UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For the following qualifications :-

M.Sci.

Astronomy 4C16: Advanced Topics in Stellar Atmospheres and Evolution

COURSE CODE	:	**ASTR4C16**
UNIT VALUE	:	**0.50**
DATE	:	**29-APR-02**
TIME	:	**14.30**
TIME ALLOWED	:	**2 hours 30 minutes**

02-C0090-3-40
© *2002 University of London*

TURN OVER

Answer THREE questions

The numbers in square brackets in the right hand margin indicate the provisional allocation of maximum marks per sub–section of a question.

You may assume the following (using standard notation):

Planck Function:

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left(\exp(\frac{hv}{kT}) - 1\right)^{-1}$$

Detailed Balance Relationship for collisional rate coefficients:

$$\frac{C_{12}}{C_{21}} = \frac{g_2}{g_1}\exp\left(-\frac{h\nu}{kT}\right)$$

The complementary function of a 2nd order differential equation:

$$\frac{d^2y}{d^2x} - ax = b$$

is of the form

$$y = Ae^{\alpha x} + Be^{\gamma x}$$

To derive the particular integral of a 2nd order differential equation:

$$\frac{d^2y}{d^2x} - ax = b$$

it is advisable to try an initial solution of the form

$$y = \lambda x + \mu$$

Einstein Relations:

$$A_{21} = \frac{2hv^3}{c^2}B_{21} \qquad g_1 B_{12} = g_2 B_{21}$$

Equation of Mass Continuity:

$$\dot{M}(r) = 4\pi r^2 \rho\, v(r)$$

1. a) Describe, qualitatively, the physical processes that operate in line-driven stellar winds? Which lines, and under what conditions, play a major role in driving the winds. [8]

b) Show that the upper limit for the mass loss rate $\dot{M}(r)$ at a distance r from the star is given by

$$\dot{M}(r) = \frac{L_{star}}{v(r)\,c}$$

Define all terms and state all assumptions. [3]

c) If

$$\log_{10} \dot{M} = \alpha \log_{10} L_{star} + \log_{10} C$$

derive an expression that gives the total mass lost M_{lost} in a time t assuming L_{star} is constant. [1]

What is the expression for the total mass lost M_{lost} in a time t if

$$L_{star} = \beta M_{star},$$

where β is a constant and

$$M_{star} = M_{initial} - M_{lost}$$

[5]

d) Use the expression

$$\dot{M}(r) = \frac{L_{star}}{v(r)\,c}$$

and the mass continuity equation to calculate the ratio of the wind density ρ_r at $1.1R_{star}$ and $20R_{star}$. The radial velocity law is given by

$$v(r) = v_o \ln[\frac{r}{R_{star}}]$$

where R_{star} is the stellar radius and v_o is the initial velocity of the wind. [3]

2. a) List the *six* equations of Local Thermodynamic Equilibrium (LTE) and *three* state variables that characterize a stellar atmosphere in hydrostatic and radiative equilibrium. You must define all terms. [8]

b) Starting with the Eddington approximation

$$\frac{dJ_\nu}{d\tau_\nu} = 3H_\nu$$

derive the first Unsöld Lucy Equation

$$J_\nu = \int_0^\infty 3\frac{\kappa_F}{\kappa_P} H_\nu \, d\tau_P + 2H_\nu(0)$$

Show all workings and define all terms. [3]

c) Use the first Unsöld Lucy Equation, given above in part b), to derive the Unsöld Lucy Equations for B_ν and $-\Delta B_\nu$ and thus show that they are equal to

$$B_\nu = \frac{\kappa_J}{\kappa_P}\left[\int_0^\infty \frac{3\kappa_F}{\kappa_P} H_\nu \, d\tau_P + 2H_\nu(0)\right] - \frac{dH_\nu}{d\tau_P}$$

and

$$-\Delta B_\nu = \frac{\kappa_J}{\kappa_P}\left[3\int_0^\infty \frac{\kappa_F}{\kappa_P} \Delta H_\nu \, d\tau_P + 2\Delta H_\nu(0)\right] - \frac{d(\Delta H_\nu)}{d\tau_P}$$

Show all workings and define all terms. [6]

d) Derive expressions for J_ν and B_ν assuming κ_J and κ_P are constants and

$$H_\nu = ae^{-\tau_P}$$

[3]

