Part C Major Option Astrophysics

High-Energy Astrophysics

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Michaelmas 2008 Lecture 2

Today's lecture: Supernova blast waves and shocks.

- Taylor-Sedov solution for powerful explosions.
- Introduction to strong shocks.
- Derivation of strong shock jump conditions.
- Shocks in accretion onto compact objects and supernova explosions.
- Practical accretion: binary systems.

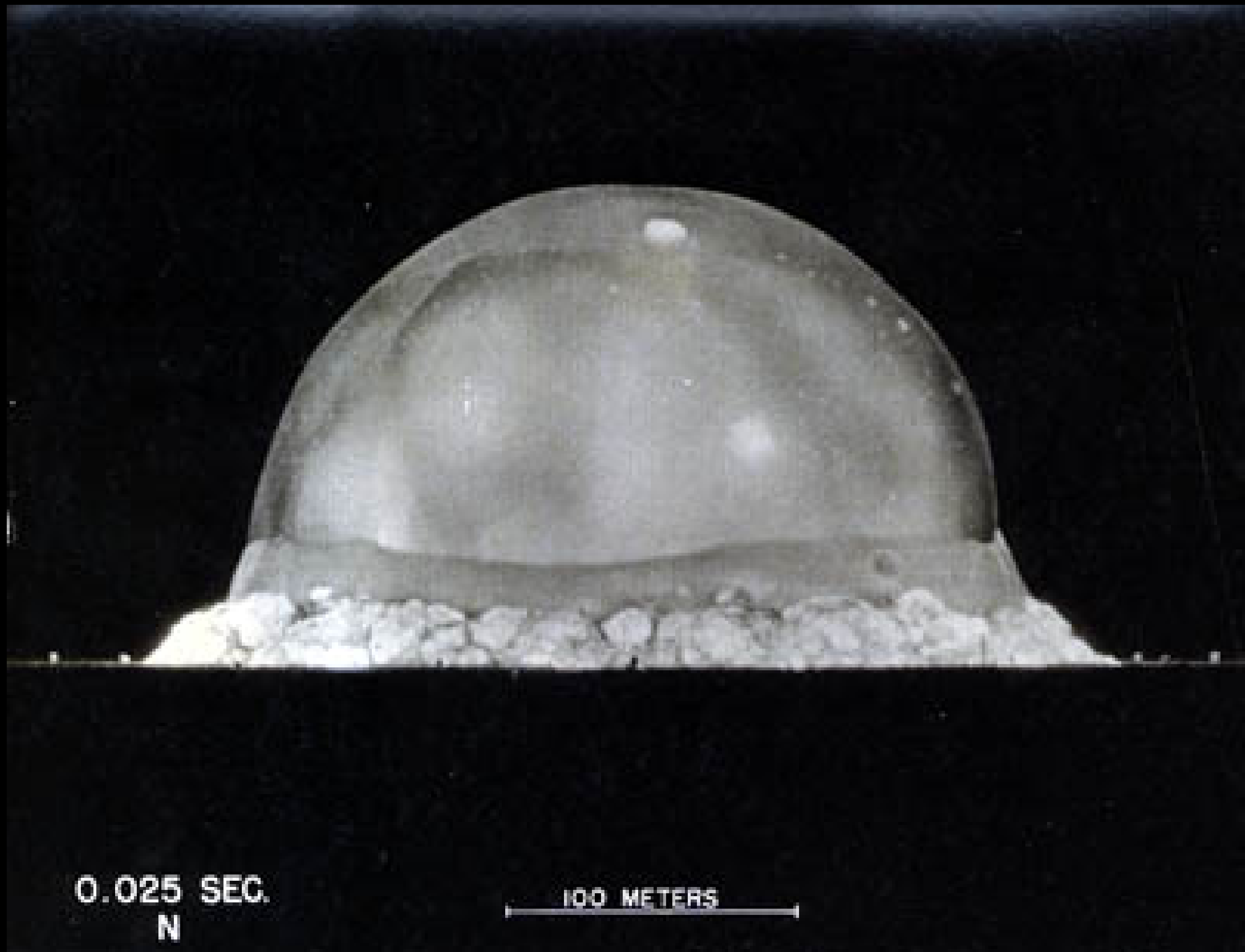
Crab supernova remnant, Hubble Space Telescope image

