Disk accretion

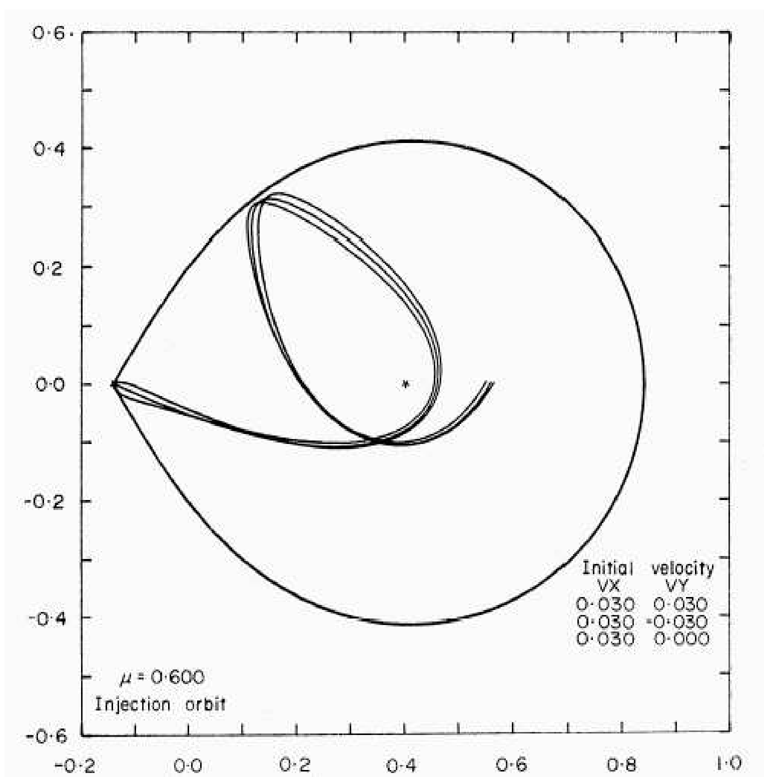
For Roche lobe overflow, specific angular momentum of gas at the L_1 point is,

$$l_{L_1} = X_1^2 \Omega_B$$

where X_1 is the distance of the L_1 point from the accreting star. X_1 is given approximately by,

$$X_1 \simeq (0.5 - 0.227 \log q)a.$$

Sound speed in the stellar atmosphere near the L_1 point is normally $\ll$ orbital velocity of the binary. Gas leaving L_1 thus follows approximately ballistic trajectories.



Particle orbits are self-intersecting → collision of the gas stream with itself, dissipation, and formation of a disk.

Define the *circularization radius* R_c as the radius where gas in Keplerian orbit has the same specific angular momentum as the gas leaving L_1,

