Part C Major Option Astrophysics

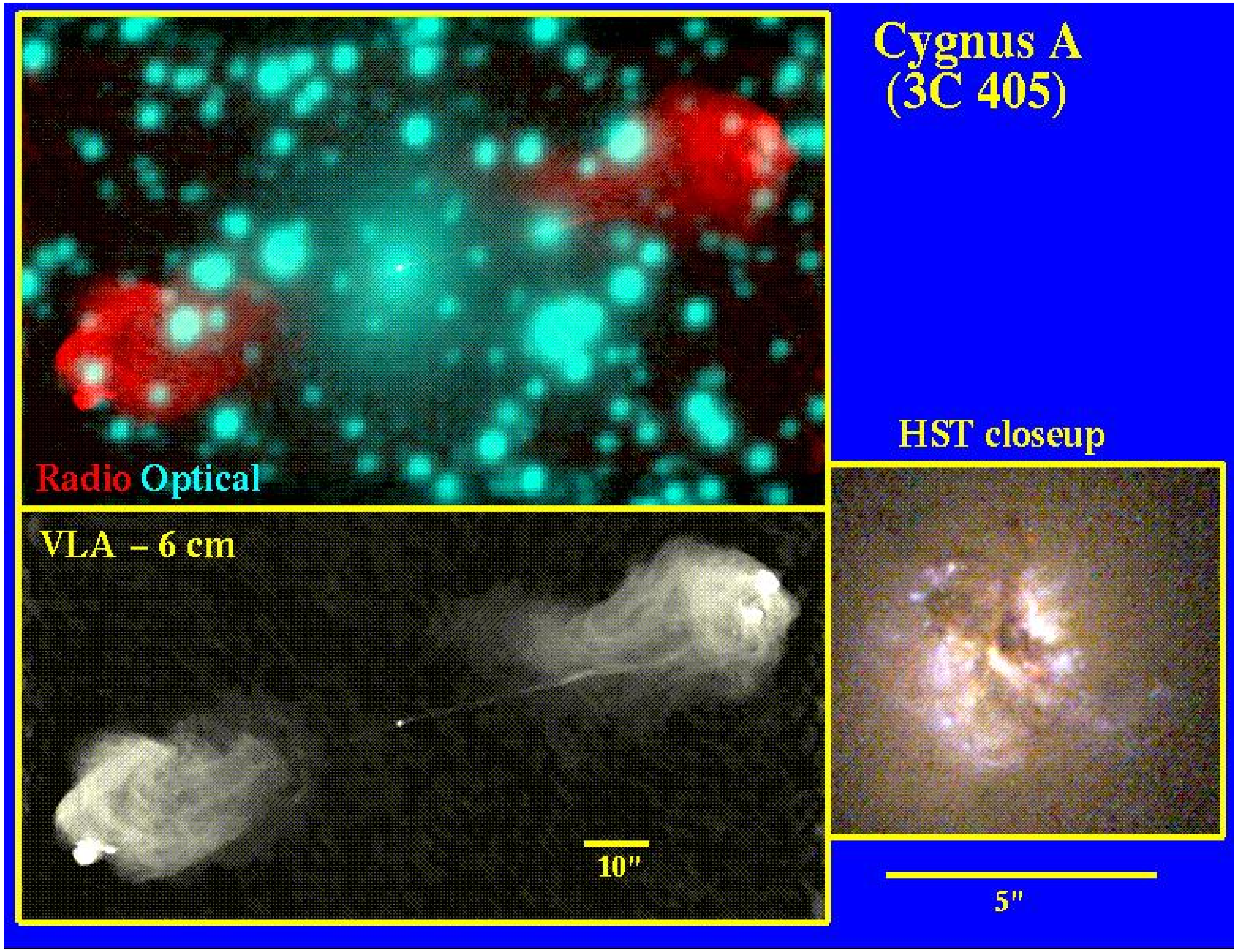
High-Energy Astrophysics

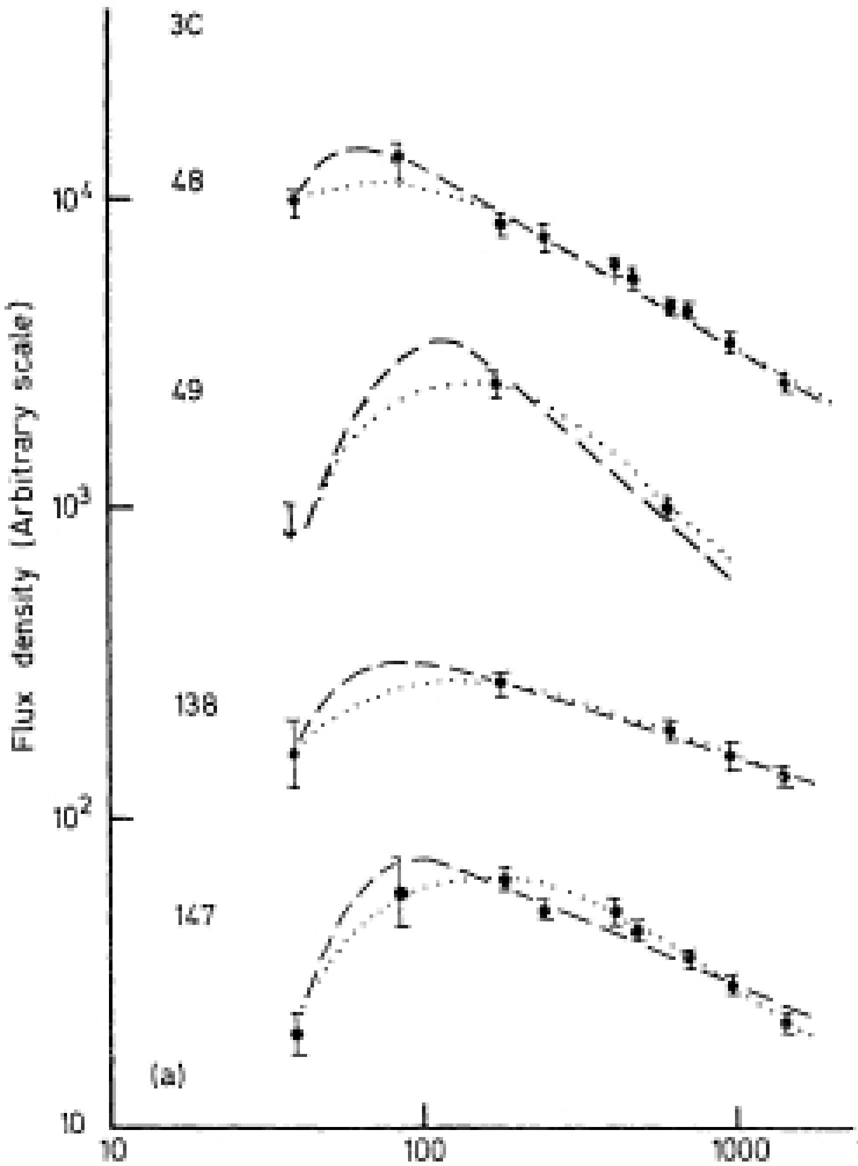
Hilary 2006

Lecture 5: Tuesday, Week 1, 9 am

Today’s lecture: Synchrotron emission Part II