Supernova blast waves

In 1950, G.I. Taylor used dimensional analysis to estimate the relationship between the energy input of an extremely powerful explosion and the growth of the resulting fireball. He showed that the relevant parameters—explosion energy E, blast radius r, time since detonation t and external density ρ—could be combined to form a dimensionless quantity

$$\frac{r^5\rho}{t^2E}.$$

Taking constants of proportionality to be of order one (verified experimentally via high-speed protography of small explosions and in a detailed analysis by Sedov), Taylor used a declassified photograph of the fireball of the first atomic bomb test to calculate the yield of the bomb. He arrived at an accurate value of ≈ 20 kilotons (a total mass-energy conversion of about one gram). Taylor's publication caused some consternation at the time since the yield was still classified.



Trinity atom bomb test, July 16 1945. 25 ms after detonation.
The fireball is about 140 m in radius.

Let us use this technique to examine the Crab supernova remnant.

$$E = \rho r^5 t^{-2}$$

The electron density in the interstellar medium is about $1\,\text{cm}^{-3} = 10^6\,\text{m}^{-3}$. The Crab supernova remnant today has a radius of about 3 pc and the time since the explosion is $2008 - 1054 = 954$ yr. We arrive at

$$E \approx 10^{42}\,\text{J}$$

Which is equivalent to the total mass-energy conversion of 10^{25} kg. Also highly significant is that the speed at which the blast wave propagates into the interstellar medium is highly supersonic. We next move on to the physical changes which occur at strong supersonic shocks.

Shocks

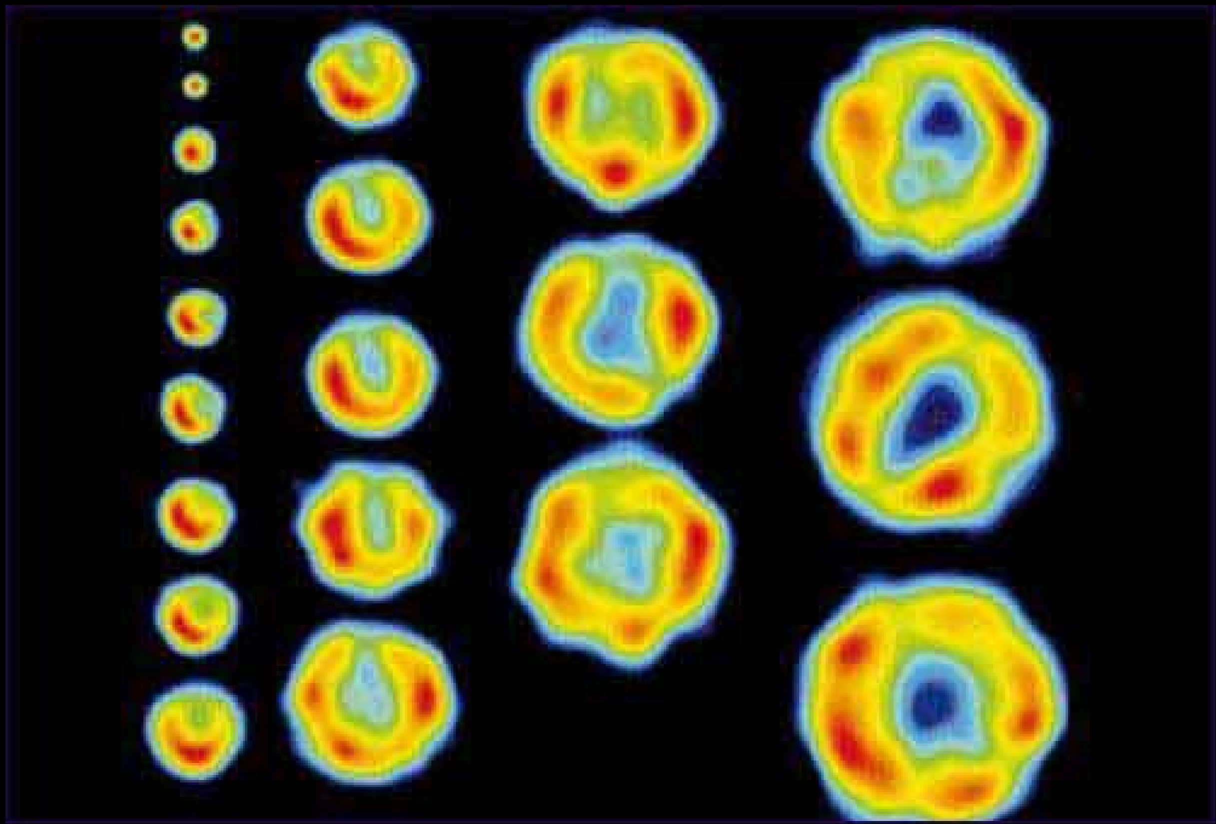
A *shock* occurs when a disturbance moves through a medium faster than the sound speed in the medium, i.e., sufficiently fast that a pressure wave cannot precede the disturbance.

