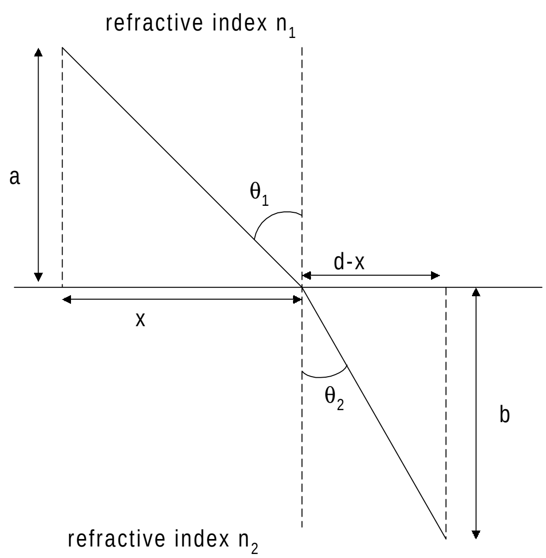

Figure L20.5: The propagation of a general wave pulse.

For refraction, as shown in figure L20.5, the path lengths must incorporate the different refractive indexes:

$$
\begin{aligned}
l &= n_1\sqrt{a^2+x^2} + n_2\sqrt{b^2+(d-x)^2} \\
\frac{\mathrm{d}l}{\mathrm{d}x} &= n_1\frac{x}{\sqrt{a^2+x^2}} - n_2\frac{d-x}{\sqrt{b^2+(d-x)^2}} \\
&= n_1\sin(\theta_1) - n_2\sin(\theta_2)
\end{aligned}
$$

and this is zero provided

$$n_1 \sin(\theta_1) = n_2 \sin(\theta_2)$$

which is Snel's[6] law.

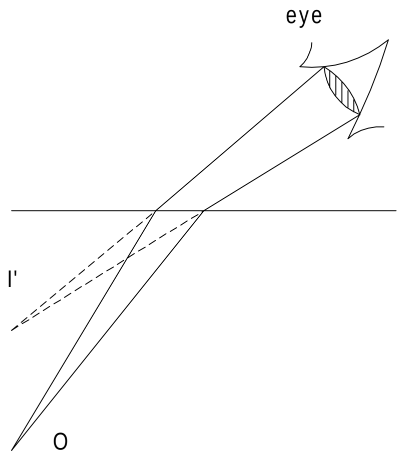

Figure L20.6: The image formed by refraction at a plane surface..

Snel's law[7] tells us that light is bent *towards* the normal when light passes from an optically less dense medium to a more dense medium. Examples of refraction include looking at objects under water (figure L20.6), mirages (figure L20.7, and shifts of star positions as a result of variations of density in the atmosphere (the regular variation with height, as shown schematically in figure L20.8, as distinct from the turbulent motion of the atmosphere which

[7]The law of refraction has an interesting history. Ptolemy, in the second century CE, showed a careful analysis of refraction in Book V of his *Optics*. His measurements were not quite accurate enough for him to discover Snel's law. Snel's law must have been known to the mathematician Ibn Sahl in Baghdad in 984, who studied the shapes of lenses. Robert Grosseteste (c1170-1253) tried to explain the focusing of the Sun's rays by a flask full of water by assuming that the angle of refraction was always half the angle of incidence. Kepler produced a complicated and inaccurate formula. The correct law is credited to Snel although he never actually published it and his contribution is known only through correspondence. There is some evidence that Thomas Harriot (c1560-1621) knew the law before, but his thousands of pages of notes never led to any publications. Harriot asked himself whether the refractive index of materials depended on the colour of the light. He showed by experiment that it did, but was not able to use this information whereas Newton, sixty years later, was.

gives rise to small cells of different density and causes stars to twinkle).

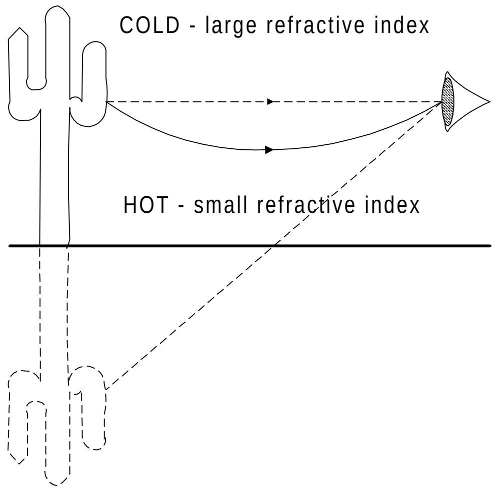

Figure L20.7: The mirage effect, formed by refraction at a layer of hot, less dense air.