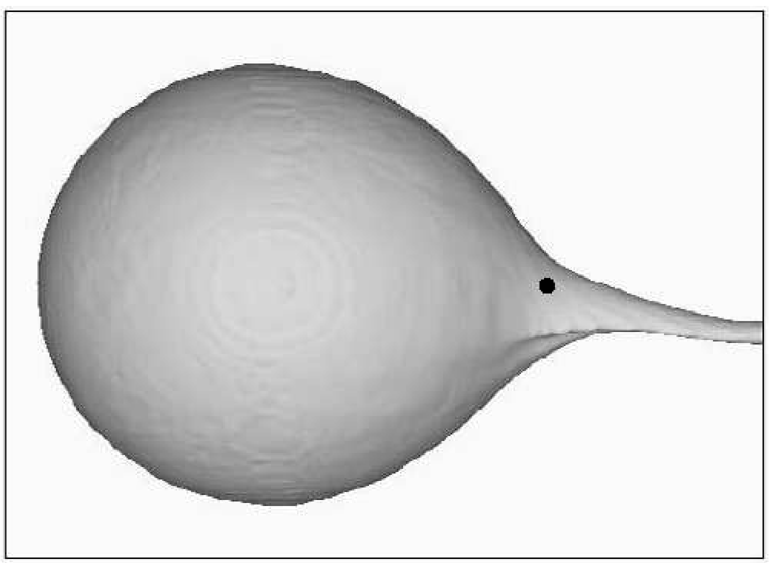
$$\sqrt{GM_{acc}R_c} = X_1^2\Omega_B$$

which gives,

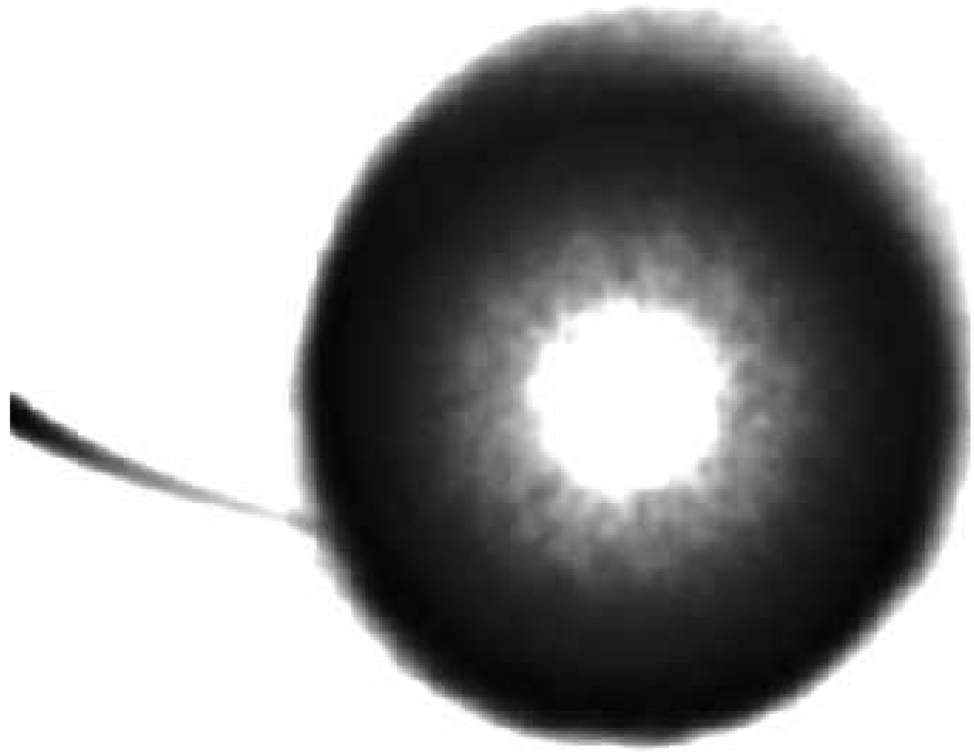
$$R_c \simeq (1+q)(0.5 - 0.227\log q)^4 a.$$

e.g. for $q = 0.5$ find $R_c/a \approx 0.16a$. Normally smaller than the Roche lobe of the accretor by a substantial margin, but larger than the radius of any compact accretor (white dwarf, neutron star, black hole).

Detailed discussion of flow through L_1 by Lubow & Shu (1975). Simulations by Oka et al. (2002):