The conditions in the medium—temperature, density, bulk velocity—thus change almost instantaneously at the shock. The material which is hit by the shock receives no forewarning.

The classic introductory example is the case where energy is injected into the ambient medium suddenly by a supersonically-moving solid object: the sonic boom. The air ahead of a supersonic aircraft doesn't have time to move out of the way before the shock hits it; temperature and density are suddenly (almost discontinuously) increased as the shock passes.

This is similar to the blast wave of a supernova. The huge energy injection heats "cold" interstellar gas suddenly as the blast moves outward.

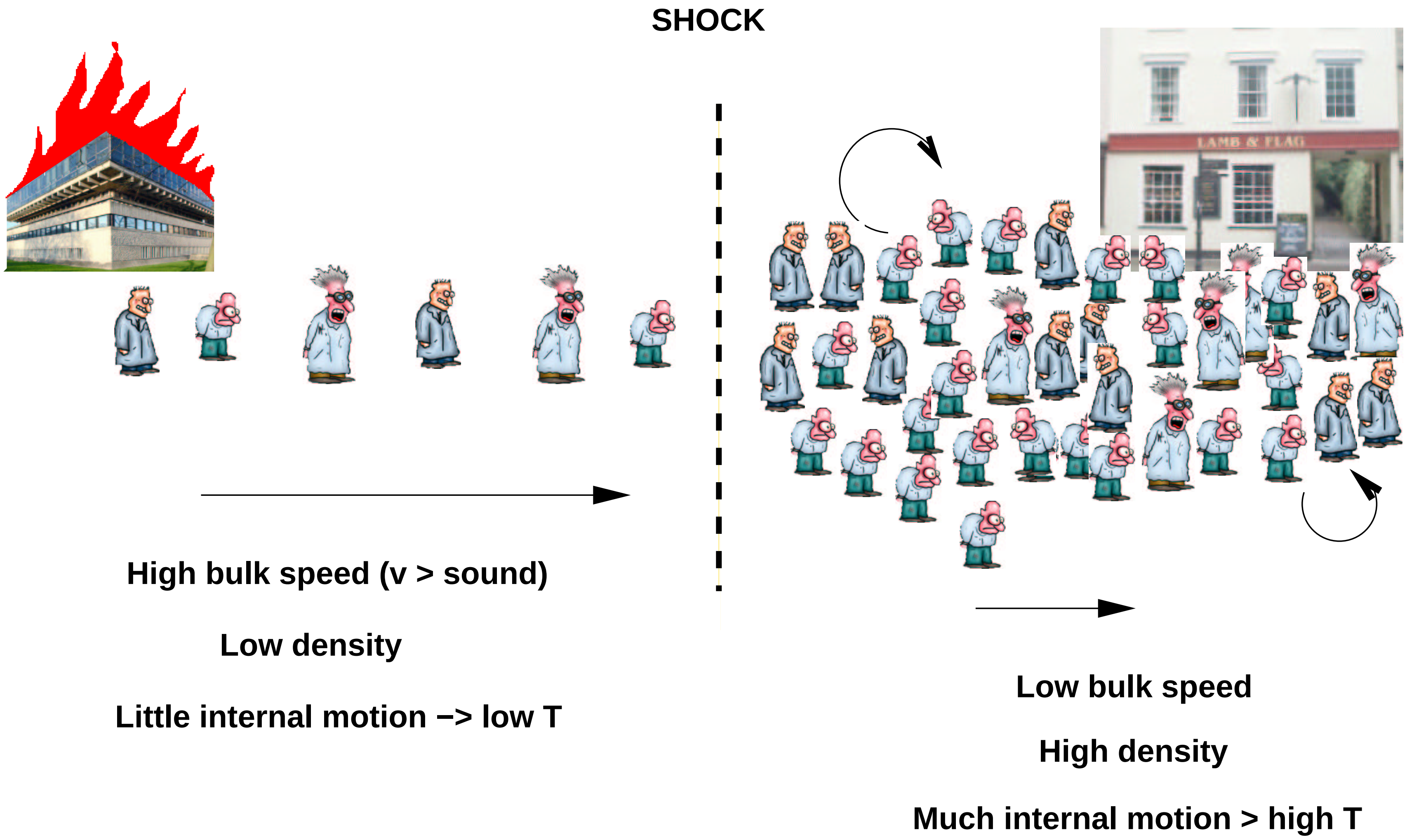
Supernova SN1993J in M81: radio maps April 1993—June 1998