3. a) Under what conditions does the assumption of Local Thermodynamical Equilibrium (LTE) break down in stellar atmospheres? [3]

State the equation of statistical equilibrium for a 2 level atom and explain all terms. Physically what does this equation represent? For a 2 level atom show that

$$\frac{N_1}{N_2} = \frac{(A_{21} + B_{21}\overline{J} + C_{21})}{(B_{12}\overline{J} + C_{12})}$$

[5]

b) Show that the line source function S_L for a 2 level atom in a non LTE environment is given by

$$S_L = \frac{\overline{J} + \epsilon' B_\nu}{1 + \epsilon'} = (1 - \epsilon)\overline{J} + \epsilon B_\nu$$

where

$$\epsilon = \frac{\epsilon'}{1 + \epsilon'}$$

and

$$\epsilon' = \frac{C_{21}}{A_{21}}[1 - \exp(-h\nu/kT)$$

Show all workings. [4]

c) Consider a ray that extends from the surface ($\tau = 0$) to a depth of $\tau = 1$. The intensity at any point along this ray is given by

$$I = I_O e^{-\tau_{total}} + \int_0^{\tau_{total}} S_L e^{-\Delta\tau'} \, d(\Delta\tau')$$

where I_O is the internal stellar intensity that illuminates the ray at a depth of $\tau = 1$. The optical depth distribution is uniform along the ray; the photon flow is only in the outward radial direction such that $\overline{J} = 1$, $\epsilon = \Delta\tau/2$ and $B_\nu = 0.5$.

Use this information to show that the output intensity I_{out} that emerges from the ray at the surface is given by

$$0.37 I_O + 0.57$$

You will need to use the integration by parts formula

$$\int U dv = UV - \int V du$$

to solve this problem. [5]

d) What would the value of I_{out} be if the total optical depth of the ray equals zero? For $\tau = 1$ what value of I_O is equal to the output generated by the $\tau = 1$ ray itself? [3]

4. a) Derive the Milne Eddington equation for radiative transfer as given by

$$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \mathcal{L}_\nu B_\nu - (1 - \mathcal{L}_\nu) J_\nu$$

where

$$\mathcal{L}_\nu = \frac{1 + \beta_\nu \epsilon_\nu}{1 + \beta_\nu}$$

and β_ν, a measure of the line strength, is the ratio of the line to continuum opacities, and ϵ_ν is the ratio of the pure absorption opacity to the total opacity for line processes. [8]

b) State the one-dimensional integral forms of the *three* moments of the radiation field, F_ν, J_ν and K_ν in terms of the intensity I_ν. Thus show that by taking moments of the Milne-Eddington equation that

$$\frac{d^2 J_\nu}{d^2 \tau_\nu} = 3L_\nu (J_\nu - B_\nu)$$

[4]

c) Using the substitution

$$B_\nu = a + \frac{b\tau_\nu}{1 + \beta_\nu}$$

where a and b are constants show by deriving the complementary function and particular integral for the expression

$$\frac{d^2 J_\nu}{d^2 \tau_\nu} = 3L_\nu (J_\nu - B_\nu)$$

that

$$J_\nu = Ae^{\tau_\nu (3L_\nu)^{0.5}} + Be^{-\tau_\nu (3L_\nu)^{0.5}} + B_\nu$$

where A and B are constants. [5]

d) For an isothermal atmosphere calculate the value of J_ν at $\tau = 0$, 20, 50 and 100. You may assume that $B = 100A$, $L_\nu = 8.4 \times 10^{-4}$, $a + b = 2A$ and $B_\nu = 0$. Comment on the variation of J_ν with τ_ν. [3]

5. a) Describe the H-core burning process for a $3M_\odot$ star as it evolves on the Main Sequence of the Hertsprung-Russell Diagram. You should include basic stellar parameters and a discussion of the underlying physics and nucleosynthesis processes that drive this stage of stellar evolution. [8]

b) How does the main sequence evolution of a $3M_\odot$ compare with that of a $60M_\odot$ star? Highlight both differences and common physical processes. Your answer should include a comparison of basic stellar parameters. [6]

c) Theories suggest $60M_\odot$ stars were the first stars to form in the early Universe, possibly at $z = 30$. How will such stars evolve and how will their lifetimes compare to that of a $60M_\odot$ star formed at $z = 0$? [6]

END OF PAPER