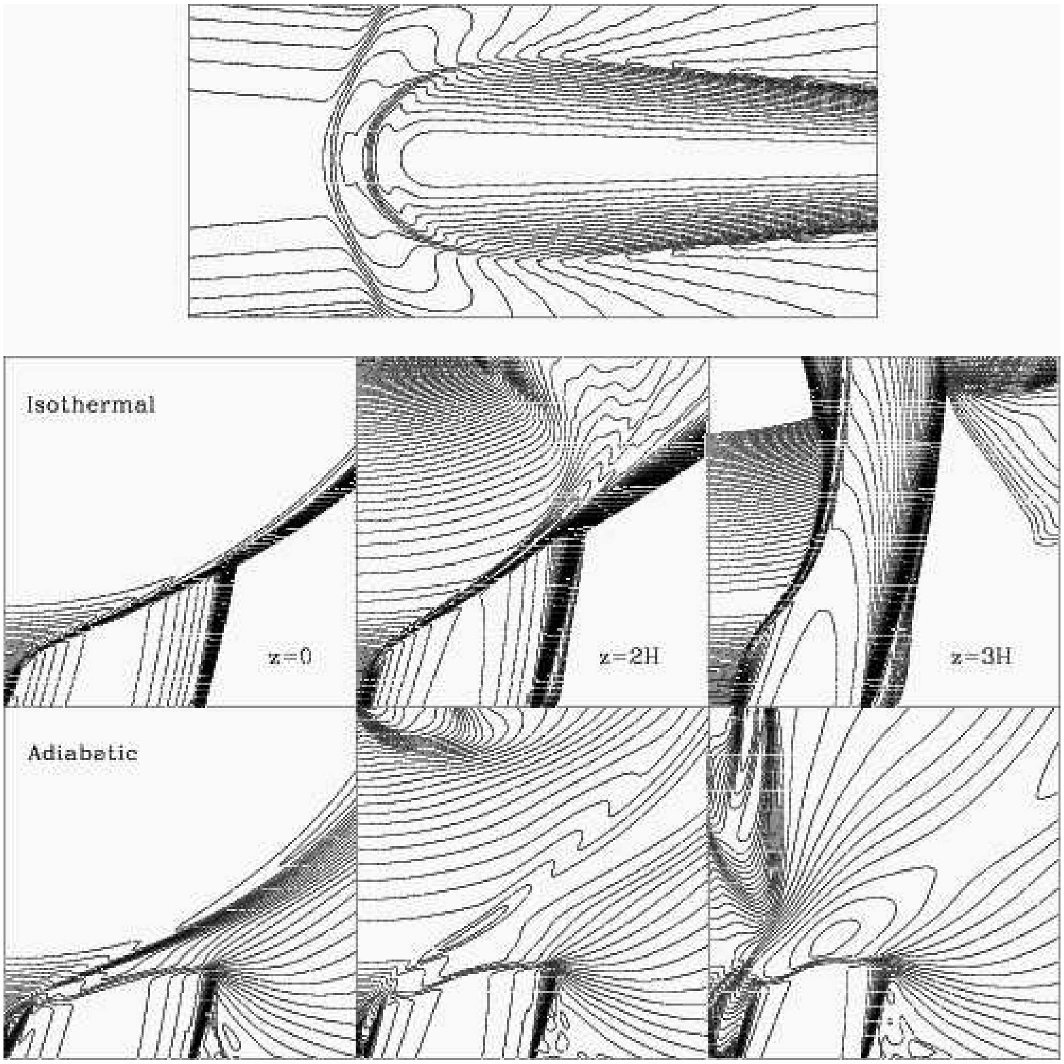


Structure of a disk accreting system:



- Gas stream from the L_1 point.
- A hotspot where the stream collides hypersonically with the edge of the disk.
- Disk distorted in the tidal potential of the binary. For $q < 0.25$, disk develops an eccentric instability and precesses (Lubow 1991).

Close-up of the hotspot region:



Observations of absorption in nearly edge-on X-ray binaries provide evidence of the flow in this region.

Disk evolution

Adopt cylindrical polar co-ordinates (R, ϕ, z). Gas in the disk at radius R has azimuthal velocity $v_\phi = R\Omega(R)$, where Ω is the angular velocity. Rate of shearing of the flow,

$$A \equiv R\frac{d\Omega}{dR}$$

is generally nonzero – ie disk rotates differentially. Any dissipation in the flow will act to damp shearing motions, converting them into heat ($\rightarrow$ radiation). Energy must come from the potential energy $\rightarrow$ accretion.

To derive equation for the evolution of the disk surface density Σ, consider an annulus with inner radius R and width ΔR. Conservation of mass gives,

$$\begin{aligned}\frac{\partial}{\partial t}(2\pi R \cdot \Delta R \cdot \Sigma) &= v_R(R,t) \cdot 2\pi R \cdot \Sigma(R,t) \\ &- v_R(R+\Delta R,t) \cdot 2\pi(R+\Delta R) \cdot \Sigma(R+\Delta R,t).\end{aligned}$$

where v_R is the radial velocity. Taking the limit,

$$R\frac{\partial \Sigma}{\partial t} + \frac{\partial}{\partial R}(R\Sigma v_R) = 0.$$