Initial expansion speed 20 000 km s^{-1}

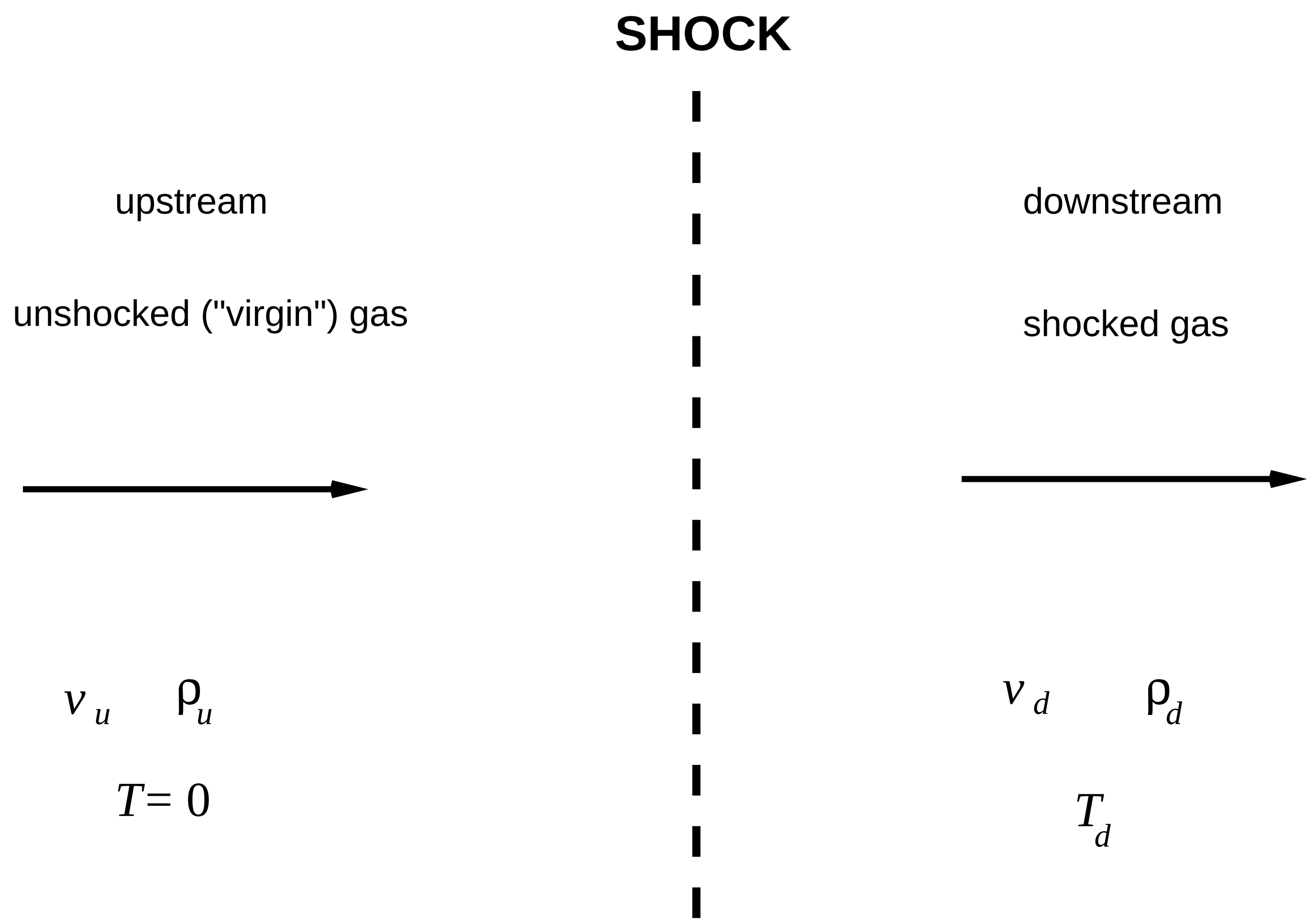
To get a better understanding of what's going on—and to allow an easier calculation of the change in conditions at the shock—we conventionally transform into the instantaneous rest frame of the shock itself.