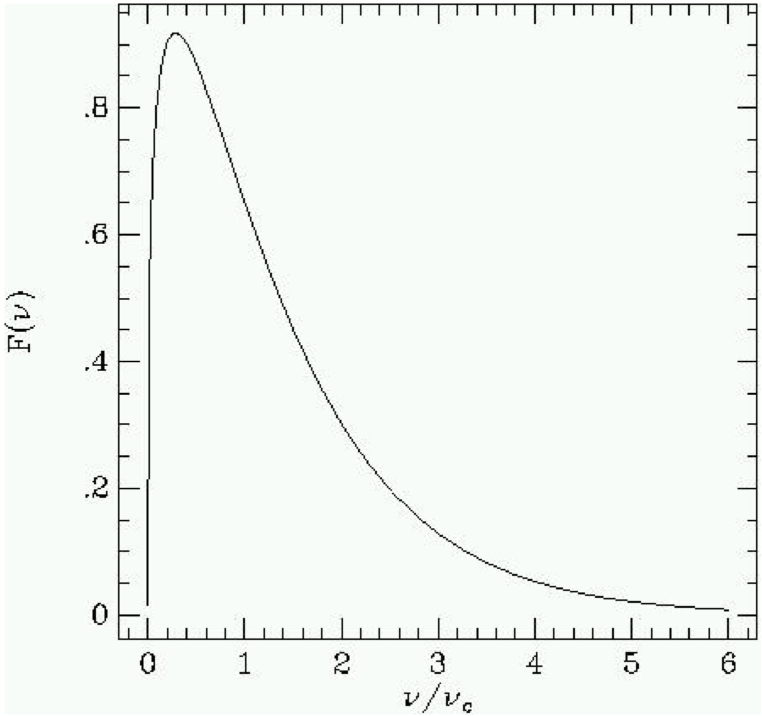
- Recap of the position at end of lecture 4.
- Relationship between the power-law synchrotron spectrum and underlying electron energies.
- Synchrotron self-absorption.


