NEUTRON STARS

1934: Baade and Zwicky proposed that supernovae represented the transition of normal stars to neutron stars

1939: Oppenheimer and Volkoff published the first theoretical model

1967: discovery of pulsars by Bell and Hewish

Maximum mass of a neutron star:

- Neutrons are fermions: degenerate neutrons are unable to support a neutron star with a mass above a certain value (c.f. Chandrasekhar mass limit for white dwarfs).
- Important differences from white dwarf case:
  - (i) interactions between neutrons are very important at high densities
  - ii) very strong gravitational fields (i.e. use General Relativity)

  N.B. There is a maximum mass but the calculation is very difficult; the result is very important for black-hole searches in our Galaxy
- Crude estimate– ignore interactions and General Relativity – apply same theory as for white dwarfs: $M_{max} \simeq 6\,M_\odot$
- interactions between neutrons increase the theoretical maximum mass
  - ▷ The interaction is attractive at distances ~ 1.4 fm but repulsive at shorter distances

  → matter harder to compress at high densities

  - ▷ but, at high densities, degenerate neutrons are energetic enough to produce new particles (hyperons and pions) → the pressure is reduced because the new particles only produce a small pressure
- Effect of gravity (gravitational binding energy of neutron star is comparable with rest mass):
- Gravity is strengthened at very high densities and pressures. Consider the pressure gradient:

$$\text{Newton: } \frac{dP}{dr} = -\frac{Gm\rho}{r^2}$$

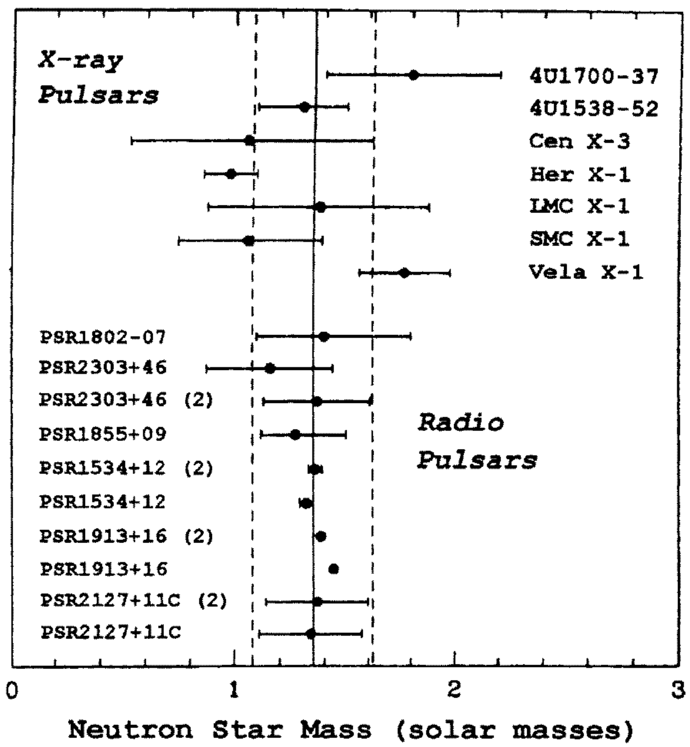
$$\text{Einstein: } \frac{dP}{dr} = -\frac{Gm\rho}{r^2} \times \frac{(1+P/(\rho c^2))(1+4\pi r^3 P/(mc^2))}{1-2Gm/(rc^2)}$$

- Pressure P occurs on RHS. Increase of pressure, needed to oppose gravitational collapse, leads to strengthening of gravitational field
- need an equation of state $P(\rho)$ that takes account of $n-n$ interactions
- Oppenheimer and Volkoff (1939) calculated M_{max} for a star composed of non-interacting neutrons.
  Result: $M_{max} = 0.7\,M_\odot$
- Enhanced gravity leads to collapse at finite density when neutrons are just becoming relativistic – not ultrarelativistic
- Various calculations, using different compressibilities for neutron star matter, predict

$$\boxed{M_{max} \in [1.5, 3]\,M_\odot} \qquad R_{NS} \simeq 7-15\,\text{km}$$

- Observed neutron star masses (from analysis of binary systems) are mostly around $1.5\,M_\odot$