This may seem a bit contrived but a simple analogy can be used to illustrate the process...


SHOCK
LAMB & FLAG
High bulk speed (v > sound)
Low density
Little internal motion –> low T
Low bulk speed
High density
Much internal motion > high T

Derivation of strong shock “jump conditions”

In restframe of shock



Assumptions:

- Monatomic gas or, e.g., fully-ionized hydrogen plasma.
- Thermal (internal) energy of unshocked gas is negligible; approximate pressure and temperature of the unshocked gas to zero.

Conserve mass: the mass per unit area flowing across the shock is conserved.

$$\rho_u v_u = \rho_d v_d$$

Conserve momentum: the shock does not accelerate in its rest frame. The difference between upstream and downstream ram-pressures must be provided by the gas pressure downstream.

$$\rho_u v_u^2 = P_d + \rho_d v_d^2$$

Conserve energy: PdV work is being done on the gas at the shock. The rate (per unit area) at which this work is done is $v_d P_d$

$$v_u(\frac{1}{2}\rho_u v_u^2) - v_d(\frac{1}{2}\rho_d v_d^2 + \frac{3}{2}P_d) = v_d P_d$$

Eliminating P_d and using $\rho_u v_u = \rho_d v_d$ we can get

$$\frac{\rho_d^2}{\rho_u^2} v_u^3 - 5\frac{\rho_d}{\rho_u} v_u^3 + 4v_u^3 = 0$$

This rearranges to...

$$(\frac{\rho_d}{\rho_u} - 4)(\frac{\rho_d}{\rho_u} - 1) = 0$$

...which has the trivial solution $v_u = v_d$, but which otherwise gives us the

Strong shock jump conditions

$$\boxed{\frac{\rho_d}{\rho_u} = 4; \frac{v_u}{v_d} = 4}$$

This is a highly-simplified case but is a reasonable approximation in many astrophysical situations because of the extreme conditions. For a full discussion of shocks for arbitrary Mach numbers, temperatures etc. see e.g. Blundell & Blundell.

Practical accretion onto stellar-mass compact objects.

Violent accretion onto stellar-mass compact objects happens in *binary* star systems: the accreting mass has to come from somewhere, and if there is a binary companion then it can “donate” fuel for the accretion.

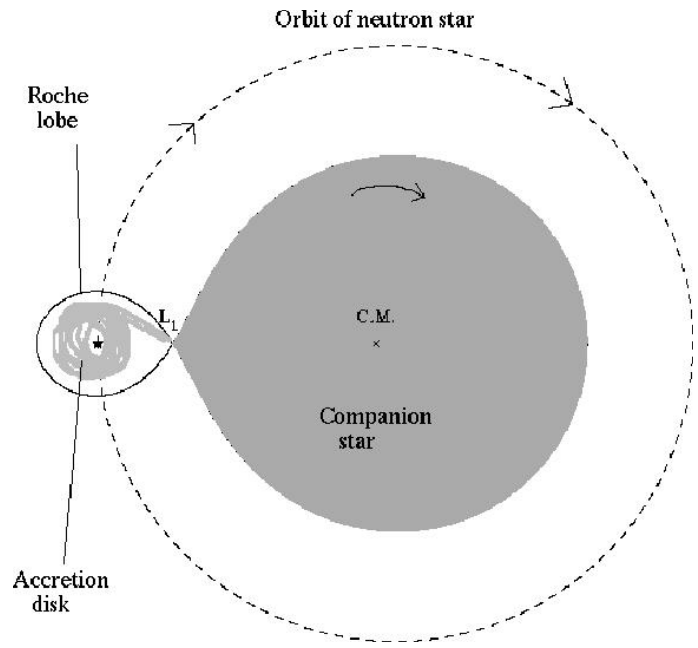
Practical accretion contd.