Cygnus A
(3C 405)
Radio Optical
VLA – 6 cm
10″
HST closeup
5″


3C
48
49
138
147
(a)
Flux density (Arbitrary scale)
10^4
10^3
10^2
10
10
100
1000

Synchrotron spectrum from a single electron

Before the vacation we outlined an argument that the spectrum of radiation from an single synchrotron electron is strongly peaked near a frequency γ^3 times the gyro-frequency $\omega_{\rm rel}$. A linear plot of the precise solution shows how nearly all the power is emitted very close to this characteristic frequency.



.8
.6
.4
.2
0
F(ν)
0
1
2
3
4
5
6
ν/ν_c

Synchrotron spectrum from a single electron energy contd.

We can usefully write this characteristic frequency as

$$\nu_{\mathrm{c}} = \frac{\gamma^2 e B}{2\pi m_{\mathrm{e}}}$$

Note here that since the total energy of the electron E is given by

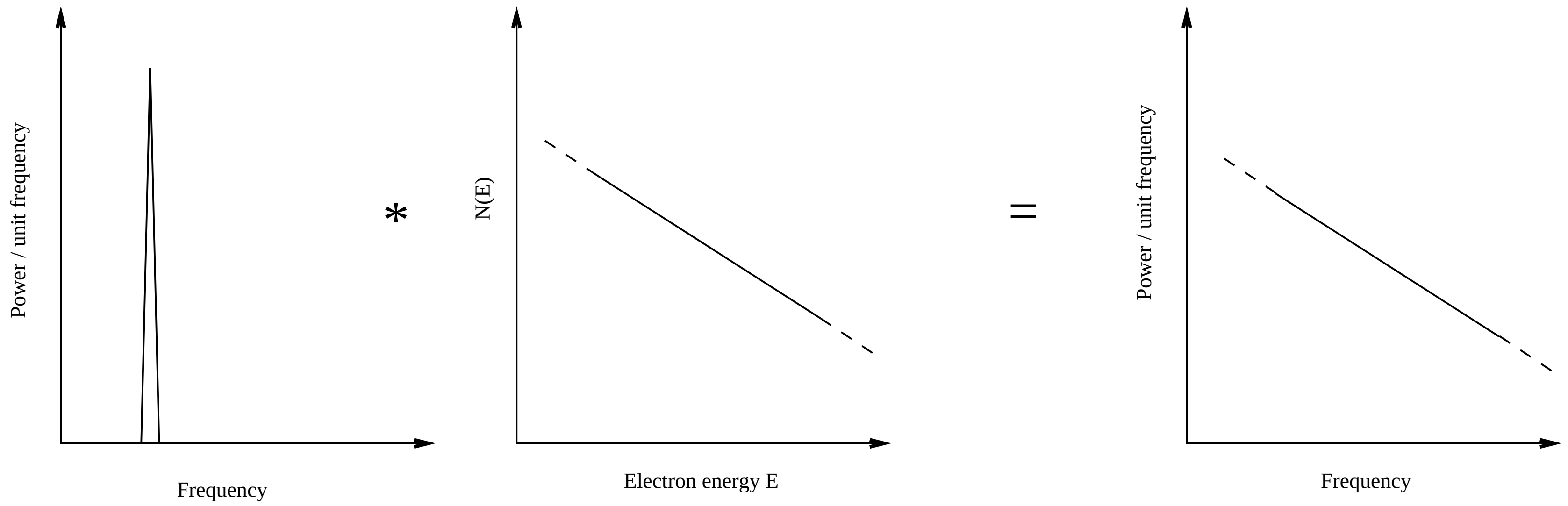
$$E = \gamma m_{\mathrm{e}} c^2$$

it immediately follows that

$$\nu_{\mathrm{c}} \propto E^2$$

i.e., in a population of electrons of different energies, but with some (on average) uniform **B**, the higher-energy electrons will radiate at higher frequencies in proportion to E^2.

