Topic 19 — The Doppler Effect: Moving sources and Receivers

The phenomena which occur when a source of sound is in motion are well known. The example which is usually cited is the change in pitch of the engine of a moving vehicle as it approaches. In our treatment we shall not specify the type of wave motion involved, and our results will be applicable to sound or to light.

The treatment we adopt assumes that the wave is propagated in some medium, with respect to which the source, or the observer, or both, are moving. This will introduce an asymmetry between the source and the observer. Although we shall use this same formalism for light this is an approximation, as we know from the principle of relativity that there is no preferred frame of reference (light does not need a medium in which to propagate), and that all that matters is the *relative* motion of the source and the observer. Nevertheless, as long as the velocities involved are not too close to light speeds, our result will be quite accurate for light.

L19.1 Doppler effect *FGT394-398, AF775-778, AF799-802*

Suppose that a source is moving with velocity v_s towards an observer in a medium in which the wave speed is c. If the frequency of the source is f, in a time t it will have emitted ft waves, but in the region in front of the source these will have been emitted into a distance $(c - v_s)t$. Thus the wavelength is

$$\lambda' = \frac{(c - v_s)t}{ft}$$

or

$$\lambda' = \frac{(c - v_s)}{f}.$$

In a time t a stationary observer will receive the waves in a distance ct, which will be ct/λ' wavelengths, so the observed frequency will be

$$f' = fc/(c - v_s)$$

when the source moves towards the observer at a speed v_s.

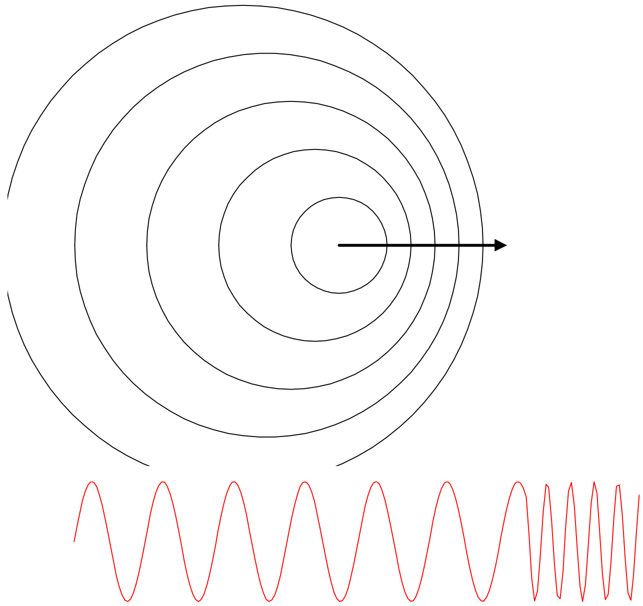

Figure L19.1: The waves spreading out from a moving source, showing the wavelengths compressed in front of the source, extended behind it.

Note that c here is the speed of the wave in the medium in which the source is moving.

Now suppose that the observer is moving with a velocity v_o away from the source[1]. We can tackle this case directly in the same way as we treated the moving source. If the observer moves with a speed v_o away from the source (note the sign convention — both source and observer speeds have been measured along the direction of travel of the signal, from the source to the observer), then in a time t the number of waves which reach the observer are those in a distance $ct - v_o t$, so the number of waves observed is $(ct - v_o t)/\lambda$,

[1]We can make this the same as the previous case if we superimpose on the whole system a velocity $-v_o$, which will give the source a velocity $-v_o$, the waves a velocity $c - v_o$, and bring the observer to rest, so that now

$$f' = f\frac{(c - v_o)}{c}.$$

giving an observed frequency

$$f' = f\frac{(c - v_o)}{c}$$

when the observer is moving away from the source at a speed v_o.

It is clear that we can calculate the wavelength set up by the moving source, and then the effect this has at a moving observer, so

when both the source and the observer are moving

$$f' = f(c - v_o)/(c - v_s),$$

taking a positive velocity as being along the source-observer distance.

Doppler effect in acoustics

For example, consider a train hooter of frequency 150 Hz on an intercity train travelling at 120km/h. If the train is approaching, the frequency which is heard will be

$$f' = 150 \times 330/(330 - 120 \times 10^3/60^2) = 167 \text{ Hz}.$$

What if you, the observer, were moving towards a stationary train?

$$f' = 150 \times (330 + 120 \times 10^3/60^2)/330 = 165 \text{ Hz},$$

showing that there is a slight lack of symmetry, when the velocity is 10 percent of the sound velocity.

Doppler effect in astronomy *FGT1090, AF799-800*

In astronomy, red shifts are central in determining the motion of objects, either the rate of recession of remote galaxies or the rotation of binary stars.

Doppler effect and line widths *AF802*

The velocity of atoms in hot gases gives Doppler broadening of the spectral lines of atomic emission.

Relativistically correct result for light

When relativistic effects are included, it is only the relative velocity of approach v which matters, leading to an observed frequency

$$f' = f\sqrt{\frac{1 + v/c}{1 - v/c}}.$$

It is an interesting exercise to show that the relativistic and non-relativistic expressions for f'/f only differ by terms involving the square of v/c. As far as radar speed traps go, the classical expression suffices!