Identical procedure for the angular momentum gives,

$$R\frac{\partial}{\partial t}(\Sigma R^2\Omega) + \frac{\partial}{\partial R}(R\Sigma v_R \cdot R^2\Omega) = \mathcal{G}$$

where $\mathcal{G}$ is the *net* effect of the viscous torques from interior and exterior annuli. If the torque of an outer annulus acting on a neighboring inner one at radius R is $G(r,t)$, then,

$$\mathcal{G} = \frac{1}{2\pi}\frac{\partial G}{\partial R}.$$

(note 2π from definition of $\mathcal{G}$). From the definition of the kinematic viscosity ν, the viscous force per unit length along the boundary between two annuli is $\nu\Sigma A$. Hence,

$$G(R,t) = 2\pi R \cdot \nu\Sigma A \cdot R.$$

Substituting this expression for G back into the angular momentum equation, and then eliminating v_R using the continuity equation, gives an equation for disk evolution,

$$\frac{\partial}{\partial t}(\Sigma R^2\Omega) + \frac{1}{R}\frac{\partial}{\partial R}(\Sigma R^3\Omega v_R) = \frac{1}{R}\frac{\partial}{\partial R}(\nu\Sigma R^3\Omega')$$

where $\Omega' \equiv d\Omega/dR$.

Specializing to the case of a point mass potential,

$$\Omega = \left(\frac{GM_{acc}}{R^3}\right)^{1/2}$$

obtain,

$$\frac{\partial\Sigma}{\partial t} = \frac{3}{R}\frac{\partial}{\partial R}\left[R^{1/2}\frac{\partial}{\partial R}(\nu\Sigma R^{1/2})\right].$$

If ν is a general function of the local conditions in the disk (ie, R, Σ, possibly t), then this is a nonlinear diffusion equation for the surface density. In the special case where ν is a power law in R only, it is a linear diffusion equation for which analytic solutions are known.

e.g. for constant ν, solution for the evolution of a ring of mass m at initial radius R_0 is,

$$\Sigma(x, \tau) = \frac{m}{\pi R_0^2 \tau x^{1/4}} e^{-(1+x^2)/\tau} I_{1/4}(2x/\tau)$$

where,

- $x \equiv R/R_0$
- $\tau \equiv 12 \nu t R_0^{-2}$
- $I_{1/4}$ is a modified Bessel function of order 1/4.

Solution for constant viscosity,

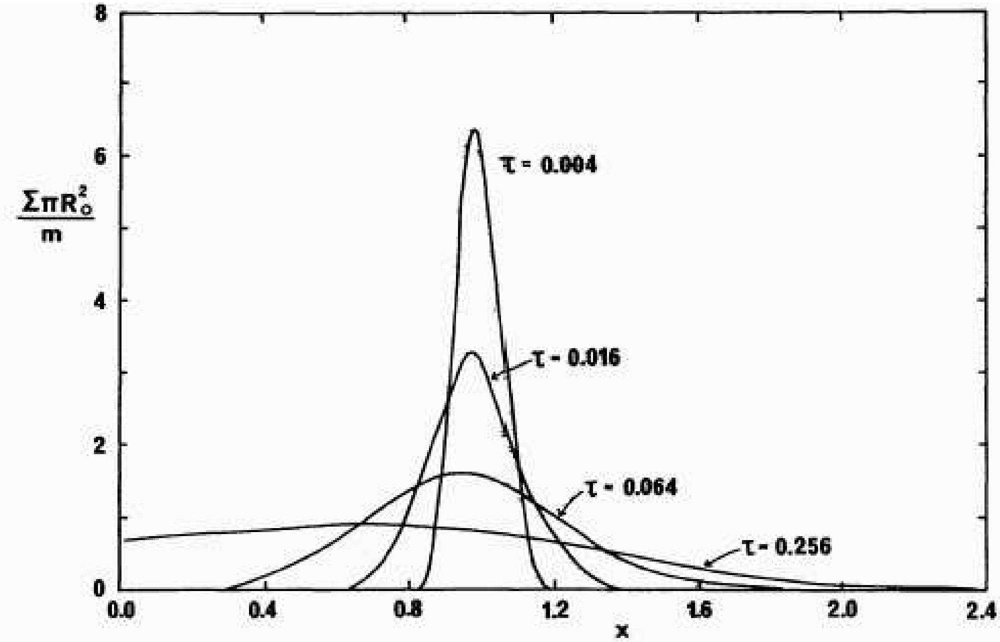


Figure 1 The viscous evolution of a ring of matter of mass m. The surface density Σ is shown as a function of dimensionless radius $x = R/R_0$, where R_0 is the initial radius of the ring, and of dimensionless time $\tau = 12\nu t/R_0^2$ where ν is the viscosity.