Relationship between single-energy spectrum and typical radiosource power-law spectrum



The frequency range over which each electron energy emits is very narrow compared with the total range of frequencies observed.

We infer that the power-law spectrum is the result of an underlying power-law distribution in *electron energy*, with the higher energy electrons causing the high-frequency emission.

The observed spectrum is the *convolution* of the electron energy distribution with the spectrum from a single electron.

Aside: how do we know that B is uniform?

Of course there are small-scale variation in the magnetic flux density in any plasma, and we see clear evidence for turbulent flows in high-resolution maps of radiosources. But we are making an assumption here that **B** is on average homogeneous, in order to convolve the mono-energetic electron spectrum with the electron energies. Are we justified in doing this?

One argument in favour of homogeneous **B** is that the sound speed in relativistic plasma is very high: $c/\sqrt{3}$, in fact. Any large variations in **B** would cause variations in internal energy density, i.e. pressure, and these would quickly be smoothed out by pressure waves.

There may, however, be reasonably smooth bulk motions of plasma inside the radio lobes, which could support gradual pressure changes across the lobes on the largest scales.

In this case we'd still locally see the spectrum as a convolution of mono-energetic synchrotron spectrum with the electron energy distribution; but from point to point within the source, radiation of a particular frequency might be emitted by electrons of different energies.

Electron energy distribution

So, we infer that there is a power-law distribution of electron energies over a wide range in energy E, which we will describe as

$$N(E)dE \propto E^{-k}dE$$

We can approximate the convolution by taking the single-electron spectrum to be a delta-function and transforming energy into frequency.

The power emitted by a single electron is proportional to B^2 and to the square of the electron energy (last lecture).

$$P(E) \propto E^2 B^2$$

And we have just seen that the characteristic frequency emitted by an electron must scale as the square of the electron energy,

$$E \propto \nu^{1/2} \quad \text{and} \quad dE \propto \nu^{-1/2} d\nu$$

Electron energy distribution contd.

Now, the power radiated between ν and $\nu + d\nu$ will be equal to the power radiated by electrons with energies between the corresponding E and $E + dE$. Hence we can substitute ν for E and we have a form for the spectrum...

$$
\begin{aligned}
S(\nu)d\nu &= P(E)N(E)dE \\
&\propto E^2 B^2 E^{-k} dE \\
&\propto E^{2-k} dE \\
&\propto \nu^{(1-k)/2} d\nu
\end{aligned}
$$

For our observed spectrum with a form $S(\nu) \propto \nu^{-\alpha}$, we can infer the energy spectral index is given via

