UNIVERSITY COLLEGE LONDON

University of London

EXAMINATION FOR INTERNAL STUDENTS

For The Following Qualification:-

M.Sci.

Astronomy 4C15: High Energy Astrophysics

COURSE CODE	:	**ASTR4C15**
UNIT VALUE	:	**0.50**
DATE	:	**29-APR-05**
TIME	:	**10.00**
TIME ALLOWED	:	**2 Hours 30 Minutes**

05-C0084-3-40

TURN OVER

Answer THREE questions.

The numbers in square brackets indicate the provisional allocation of maximum marks per sub-section of a question

Symbols and quantities used in expressions:

Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-8}$ W m^{-2} K^{-4}
Gravitational constant	$G = 6.67 \times 10^{-11}$ m^3 kg^{-1} s^{-2}
Velocity of light	$c = 3 \times 10^8$ m s^{-1}
Mass of the Sun	$M_\odot = 2 \times 10^{30}$ kg
Radius of the Sun	$R_\odot = 7 \times 10^8$ m
Parsec	pc = 3.08×10^{16} m
Electron volt	eV = 1.6×10^{-19} J
Magnetic permeability of free space	$\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}

Numerical constants (in CGS units):

Gravitational constant	$G = 6.67 \times 10^{-8}$ dyn cm^2 g^{-2}
Speed of light	$c = 3 \times 10^{10}$ cm s^{-1}
Reduced Planck constant	$\hbar = 1.0544 \times 10^{-27}$ erg s
Stefan-Boltzmann constant	$\sigma = 5.67 \times 10^{-5}$ erg cm^2 s^{-1} K^{-4}
Electron charge	$e = 4.803 \times 10^{-10}$ esu
Electron mass	$m_e = 9.11 \times 10^{-28}$ g
Solar mass	$M_\odot = 2 \times 10^{33}$ g
Electron volt	eV = 1.6×10^{-12} erg
Parsec	pc = 3.08×10^{18} cm

PLEASE TURN OVER

1. (a) The macroscopic Maxwell Equations are

$$\nabla \cdot \vec{D} = 4\pi \rho$$

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t}$$

$$\nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{H} = \frac{1}{c}\frac{\partial \vec{D}}{\partial t} + \frac{4\pi}{c}\vec{J}$$

where $\vec{D} = \varepsilon \vec{E}$ and $\vec{B} = \mu \vec{H}$.

Show that $\vec{E}$ and $\vec{B}$ satisfy the wave equation in the absence of charge and current.
[Hint: use $\nabla \times \left(\nabla \times \vec{A}\right) = \nabla\left(\nabla \cdot \vec{A}\right) - \nabla^2 \vec{A}$.]

[8 marks]

(b) By considering a plane-wave solution, show that

$$\left(k^2 - \frac{n^2}{c^2}\omega^2\right)\begin{pmatrix}\vec{E}\\ \vec{B}\end{pmatrix} = 0,$$

where $\vec{k}$ is the wave propagation vector, ω is the angular frequency, and $n = \sqrt{\mu\varepsilon}$ is the refractive index.

[4 marks]

(c) The speed of light in water at rest is $v_0 = c/n_w$, with the refractive index of water $n_w = 1.33$. It was found that the speed of light in water flowing at a speed u with respect to an observer can be expressed as $v = \frac{c}{n_w} + \lambda u$, where the coefficient λ is a constant depending on n_w.

Starting from the Lorentz transform for the velocity: $v = \frac{v_0 + u}{1 + v_0 u / c^2}$.

Show, in the limit of small water flow velocity, that $v \approx \left(\frac{c}{n_w} + u\right)\left(1 - \frac{u}{n_w c}\right)$, and that the numerical value of λ is approximately 0.43.

[8 marks]

CONTINUED

2. (a) What is the main difference between a Schwarzschild black hole and a Kerr black hole? Express the Schwarzschild radius of a black hole in terms of its mass and the fundamental constants. What are these fundamental constants?

[4 marks]

(b) Now consider the convention $\hbar = G = c = 1$. The Hawking temperature T of a black hole of mass M, charge Q and angular momentum a is given by

$$k_B T = \frac{1}{2\pi} \frac{\sqrt{M^2 - a^2 - Q^2}}{r_+^2 + a^2},$$

where k_B is the Boltzmann constant, $r_+ = r_g + \sqrt{r_g^2 + a^2}$ and the gravitational radius $r_g = M$. What is the expression of the Hawking temperature for a Schwarzschild black hole?

[3 marks]

(c) Suppose that for Schwarzschild black holes numerically

$$T = 0.62 \times 10^{-7} \left(\frac{M_{Sun}}{M} \right) K.$$

What is the Hawking temperature of a Schwarzschild black hole of 10 solar masses?