Find (eg Pringle 1981, ARA&A, 19, 137),

- Viscosity tends to spread the ring out.
- Bulk of the mass moves to small radius.
- Tail moves to large radius to conserve total angular momentum.

In most (all?) cases the source of ν is probably turbulence driven by magnetohydrodynamic instabilities (Balbus & Hawley 1991).

Steady disks

The disk in a mass transfer binary is continuously replenished by mass flow from L_1. In the absence of global instabilities, useful to consider the structure of a steady disk.

Defining $\dot{M}$ as the steady inward mass flux, continuity gives,

$$\dot{M} = 2\pi R \Sigma (-v_R).$$

Integrating the angular momentum equation,

$$\nu \Sigma (-\Omega') = \Sigma (-v_R)\Omega - \frac{C}{2\pi R^3}$$

where C is a constant of integration. At a point in the flow where the shear vanishes (ie $\Omega' = 0$),

$$C = \dot{M} R^2 \Omega.$$

ie C is the **flux of angular momentum** through the disk. Normally, for a slowly rotating star, the location where the shear in a thin disk vanishes is close to the surface of the accreting star at $R = R_*$ (marginally stable circular orbit for a black hole). Thus,

$$C \simeq \dot{M} \sqrt{G M_{acc} R_*}.$$

Using this boundary condition, obtain the basic relation for steady disks,

$$\nu \Sigma = \frac{\dot{M}}{3\pi} \left[1 - \sqrt{\frac{R_*}{R}} \right].$$

A kinematic viscosity ν generates dissipation in the disk at a rate $D(R)$ per unit area per unit time, where

$$D(R) = \frac{1}{2}\nu\Sigma(R\Omega')^2.$$

Substituting for $\nu\Sigma$,

$$D(R) = \frac{3GM_{acc}\dot{M}}{4\pi R^3}\left[1 - \sqrt{\frac{R_*}{R}}\right].$$

If the disk is optically thick to its own thermal radiation, then in a steady state,

$$D = 2\sigma T_{eff}^4$$

which implies,

$$T_{eff} = \left(\frac{3GM_{acc}\dot{M}}{8\pi R^3\sigma}\left[1 - \sqrt{\frac{R_*}{R}}\right]\right)^{1/4}.$$

Note,

- The effective temperature (more generally the dissipation) **does not depend on the viscosity** in a steady state.
- $T_{eff} \propto R^{-3/4}$ at large radius ($R \gg R_*$).

Order of magnitude disk temperatures. For $R \gg R_*$,

$$T = T_* \left(\frac{R}{R_*}\right)^{-3/4}.$$

White dwarfs

$$T_* = 4.1 \times 10^4 \left(\frac{\dot{M}}{10^{16}\ \mathrm{gs}^{-1}}\right)^{1/4} \left(\frac{M_{acc}}{M_\odot}\right)^{1/4} \left(\frac{R}{10^9\ \mathrm{cm}}\right)^{-3/4}\ \mathrm{K}.$$

→ inner disk should be bright in the UV.

Neutron stars

$$T_* = 1.3 \times 10^7 \left(\frac{\dot{M}}{10^{17}\ \mathrm{gs}^{-1}}\right)^{1/4} \left(\frac{M_{acc}}{M_\odot}\right)^{1/4} \left(\frac{R}{10^6\ \mathrm{cm}}\right)^{-3/4}\ \mathrm{K}.$$

→ inner disk should be bright in X-rays.

Black holes

At the Eddington limit, $\dot{M} \propto M_{acc}$. The radius of the innermost stable orbit also scales linearly with M_{acc}. Thus,

$$T_* \propto M_{acc}^{-1/4}$$

More massive black holes should have cooler thermal X-ray spectra. But note that nonthermal emission is also often present.