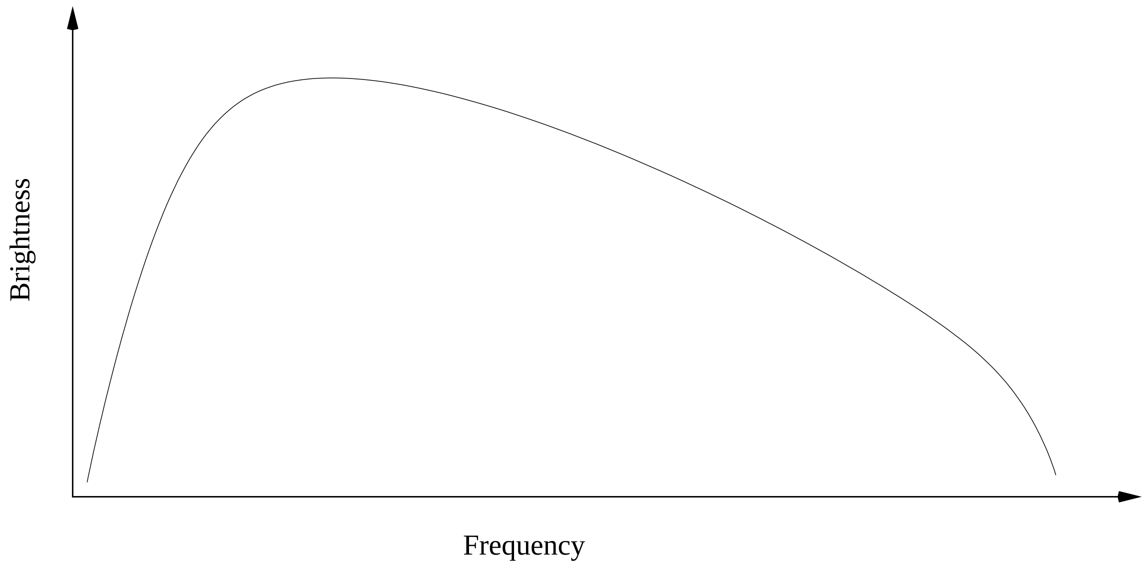
$$\alpha = (k - 1)/2$$

Taking $\alpha = 0.5$, which is around the lower limit we we see for the standard observed synchrotron spectrum in radiosource lobes and supernova remnants, we find $k = 2$, i.e. the underlying electron energy distribution has the form

$$N(E)dE \propto E^{-2}dE$$

And so we have consistency between the proposed acceleration mechanism and the radio spectral index.

Next we will examine the regions of the radio spectrum at low and high frequencies, where the spectrum "turns over"



Low-frequency cutoff: synchrotron self-absorption

Scattering within synchrotron plasma

In calculating the synchrotron spectrum so far, we have implicitly assumed that all of the synchrotron radiation emitted by each electron reaches the observer. However this is not necessarily the case: as a photon propagates through the plasma on its way out of the source, there is a chance that it will scatter off one of the synchrotron electrons. This is known as *synchrotron self-absorption*.

If such scattering occurs many times many times before the photon can get out of the source, the result is that an outside observer only “sees” emission from a thin layer near the surface of the source. Beneath this, the synchrotron electrons are simply exchanging photons, in a quasi-thermal-equilibrium fashion, and so the total flux the observer sees will be much smaller than if all the synchrotron photons escaped the source. This is analogous to looking at the surface of a dense object in thermal equilibrium—e.g. the surface of the Sun.

For a thermal distribution of particle energies, the calculation of the black-body spectrum observed is a fairly straightforward set piece. However we are dealing with a relativistic power-law distribution of particle energies, and each particle emits and receives radiation only at its characteristic wavelength. The detailed calculation of the self-absorption cross-section as a function of wavelength is difficult (Longair pp258–260) and we will not discuss it here; instead we will calculate the spectral index.

The absorption cross-section for a synchrotron electron and a low-energy photon is greater at longer wavelengths. So for a source of a given size, we see very-long wavelength emission only from a very thin shell at the surface of the source. As our observing frequency increases, we see photons coming from regions of the source that are progressively deeper and deeper, and as we do so the total flux density increases. Eventually we reach a point where we can "see all the way through" the plasma, and above this frequency we recover the underlying power-law distribution.

By analogy with the terms used in optical astronomy, we call the self-absorbed region of the spectrum, where the photon mean free path is much smaller than the source size, *optically thick*; and we call the remaining part of the spectrum, where the photon mean free path is greater than the source size, *optically thin*.

Synchrotron self-absorption: calculation

Without doing the detailed calculation we can however recover the spectral shape of the optically-thick spectrum, via a very simple (but in fact strictly correct) trick.

Let us assign an *effective temperature*, $T_{\rm Eff}$, to the electrons in the synchrotron plasma. Although the particle energy distribution is not thermal, we can still define such an effective temperature according to

$$k_{\rm B} T_{\rm Eff} \sim \gamma m_{\rm e} c^2$$

We are now familiar with the result that the electron emits at a characteristic frequency, so we have

$$k_{\mathrm{B}} T_{\mathrm{Eff}} \propto E \propto \nu^{1/2}$$

Now here is the trick: even though the electrons are not in a thermal distribution, at any frequency they cannot emit radiation more effectively than a black-body. So we use the above functional form for $T_{\mathrm{Eff}}(\nu)$ in the Planck formula! This gives us the maximum brightness that a self-absorbed synchrotron plasma can have.

