Topic 28 — The Michelson Interferometer

In the systems we have looked at so far which involve interference by division of amplitude the path length differences where fixed by the geometry of the system. In Michelson's[1] interferometer we have control over the geometry.

Note that in interference by division of amplitude we do not need a spatially coherent wave, as it is the amplitude at one point which is split and sent by different paths to interfere, not that from different points on the front.

L28.1 Michelson interferometer *FGT1038, AF484, H354*

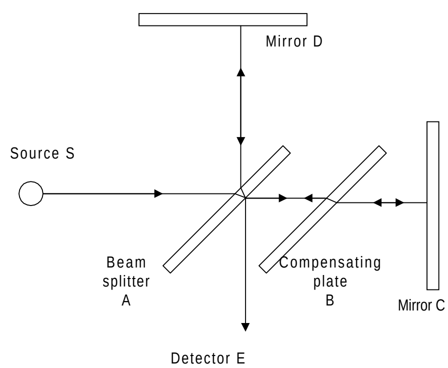

Figure L28.1: The Michelson interferometer: see the text for a detailed description.

The arrangement is very simple (see figure L28.1): a source (which may be extended) is divided by a partly-silvered mirror, travels to two mirrors, and is recombined again by the beam-splitter. Any path length difference gives rise to interference. One mirror (C in the diagram) can be moved perpendicular to its own plane: it can also be tilted.

[1]Albert Michelson was born in Strzelno, Poland, on December 19th 1852. The family left Poland to escape anti-Semitism, worked across Europe, steamed to New York, and joined the gold rush to California. Travelling via Panama, they escaped the lawless, malaria and small-pox ridden city of Porto Bello by canoe through the swamps and later by mule. Finally they reached San Francisco, and Albert's father set up a store in a mining town. Albert himself eventually went to Naval College, where he came top in optics and 25th in seamanship. The Navy then sent him to sea for two years!

compensating plate for white light *H354- 355*

One beam passes through the beam-splitter three times, the other only once. This means that there will be a path length difference which will be wavelength dependent if the glass is dispersive. We can eliminate this by using a compensator plate, which is the same as the beam-splitter but unsilvered. Obviously this is less important for a source which emits a narrow range of wavelengths than for, say, a white light source.

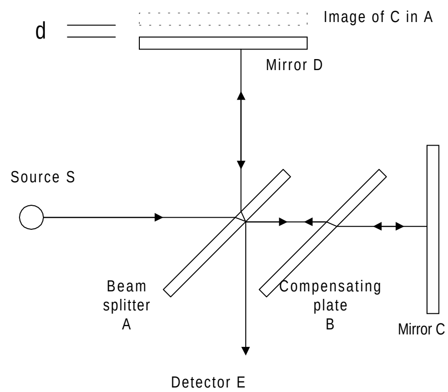

Figure L28.2: The relative positions of the mirrors in the Michelson interferometer are best thought of by thinking of the relative position of the mirrors in terms of the image of the movable mirror in the beam splitter.

The easiest way of thinking about the system is to think of the beam splitter acting as a mirror, so that one of the interfering signals comes straight through the beam-splitter from mirror D, one comes from the image of mirror C in the beam-splitter (see figure L28.2), and the mirror and the image are separated by a distance d.

circular fringes *H356*

Now suppose we look at an angle θ to the normal, and the mirror D and the image of C are d apart. The path difference is then (as proved in the context of non-normal incidence on a film in Lecture 26)

$$\Delta = 2d\cos(\theta)$$

and[2]

there will be dark rings (because of the extra phase shift in one path) at angles which satisfy $$\cos(\theta) = p\frac{\lambda}{2d}$$ where p is an integer.

If we arrange for the amplitudes of the signals travelling along the two paths to be equal, then the total amplitude may be written as

$$E_{\rm tot} = E_0\left(1 - e^{ik\Delta}\right),$$

the relative sign being negative because although the light along each path has been reflected once from a mirror, at the beam-splitter one beam was internally reflected at the part-silvered back surface and one was externally reflected. There will thus be a phase difference of π between them. In a similar manner to the intensity in Young's slits we write

$$E_{\rm tot} = E_0 e^{ik\Delta/2}\left(e^{-ik\Delta/2} - e^{ik\Delta/2}\right),$$

so that the intensity distribution will be

$$\begin{aligned} I(\theta) &= I(0)\sin^2\left(kd\cos(\theta)\right) \\ &= I(0)\sin^2\left(\frac{2\pi}{\lambda}d\cos(\theta)\right). \end{aligned}$$