In accreting binary systems, the compact object can be a white dwarf, neutron star or black hole. The donor stars are usually either high-mass (and hence young) or are low-mass stars, *but* systems in which a massive star is accreting onto a white dwarf are not observed. (Recap from B3: why?)

Jargon alert: Although all accreting binary systems emit X-rays, the phrase “X-ray binary” is commonly reserved only for those systems in which the compact object is a WD or NS. Systems in which a low-mass star is fuelling accretion onto a WD are termed “cataclysmic variables” or “dwarf novae”. (Recap from B3: what is often the fate of such objects?)

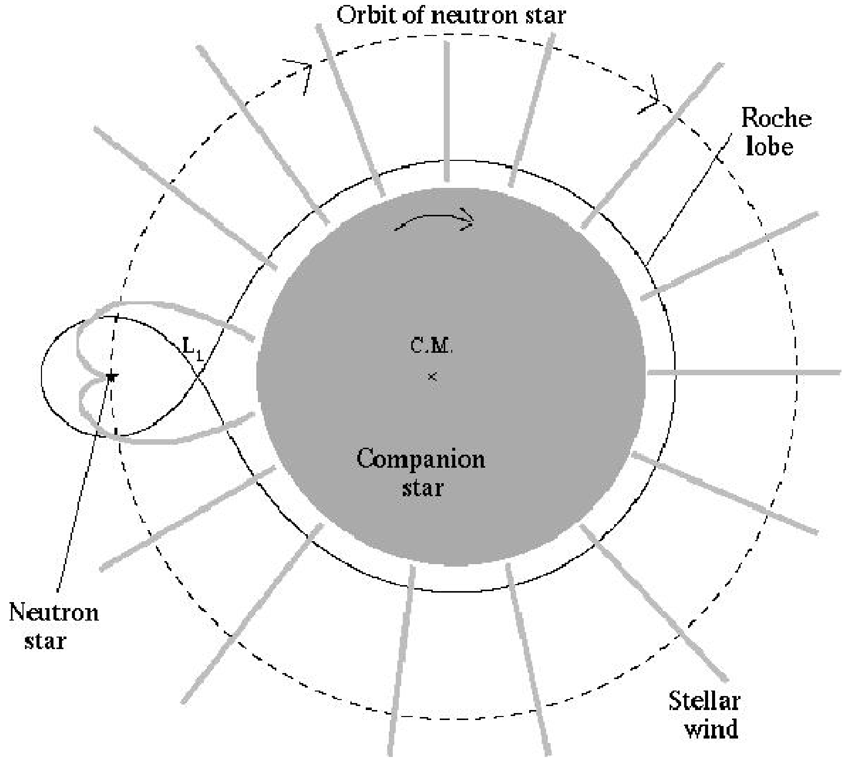
X-ray binaries are then classified as high-mass or low-mass depending on the mass of the *donor* star. This distinction is important because it broadly divides the accretion into the two mechanisms we have discussed: material falling in gradually through an accretion disc, and material falling straight to the surface of the compact object.

Roche lobe overflow



This occurs when the envelope of the donor star expands beyond the equipotential surface between it and its compact companion (the *Roche Lobe*). The atmosphere of the donor star falls fairly smoothly down into the potential of the compact object and forms an accretion disc. This process can happen even with low-mass donor stars once they expand off the main sequence.

Stellar wind capture



This occurs where the donor star is very high mass (typically $M > 15M_{\odot}$), and the stellar wind of the high-mass star can be captured by the compact object. In these systems the luminosity depends on the mass outflow rate from the donor.