- Modern calculations suggest that M_{max} is probably less than $3M_\odot$ and definitely less than $5M_\odot$. See Phillips, "The Physics of Stars", for an example calculation: incompressible star of constant density



PULSARS

Expected properties of neutron stars:

(a) Rotation period (c.f. white dwarfs, e.g. 40 Eri B, $P_{WD} = 1350\,s$)

From simple theory : $\frac{R_{WD}}{R_{NS}} \simeq \frac{m_n}{2^{5/3} m_e} \simeq 600$

conservation of angular momentum: with $M_{WD} \sim M_{NS}$ and $I = (2/5)MR^2$ (uniform sphere)

$$I_i \omega_i = I_f \omega_f$$

$$\omega_f = \omega_i (R_i/R_f)^2$$

$$P_{NS} \simeq 3 \times 10^{-6}\, P_{WD} \simeq 4\,ms$$

→ neutron stars rotate rapidly when they form

▷ but angular momentum is probably lost in the supernova explosion

▷ rotation is likely to slow down rapidly

(b) magnetic field

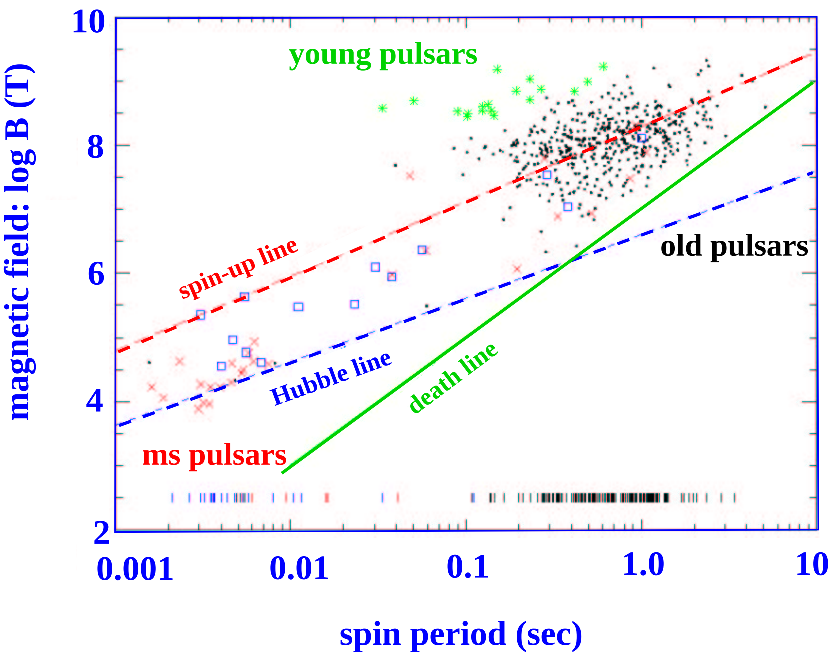
▷ Flux conservation requires $\int B\, dS$ = constant

$$B_i\, 4\pi R_i^2 = B_f\, 4\pi R_f^2$$

▷ Take largest observed white dwarf field, $B_{WD} \simeq 5 \times 10^4$ T

$$B_{NS} \simeq B_{WD} (R_{WD}/R_{NS})^2 \simeq 10^{10}\,T \qquad \text{(upper limit)}$$