phase change on reflection ***AF864***

A full treatment of the reflection and refraction of light needs electromagnetic theory. This will tell us

- the reflection and transmission coefficients vary with angle
- the reflection and transmission coefficients depend on polarization
- the phase shift can also vary with angle, but for near-normal incidence (in the cases we'll be looking at, angles less than about 30 degrees) is zero for reflection from a less optically dense material, π for reflection from a more optically dense material (a result we have already deduced, using the formula for the reflection coefficient $r = (n_1 - n_2)/(n_1 + n_2)$).

For normal incidence, we have reflection and transmission coefficients which do not depend on polarisation, and

$$r = \frac{n_1 - n_2}{n_1 + n_2}$$

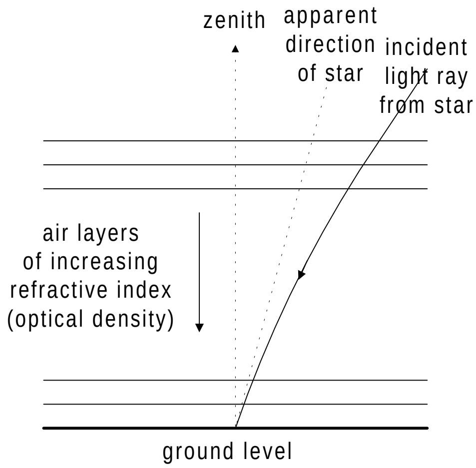

Figure L20.8: The effect of atmospheric density variation on the apparent positions of stars.

$$t = \frac{2n_1}{n_1 + n_2}.$$

Note that this means that there is a phase change of π at a reflection from an interface with an optically more dense medium (one with a larger refractive index), of zero on reflection from an optically less dense medium.

polarised light - Brewster's law *AF865*

The amplitude of reflection depends on polarisation[8] an it turns out that at one angle, known as the Brewster angle or the Polarisation angle, the reflection coefficient for light with the electric vector *parallel* to the plane of incidence falls to zero.

This happens for $\tan\theta_1 = n_2/n_1$.

At any angle, though, the reflected light will be partly polarised, with more light reflected with electric vector perpendicular to the plane of incidence than parallel to the plane of incidence.

[8]This needs a full electromagnetic treatment for its derivation: for the moment we just have to accept it.

Polaroid and similar materials ***AF867***

This polarisation of reflected light is what makes polaroid sunglasses work. Polaroid, in one of its forms (the form that does not involve dropping iodine into the urine of a dog that had been fed quinine - see Hecht page 281!) is effectively a sheet containing parallel conducting long chain molecules, which will let through light with electric vector perpendicular to the molecules whilst light parallel to the chains does work on the electrons in the chains leading to absorption.

Polaroid sunglasses cut down reflected glare. You can check this by turning your lenses through 90 degrees - the reduction of glare is lost. As usual, nature has got there first – the pond skater *Gerris lacustris* has eyes that are only sensitive to vertically polarised light, so that it is insensitive to glare (O. Trujillo-Cénoz (1972) The structural organization of the compound eye in insects, in M.F.G. Fuortes, ed., *Physiology of Photoreceptor Organs* pp 5-62, Springer-Verlag). Water beetles, on the other hand, are often deceived by the light reflected from, for example, well polished car bonnets into thinking they have found ponds – they use the polarisation of the light as a signature of reflection.

total internal reflection

Note from Snel's law that if

$$\sin(\theta_2) = \frac{n_1}{n_2}\sin(\theta_1) > 1$$

then θ_2) is not a real angle. No energy propagates away from the interface on the transmitted side – the wave is totally internally reflected. As an example, for light in glass with refractive index 1.4 encountering an interface with air there is total internal reflection if the angle of incidence exceeds 45.6 degrees.

To be more precise, there is a disturbance beyond the interface with an amplitude that decays exponentially with distance. If one can put a second interface close enough to the first to pick up an appreciable signal in the exponential tail some light will then couple into the second slab. This is a phenomenon similar to quantum mechanical tunneling: in fact, tunneling is a wave phenomenon, not a quantum phenomenon.

Note that this is evidence for the fact that when we are dealing with waves things do not change abruptly - if we try to stop a wave by total internal reflection it still carries making its presence felt for a distance of the order of a wavelength.