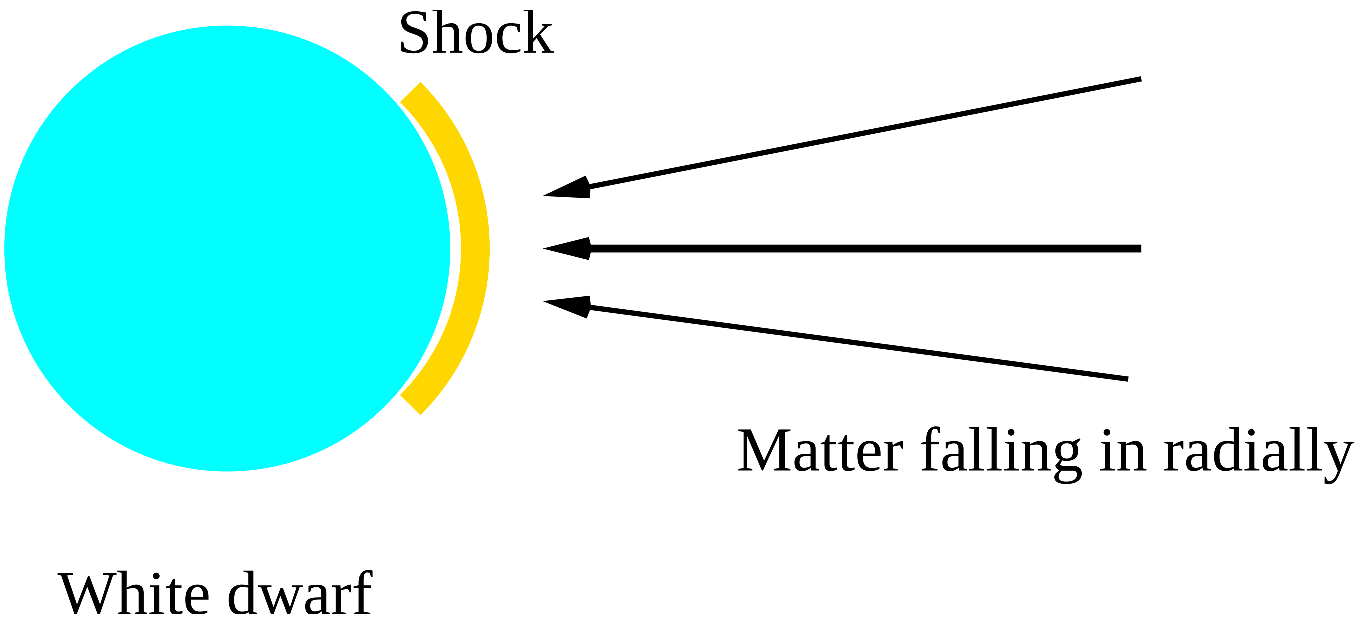
We must of course be aware that high-mass donor stars can also have sufficiently large envelopes that they fill their Roche lobes, so these two pictures represent the extreme cases. The details of binary star evolution will be given in Prof Podsiadlowski's course.

A further complication in white dwarf and neutron star accretion is the *magnetic field* of the compact object. The extremely strong fields found in collapsed stars can have a profound influence on the flow of accreting material: in extreme cases this is a method of preventing an accretion disc forming.

Shocks: implications for accreting objects

We will consider accretion discs in a later lecture. For the time being, let's simply consider material falling radially onto a compact object.

Note that we cannot convert all of the bulk kinetic energy of the upstream gas into thermal energy downstream. So, when "cold" material falls at great speed onto a compact object, it is decelerated and heats up in a shock a little way above the surface.


Shock
Matter falling in radially
White dwarf

Shocks: calculation

We can use the ideal gas law $P = nk_{\text{B}}T$ to determine the conditions of the shocked gas. For a gas with mean particle mass m and density ρ the ideal gas law can be written as

$$P = \frac{\rho}{m} k_{\text{B}} T$$

Which for our strong shock conditions can be used to obtain

$$T_d = \frac{3}{16} \frac{m v_u^2}{k_{\text{B}}}$$

For material falling onto the surface of a white dwarf, the temperatures which arise are sufficient to generate hard X-ray emission.