What is the radius of the event horizon of this black hole?

What is the Hawking temperature of a Schwarzschild black hole of 10^9 solar masses?

What is the radius of the event horizon of this black hole?

[4 marks]

(d) The event horizon of the black hole is a perfect black body radiator. Estimate numerically the energy flux of the Hawking radiation from a 10 solar mass Schwarzschild black hole. How long can this black hole live? Can this black hole have survived till now if it was born 3 minutes after the Big Bang?

[9 marks]

3. Supernova explosions occur at the end of the evolutionary history of stars. In the context of supernovae, outline the steps that are believed to occur in the evolution of a star with mass > 8 $M_{\odot}$. You should describe the sequence from the original formation of a gas cloud to the final collapse of the star.

[5 marks]

For stellar masses > 8 $M_{\odot}$, describe the two possible outcomes of the supernova. What outcome is believed to result for stellar masses < 2 $M_{\odot}$?

[3 marks]

What is a neutron star? Sketch the internal cross-section of a neutron star and indicate the four main regions of the interior.

[5 marks]

Describe a recent spectral observation of an X-ray burst source by the XMM-Newton spacecraft and say which fundamental properties of neutron stars were verified by this observation.

[3 marks]

If a star of radius 1 $R_{\odot}$ and a polar magnetic field of ~ 0.01 Tesla collapses to form a neutron star of radius 10 km, what will be the resulting neutron star magnetic field strength?

[2 marks]

If the main energy loss mechanism for a rotating neutron star is by magnetic dipole radiation, indicate a method for estimating the magnetic field of the object.

[2 marks]

CONTINUED

4. What are the principal observational properties of Active Galactic Nuclei (AGN)?

[4 marks]

If variability in the radiation output of these sources is observed on a timescale $\tau \geq 10^4$ seconds, deduce the approximate size of the emitting region. What kind of central object does this suggest is present in the active nucleus?

[2 marks]

Given that a typical AGN luminosity is $L \sim 10^{40}$ J s^{-1}, calculate the mass accretion rate in $M_\odot$/year required to sustain this luminosity. Why does the accreting material form a disc around the central object?

[2 marks]

Since disc self-gravitation is negligible, the accreting material is in Keplerian rotation with angular velocity $\Omega_K = (GM/R^3)^{1/2}$ where R is the radius of an individual ring of gas in the disc. If ν is the kinematic viscosity for rotating rings of gas, write an expression for the viscous torque, $Q(R)$, exerted by a ring of gas at $R + dR$ on the neighbouring ring at radius R. Hence deduce the radiative energy dissipation per unit area of the disc.

[4 marks]

For an accretion rate $\dot{M}$, it can be shown that $\nu\Sigma = (\dot{M}/3\pi)\,[1 - (R_*/R)^{1/2}]$ where R_* is the radius of the central accreting object and $\Sigma = H\rho$ is the surface density of the disc with H being the disc scale height in the z direction. Show that the radiative energy flux through the faces of the disc is independent of viscosity.

[4 marks]

If the accretion disc can be considered as a series of rings radiating as black bodies, derive an expression for $T(R)$, the disc temperature as a function of radius.

[4 marks]

CONTINUED

5. What are Radio Galaxies? With the help of a diagram, briefly describe their observational features and show how their morphology is thought to relate to the standard model of Active Galaxies.

[4 marks]

Assuming that the radio luminosity is due to synchrotron emission from relativistic electrons with a power law energy spectrum and radiating in the magnetic fields of the extended lobes of the radio source, derive the minimum overall energy in the source by deducing the energy density in relativistic electrons and in magnetic fields.

[8 marks]

For the well-known radio galaxy, Cygnus A, the radio luminosity is $\sim 5 \times 10^{37}$ J s^{-1}. If the diameter of each radio lobe is ~ 75 kpc, and the emerging jet velocity is 10^3 km s^{-1}, use the minimum energy result you have derived to obtain an estimate of the magnetic field strength in the lobes.

[4 marks]

If the maximum radio frequency observed is 10^{11} Hz, use the equipartition value of the magnetic field to obtain estimates of the typical Lorentz factor and lifetime of the relativistic electrons in the radio lobes. How does the electron lifetime compare with the value required by the jet velocity and the observed source extent?

[You may assume that for synchrotron radiation, the maximum frequency is given by $\nu_m = 4.2 \times 10^{36} E^2 B$ Hz where E – the electron energy, is in J and B is in Tesla, while the time for an electron of energy E in a magnetic field B to loose its energy by synchrotron radiation is given by $\tau_{syn} = 5 \times 10^{-13} B^{-2} E^{-1}$ s]

[4 marks]

END OF PAPER