Radio Pulsars: the P-B Diagram



(c) Luminosity

- ▷ neutron star forms at $T \sim 10^{11}$ K but T drops to $\sim 10^9$ K within 1 day
- ▷ main cooling process: neutrino emission (first $\sim 10^3$ yr), then radiation
- ▷ after a few hundred years, $T_{\rm internal} \sim 10^8$ K, $T_{\rm surface} \sim$ a few $\times 10^6$ K.
- ▷ star cools at constant R for $\sim 10^4$ yr with $T_{\rm surface} \sim 10^6$ K

$$L \sim 4\pi R^2 \sigma T_s^4 \sim 10^{26}\,{\rm W}\ \ ({\rm mostly\ X-rays},\ \lambda_{\rm max} \sim 3\ {\rm nm})$$

Discovery of neutron stars

- The first pulsar was discovered by Bell and Hewish at Cambridge in 1967
  - ▷ A radio interferometer (2048 dipole antennae) had been set up to study the scintillation which was observed when radio waves from distant point sources passed through the solar wind. Bell discovered a signal, regularly spaced radio pulses 1.337 sec apart, coming from the same point in the sky every night.
- Today, about 1500 pulsars are known
  - ▷ Most have periods between 0.25 s and 2 s (average 0.8 s)
  - ▷ Extremely well defined pulse periods that challenge the best atomic clocks
  - ▷ Periods increase very gradually
  - ▷ spin-down timescale for young pulsars $\sim P/\dot{P} \sim 10^6 - 10^7$ yr

SUPERNOVA REMNANTS

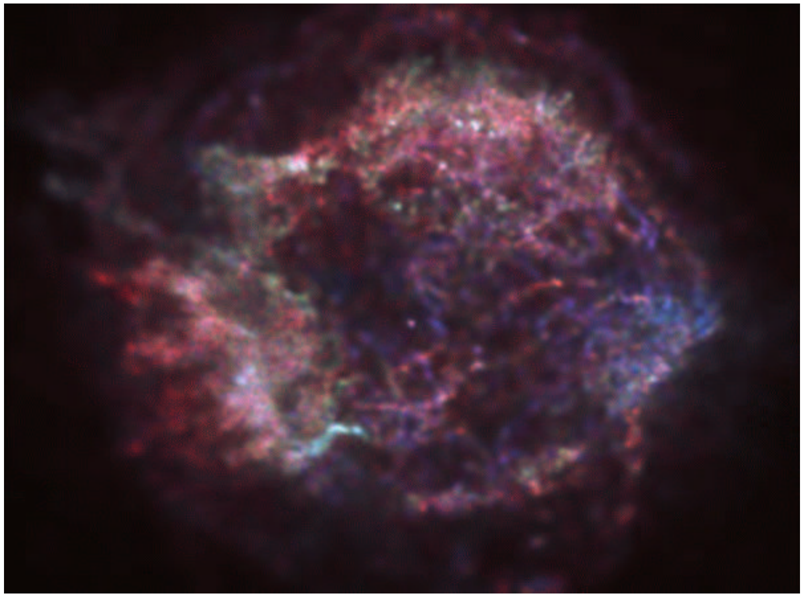
The Crab Nebula (plerionic/filled)

VLT

Cassiopeia A (shell-like)

Chandra

(X-rays)



The Crab Pulsar

- The Crab nebula is the remnant of a supernova explosion observed optically in 1054 AD.
- The Crab pulsar is at the centre of the nebula, emitting X-ray, optical and radio pulses with $P = 0.033\,s$.
- The Crab nebula is morphologically different from two other recent supernova remnants, Cas A and Tycho (both $\sim 400\,yr$ old) which are shell-like.
- The present rate of expansion of the nebula can be measured: uniform expansion extrapolates back to 90 years after the explosion, i.e. the expansion must be accelerating
- The observed spectrum is a power law from $\sim 10^{14}\,Hz$ (IR) to $\sim 1\,MeV$ (hard X-rays); also, in the extended nebulosity, the X-rays are 10-20 % polarised
  $\rightarrow$ signature of synchrotron radiation (relativistic electrons spiralling around magnetic field lines with $B \sim 10^{-7}\,T$).
- Synchrotron radiation today requires (i) replenishment of magnetic field and (i) continuous injection of energetic electrons.
- Total power needed $\sim 5 \times 10^{31}\,W$
- Energy source is a rotating neutron star ($M \simeq 1.4\,M_\odot$, $R \simeq 10\,km$)