Here is the full form for the brightness from a black-body (recall from lecture 1):

$$B_\nu = \frac{2h\nu^3}{c^2}\left\{\exp\left(\frac{h\nu}{k_\mathrm{B}T}\right) - 1\right\}^{-1}$$

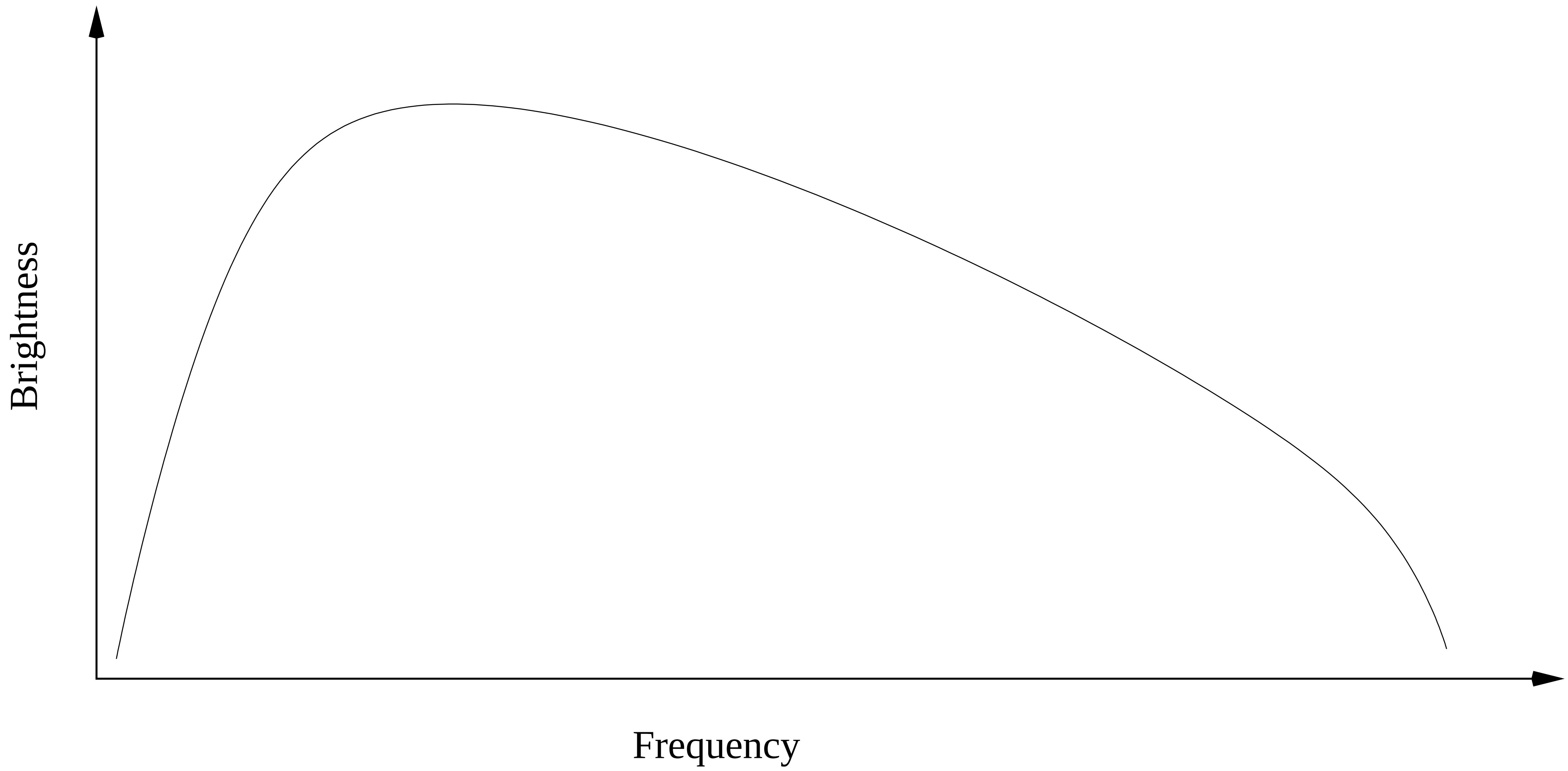
We will make a simplification and use the Rayleigh-Jeans limiting form (as we are explicitly on the low-frequency tail here...)

$$B_\nu = \frac{2k_\mathrm{B}T\nu^2}{c^2}$$

Which, using $T_\mathrm{Eff} \propto \nu^{1/2}$, gives us

$$B_\nu \propto \nu^{5/2}$$

So the spectrum *rises* steeply at low frequencies, until the mean free path between scatterings reaches the size of the source.


Brightness
Frequency

Next lecture: Synchrotron “spectral aging” and the high-frequency cutoff.