[2]Note that if there is a dark central fringe, with $p_0 = 2d/\lambda$, then this p_0 is likely to be *large* (for example, if d is 100 mm and λ is 500 nm, p_0 will be 400,000). It may be more convenient to count rings in order away from the centre, by

$$\begin{aligned} 2d\cos(\theta_1) &= (p_0 - 1)\lambda \\ 2d\cos(\theta_2) &= (p_0 - 2)\lambda \\ .... & \quad .. \\ 2d\cos(\theta_m) &= (p_0 - m)\lambda \end{aligned}$$

and then if we write

$$\begin{aligned} m\lambda &= 2d(1 - \cos(\theta_m)) \\ &\approx 2d(1 - (1 - \frac{1}{2}\theta_m^2 + ...)) \\ \theta_m &= \sqrt{\frac{m\lambda}{d}}. \end{aligned}$$

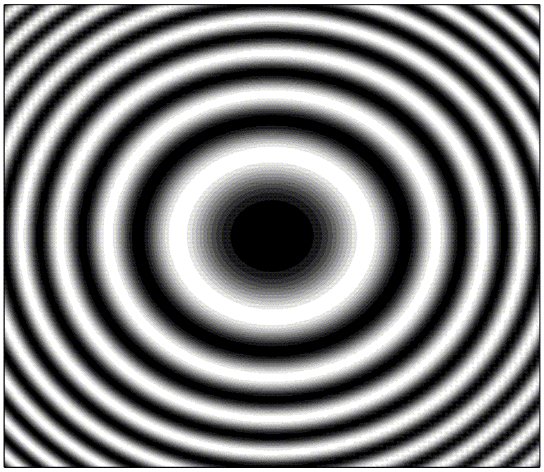

Figure L28.3: The pattern of circular fringes produced in the Michelson interferometer with both mirrors perpendicular to the beams and with a spacing of 100 wavelengths.

If we alter the spacing between the mirrors, so as to increase the spacing, as d gets larger for a given fringe θ must get larger – that is a ring will appear from the centre and expand. One fringe appears for each movement of $\lambda/2$ of the mirror.

measurement of refractive index

Any transparent object inserted in one arm of the interferometer, with refractive index n and filling a length t of the arm, will alter the optical length of that arm by $(n - n_{\text{air}})\,t$. As the material will be traversed twice by the light, this changes the optical path length by $2\,(n - n_{\text{air}})\,t$, causing the fringe pattern to shift by

$$\frac{2\,(n - n_{\text{air}})\,t}{\lambda} \text{ fringes.}$$

This can be used to measure, for example, the refractive indexes of gases.

straight fringes *H357*

If we set one mirror at a very slight angle, we are back to the same situation as the wedge. Fringes will be seen at positions which correspond to increments of $\lambda/2$ in the wedge thickness.

If we use white light with the mirror at the same distance but slightly angled, we get coloured fringes, fading to white as the different coloured fringes systems overlap.

Doublet source

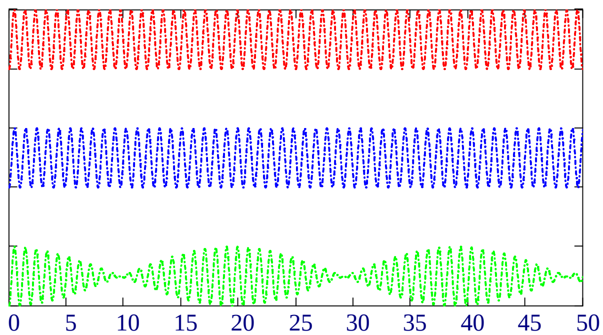

Figure L28.4: The variation of fringe intensity with distance for (top) a wavelength λ, (centre) a wavelength of 1.05λ and (bottom) a doublet source containing both wavelengths.

If the light comes from a source with two spectral lines close together (sodium is the typical example) each line will have a similar but not identical fringe system. If $2d\cos(\theta)$ is an integer number of wavelengths for *both* lines, the fringes will coincide and the visibility will be good. As the mirror is moved, the fringe visibility will vary, as shown in figure L28.4.

When the visibility of the fringes is good, and $\cos(\theta) \approx 1$,

$$2d = p_1\lambda_1 = p_2\lambda_2$$

or

$$p_1 - p_2 = 2d\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right).$$

Of course, this will happen for *large* values of p if the two wavelengths are close together.

If d is changed to $d + \Delta d$ to reach the next peak in visibility, we must have changed $p_1 - p_2$ by 1 (see the close-up in figure L28.5), so

$$p_1 - p_2 + 1 = 2(d + \Delta d)\left(\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right).$$

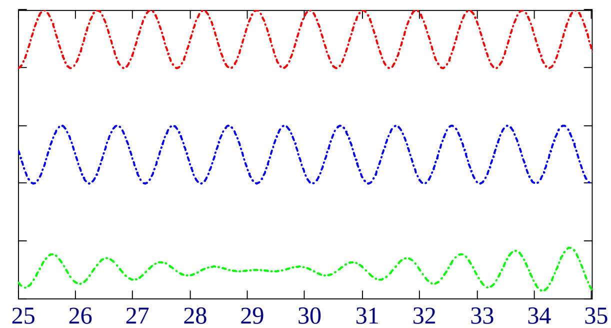

Figure L28.5: Close-up of the variation of fringe intensity with distance for (top) a wavelength λ, (centre) a wavelength of 1.05λ and (bottom) a doublet source containing both wavelengths.