$$U = (1/2)\,I\omega^2 = 2\pi^2\,I/P^2$$

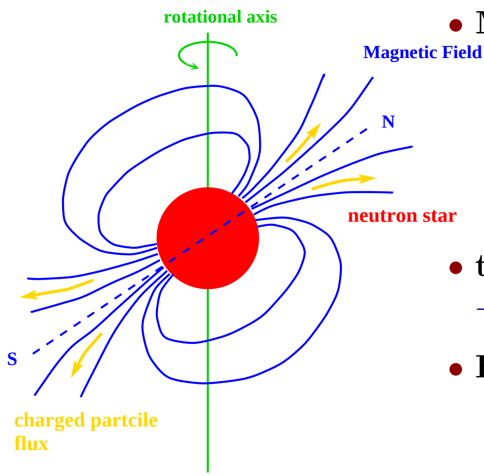
$$\frac{dU}{dt} = -\frac{4\pi^2 I\dot{P}}{P^3}$$

Taking $I = (2/5)\,MR^2 \sim 1 \times 10^{38}\,kg\,m^2$; $P = 0.033\,s$;

$$\dot{P} = 4.2 \times 10^{-13} \quad \rightarrow \quad dU/dt \simeq 5 \times 10^{31}\,W$$

A simple pulsar model

- A pulsar can be modelled as a rapidly rotating neutron star with a strong dipole magnetic field inclined to the rotation axis at angle θ.
- Pulsar emission is beamed (like a lighthouse beam) $\rightarrow$ observer has to be in the beam to see pulsed emission.



- Magnetic dipole radiation:

$$\frac{dU_{mag}}{dt} = -\frac{2}{3c^3}\left(\frac{\mu_0}{4\pi}\right) m^2\omega^4 \sin^2\theta = -\frac{32\pi^4}{3c^3}\left(\frac{\mu_0}{4\pi}\right)\frac{m^2\sin^2\theta}{P^4}$$

- taking $dU/dt = -5 \times 10^{31}$ W $\rightarrow \quad m\sin\theta \simeq 4 \times 10^{27}$ A m^2.
- Hence, surface magnetic field

$$B \simeq \frac{\mu_0 m}{4\pi R^3} \simeq 10^8\,\mathrm{T}.$$

- Further argument for magnetic dipole radiation:

$$-\frac{dU_{rot}}{dt} = -I\omega\frac{d\omega}{dt} \propto \omega^4$$

$$\rightarrow \quad \frac{d\omega}{dt} = -C\omega^3.$$

Integrate $\quad t = \frac{1}{2C}\left(\frac{1}{\omega^2} - \frac{1}{\omega_0^2}\right)$

So $t < \frac{1}{2C\omega^2}$ with $C = 3.5 \times 10^{-16}$ s and $\omega = 190\,\mathrm{s}^{-1}$

$\rightarrow t < 1250$ yr (comparable to the known age $\simeq 950$ yr)

N.B. The physics underlying pulsar emission mechanisms is very complicated and not well understood

Pulsar dispersion measure

- Consider an electromagnetic wave of the form $E = E_0\cos(kx \pm wt)$ propagating through an ionised medium where the number density of electrons is n_e.
- The dispersion relation is

$$\omega^2 = k^2c^2 + \omega_p^2, \quad \omega_p = (n_e e^2/(m\epsilon_0))^{1/2} \text{ (plasma frequency)}.$$

- If $\omega < \omega_p \rightarrow$ no wave propagation (e.g. $\sim$ few MHz cut-off to radio waves through the ionosphere). If $\omega > \omega_p$, propagation occurs with group velocity v_g given by:

$$v_g = \frac{d\omega}{dk} = c\left(1 + \frac{w_p^2}{c^2k^2}\right)^{-1/2} = c\left(1 + \frac{\omega_p^2}{\omega^2 - \omega_p^2}\right)^{-1/2}$$

