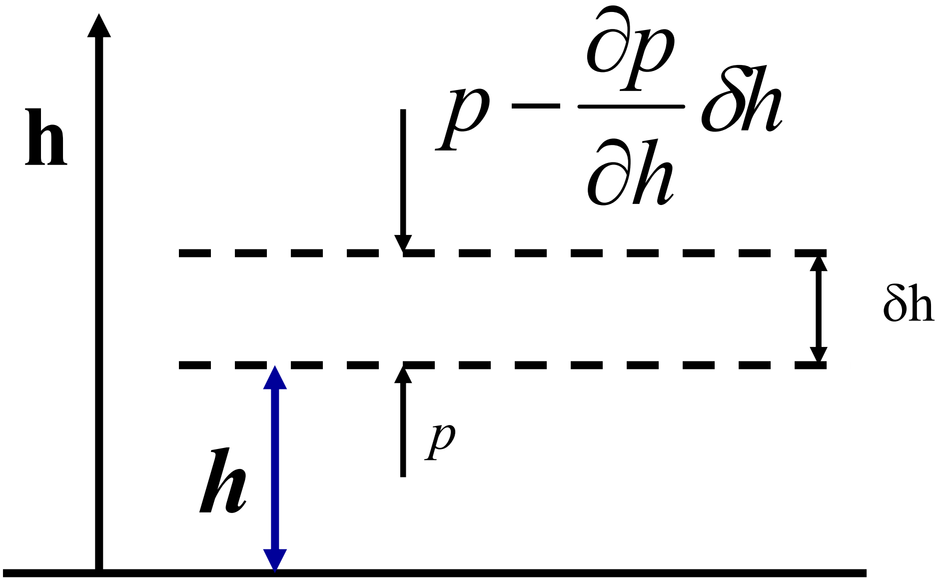
4C15/SS7 Homework Sheet 2

<u>*Answers*</u>

*Please hand in by Thursday, 14th March*
*Late submissions will incur a 10% penalty for each day past the due date.*

1.
a) For a star's atmosphere of uniform temperature and in hydrostatic equilibrium, derive an expression for the variation of pressure with height.




The pressure difference is given by:

$$\frac{\partial p}{\partial h}\delta h = -\delta h \times \rho \times g$$

where ρ is the density. But $p = \left(\frac{\rho}{m}\right)kT$ where m is the mass of the constituent particles. Thus

$$\rho = \frac{pm}{kT} \quad \text{and} \quad \frac{\partial p}{\partial h} = -p\frac{mg}{kT}$$

Integrating: $$p = p_0 \exp\left(-\frac{mg}{kT}h\right)$$

Thus pressure falls off exponentially with height in an atmosphere with uniform temperature

**[4]**

b) What is meant by the term "scale height"?

$h_0 = \left(\frac{kT}{mg}\right)$ has the dimensions of length and is called a "scale height".

**[1]**

c) Obtain the value of atmosphere scale height for a neutron star of mass $1M_\odot$, radius 10 km and surface temperature $10^6$ K. You may assume that the atmosphere is fully ionised and, for the purpose of the calculation, consists only of protons and electrons. [Boltzman's constant, $k = 1.38 \times 10^{-23}$ J deg $K^{-1}$; proton mass, $m_p = 1.67 \times 10^{-27}$ kg]

For a neutron star, $g = 10^{12}$ m/s$^2$ and $T \sim 10^6$ K. If hydrogen is the only constituent of the gas, then each proton and electron act as an independent particle of mass:

$$(m_p+m_e\text{-})/2 \sim m_p/2.$$

Thus $p = p_o \exp(-\, m_p gh/2kT)$ and $h_o = 2kT/m_p g \sim 0.01$m.

**[2]**
**[7 marks]**

2.

a) For a spectral feature, of rest wavelength $\lambda_o$, generated in a thin layer of the atmosphere of a neutron star near its surface, write an expression for the gravitationally red-shifted wavelength of the feature.

$$\lambda = \lambda_o (1 - 2GM/c^2R)^{-1/2}$$

**[1]**

b) If $\lambda_o = 18.97$ Å (O VIII Lyman α) is the rest wavelength of such a feature, what is the value of the gravitationally red-shifted wavelength in the case of a neutron star of mass 1.5 $M_\odot$ and radius 10 km?

$$\lambda = 18.97\,(1 - 2 \times 6.67.10^{-11} \times 1.5 \times 2.10^{30}/9.10^{16} \times 10^4)^{-1/2}$$
$$= 18.97 \times 1.342$$
$$= 25.46 \text{ Å}$$

**[2]**
**[3 marks]**

3.

a) If a 1 $M_\odot$ neutron star of radius 10 km has an observed X-ray luminosity, $L_X = 10^{31}$ J/s, what is the mass accretion rate, in $M_\odot$/year, needed to sustain this luminosity?

$$L_{acc} = GM\dot{m}/R$$

$$= 6.67.10^{-11} \times 2.10^{30} \times \dot{m}/1.10^4 \text{ or}$$

$$\dot{m} = 10^{31}/10^{16} = 10^{15} \text{ kg/s}$$

$$= 3.10^{22} \text{ kg/year}$$

$$= 10^{-8}\, M_\odot/\text{year}$$

**[3]**

b) Calculate the accretion yields or efficiencies (η) in units of $mc^2$ for the following 1 $M_\odot$ objects –

For $M_\odot$, G x M = $6.67.10^{-11}$ x $2.10^{30}$ = 1.33 x $10^{20}$

i. a neutron star

$R = 10^4$ m, η = 0.15

ii. a white dwarf

$R = 10^7$ m, η = 1.5 x $10^{-4}$

iii. the Sun

$R = 7.10^8$ m, η = 2.0 x $10^{-6}$

How do these compare with the typical value of η for nuclear fusion?

η = 0.007

**[3]**

c) Explain what is meant by the Eddington luminosity, $L_E$, and derive an expression for its value.

If L is the accretion luminosity, then the number of photons crossing unit area per sec at a distance r from the source is

$$= \frac{L}{4\pi r^2}\frac{1}{h\nu}$$

If the scattering cross-section is the Thomson cross-section, $\sigma_e$, then the number of scatterings per second will be

$$= \frac{L\sigma_e}{4\pi r^2 h\upsilon}$$

The momentum transferred from a photon to a particle is hν/c and thus the momentum gained per second by the particles is the force exerted by photons on particles which is therefore

$$= \frac{L\sigma_e}{4\pi r^2 h\upsilon}\frac{h\nu}{c} = \frac{L\sigma_e}{4\pi r^2 c} Newton$$

The source luminosity for which the radiation pressure balances the gravitational force on the accreting material is called the Eddngton luminosity and emerges from the equation

$$\frac{L\sigma_e}{4\pi r^2 c} = G\frac{Mm}{r^2}$$ which gives $L_{Edd} = \frac{4\pi cGMm}{\sigma_e}$

**[2]**

d) What are the values of $L_E$, in J/s for –

$$L_{Edd} = \frac{4\pi\left(3\times10^{8}\right)6.67\times10^{-11}.1.67\times10^{-27}}{6.65\times10^{-29}} \quad \text{x M J/s}$$

where m is taken as the proton mass since electrons and protons hold together through electrostatic forces

i. a 1 $M_{\odot}$ neutron star in a galactic binary system

$L_{Edd}$ ~ 6.3 x M J/s = $1.3.10^{31}$ J/s

ii. a $10^{8}$ $M_{\odot}$ black hole at the nucleus of an active galaxy

$L_{Edd}$ ~ 6.3 x M J/s = $1.3.10^{39}$ J/s

**[2]**

**[10 marks]**