Subtracting and rearranging

$$\Delta\lambda = \lambda_2 - \lambda_1 = \frac{\lambda_1\lambda_2}{2\Delta d} \approx \frac{\lambda^2}{2\Delta d}.$$

Thus by counting the number of variations of visibility of the fringes in the Michelson interferometer as the mirror is moved, to get an average value of Δd, we can measure the splitting of a doublet spectral line.

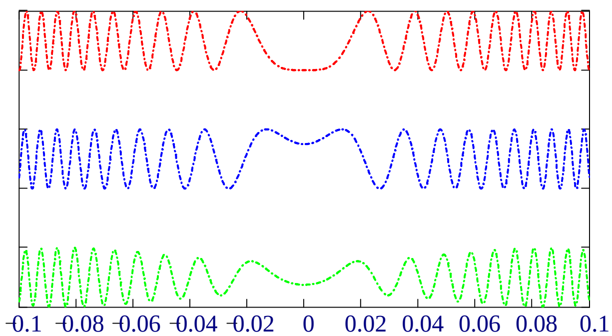

Figure L28.6: The variation of fringe intensity with angle for (top) a wavelength λ, (centre) a wavelength of 1.05λ and (bottom) a doublet source containing both wavelengths, at a mirror spacing of 1000 wavelengths.

Figures L28.6 and L28.7 show the variation with angle of the same lines and the doublet, as the spacing d is changed from 1000 wavelengths to 1000.1 wavelengths.

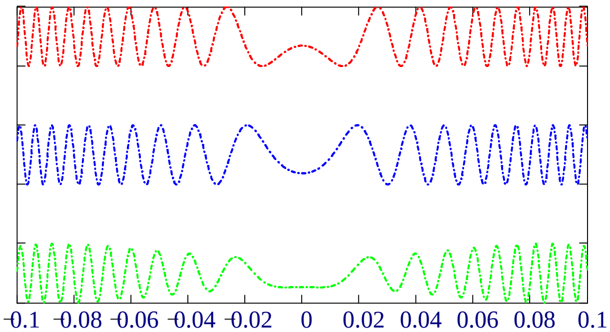

Figure L28.7: The variation of fringe intensity with angle for (top) a wavelength λ, (centre) a wavelength of 1.05λ and (bottom) a doublet source containing both wavelengths, at a mirror spacing of 1000.1 wavelengths.

stellar interferometer - sizes of stars *H530-532*(for interest only)

An important feature of light from sources of finite size is the degree of spatial coherence of the light. It is fairly clear (and the van Cittert - Zernicke theorem is a formal proof) that in the central maximum of the diffraction pattern all the signals are adding up in phase, so the width of the central peak is a measure of the spatial coherence.

Now the angular half-width of the diffraction pattern from a circular source is (Rayleigh's criterion again)

$$\theta = 1.22\frac{\lambda}{d}$$

where d is the source diameter.

In other words, if we take light from such a source at angles more than about θ apart, we expect to lose coherence and not to be able to make interference patterns.

This is what the Michelson stellar interferometer does. The mirror separation is increased until the fringes are no longer visible: this gives the angular diameter of the source, or, if we know its distance, its actual diameter.

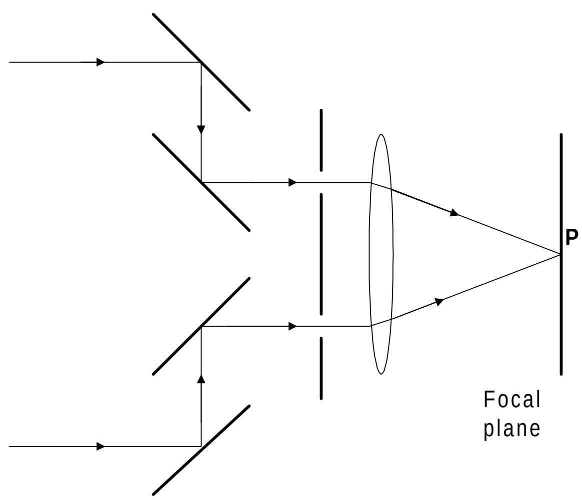

Figure L28.8: Michelson's stellar interferometer: the mirrors are adjustable so as to vary the aperture with which a distant star is viewed.