- i.e. frequency dependent: longer wavelength has lower velocity.
- A pulse of radiation travels a distance l in time $t = l/v_g$.
- Frequency dependence of arrival time is given by:

$$\frac{\Delta t}{\Delta\omega} = -\frac{l}{v_g^2}\frac{\Delta v_g}{\Delta\omega} \simeq -\frac{l}{c^2}\frac{\Delta v_g}{\Delta\omega} \text{ since } v_g \simeq c \text{ for } \omega >> \omega_p.$$

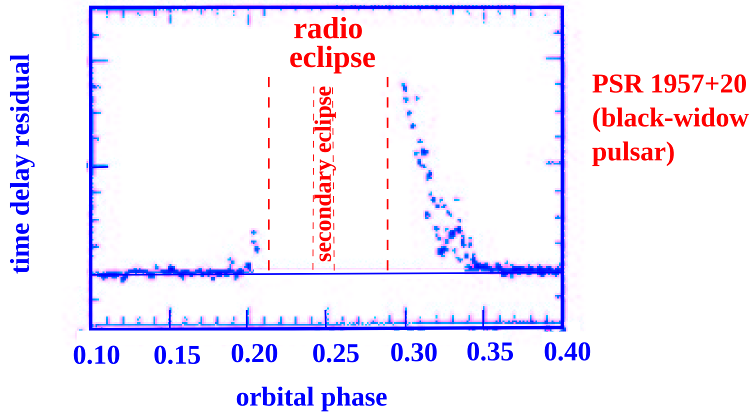
But $$\frac{\Delta v_g}{\Delta\omega} = \frac{\omega\omega_p^2 c}{(\omega^2 - \omega_p^2)^2}\left(1 + \frac{\omega_p^2}{\omega^2 - \omega_p^2}\right)^{-3/2} \simeq \frac{\omega_p^2 c}{\omega^3}.$$

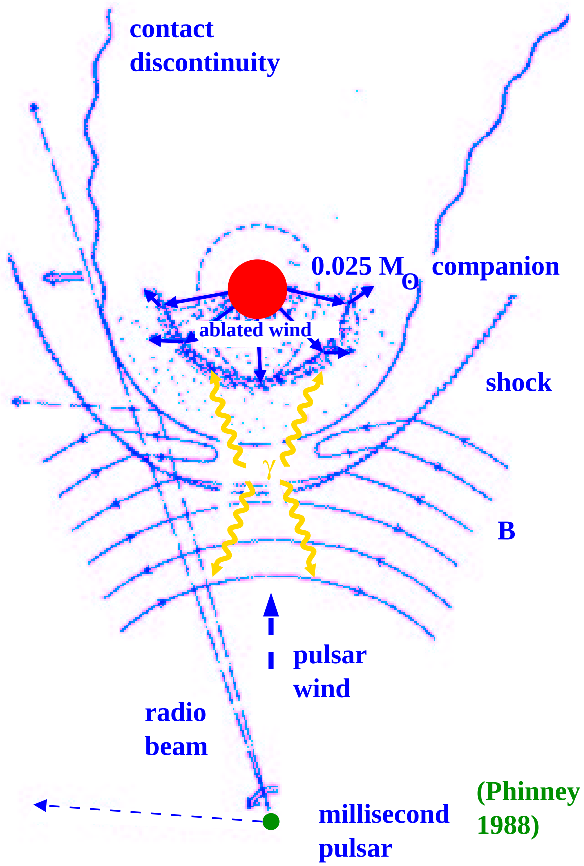
Therefore $$\frac{\Delta\nu}{\Delta t} = \frac{1}{2\pi}\frac{\Delta\omega}{\Delta t} = -\frac{4\pi^2 mc\ \epsilon_0}{e^2}\frac{\nu^3}{n_e l}$$

Strictly $$\frac{\Delta\nu}{\Delta t} = -\frac{4\pi^2 mc\ \epsilon_0}{e^2}\frac{\nu^3}{\int n_e dl}$$ since n_e varies with l.

- $\int n_e dl$ is known as the DISPERSION MEASURE. It is a useful distance indicator if n_e is uniform (typical value for the Galactic plane: $1 - 3 \times 10^5\,\mathrm{m}^{-3}$)

(Fruchter, Stinebring, Taylor 1988)

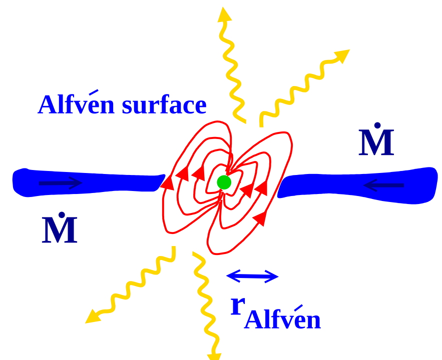




(Phinney et al. 1988)

MILLISECOND (RECYCLED) PULSARS

- a group of ~ 100 radio pulsars with very short spin periods (shortest: 1.6 ms) and relatively weak magnetic fields ($B \lesssim 10^6$ T)
- they are preferentially members of binary systems,
- they have spin-down timescales comparable or longer than the Hubble time (age of the Universe)
- standard model
  - ▷ these pulsars are neutron stars in binary systems that spin-down first, lose their strong magnetic field (due to accretion?)
  - ▷ and are spun-up by accretion from a companion



- ▷ magnetospheric accretion: magnetic field becomes dominant when magnetic pressure > ram pressure in flow → flow follows magnetic field lines (below r_A)
- ▷ spin-up due to accretion of angular momentum

- equilibrium spin period: $v_{rot}(r_A) = v_{Kepler}(r_A)$

$$\rightarrow \boxed{P_{eq} =\simeq 2\,\text{ms}\,\left(B/10^5\,\text{T}\right)^{6/7}\left(\dot{M}/\dot{M}_{Edd}\right)^{-3/7}}$$

- a significant fraction of millisecond pulsars are single

  → pulsar radiation has evaporated the companion
  - ▷ example: PSR 1957+20 (the black-widow pulsar): companion mass: only $0.025\,M_\odot$
  - ▷ direct evidence for an evaporative wind from the radio eclipse (much larger than the secondary)

  → comet-like evaporative tail

SCHWARZSCHILD BLACK HOLES

- event horizon: (after Michell 1784)
  - ▷ the escape velocity for a particle of mass m from an object of mass M and radius R is $v_{esc} = \sqrt{\frac{2GM}{R}}$ ($11\,km\,s^{-1}$ for Earth, $600\,km\,s^{-1}$ for Sun)
  - ▷ assume photons have mass: $m \propto E$ (Newton's corpuscular theory of light)
  - ▷ photons travel with the speed of light c

  → photons cannot escape, if $v_{esc} > c$

  → $\boxed{R < R_s \equiv \frac{2GM}{c^2}}$ (Schwarzschild radius)
  - ▷ $R_s = 3\,km\,(M/M_\odot)$

    Note: for neutron stars $R_s \simeq 5\,km$; only a factor of 2 smaller than R_{NS} → GR important
- particle orbits near a black hole
  - ▷ the most tightly bound circular orbit has a radius $R_{min} = 3\,R_s = (6GM)/c^2$ (defines inner edge of accretion disk)
  - ▷ for a black hole accreting from a thin disk, the efficiency of energy generation is (usually) determined by the binding energy of the inner most stable orbit ($\sim 6\,\%$ for a Schwarzschild black hole)
- no hair theorem: the structure of a black hole is completely determined by its mass M, angular momentum L and electric charge Q