Binary evolution

Properties of several classes of binary stars are inexplicable in terms of independent single star evolution.

Algols: prototype comprises an unevolved main-sequence star (spectral type B, mass $3.7M_\odot$) in a binary with an evolved subgiant companion (spectral type G, mass $0.8M_\odot$).

Why has the lass massive star apparently evolved *faster* than the more massive star?

Compact binaries: A binary with $P < 2$ hours has a separation (for Solar mass objects) $a < R_\odot$. Obviously must have been some interaction between the progenitors of the compact objects while they were on the main sequence.

Need to consider,

- How mass can be transferred between components in a close binary, either by stellar winds or Roche lobe overflow.
- Mechanisms for altering the binary separation. Angular momentum can be exchanged between the stars due to mass transfer, and / or lost entirely due to outflows or gravitational radiation.

Roche approximation

Consider a binary in which stars of mass M_1 and M_2 rotate about the center of mass in a circular orbit. In the Roche approximation, we assume,

- That the gravitational field can be approximated as that of two point masses.
  - Valid if both stellar radii $R_1, R_2 \ll a$, the binary separation.
  - A good approximation for close binaries if the stars are highly centrally concentrated (ie have a large effective polytropic index n).
- That the stars *corotate* with the rotation of the binary.

In a coordinate system which rotates with the binary, with the origin at the center of mass, the potential (gravitational plus centrifugal) is,

$$\phi = -\frac{GM_1}{|\mathbf{r} - \mathbf{r_1}|} - \frac{GM_2}{|\mathbf{r} - \mathbf{r_2}|} - \frac{1}{2}\left(\mathbf{\Omega_B} \times \mathbf{r}\right)^2$$

where $\mathbf{r_1}$ and $\mathbf{r_2}$ are the position vectors of the stellar centers and $\mathbf{\Omega_B}$ is the angular velocity vector of the binary system.

Magnitude of the angular velocity is,

$$\Omega_B = \sqrt{\frac{G(M_1 + M_2)}{a^3}}.$$

If $\mathbf{v}$ is the velocity of a fluid element relative to the rotating frame, the equation of motion is,

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla\phi - \frac{1}{\rho}\nabla P - 2\mathbf{\Omega_B} \times \mathbf{v},$$

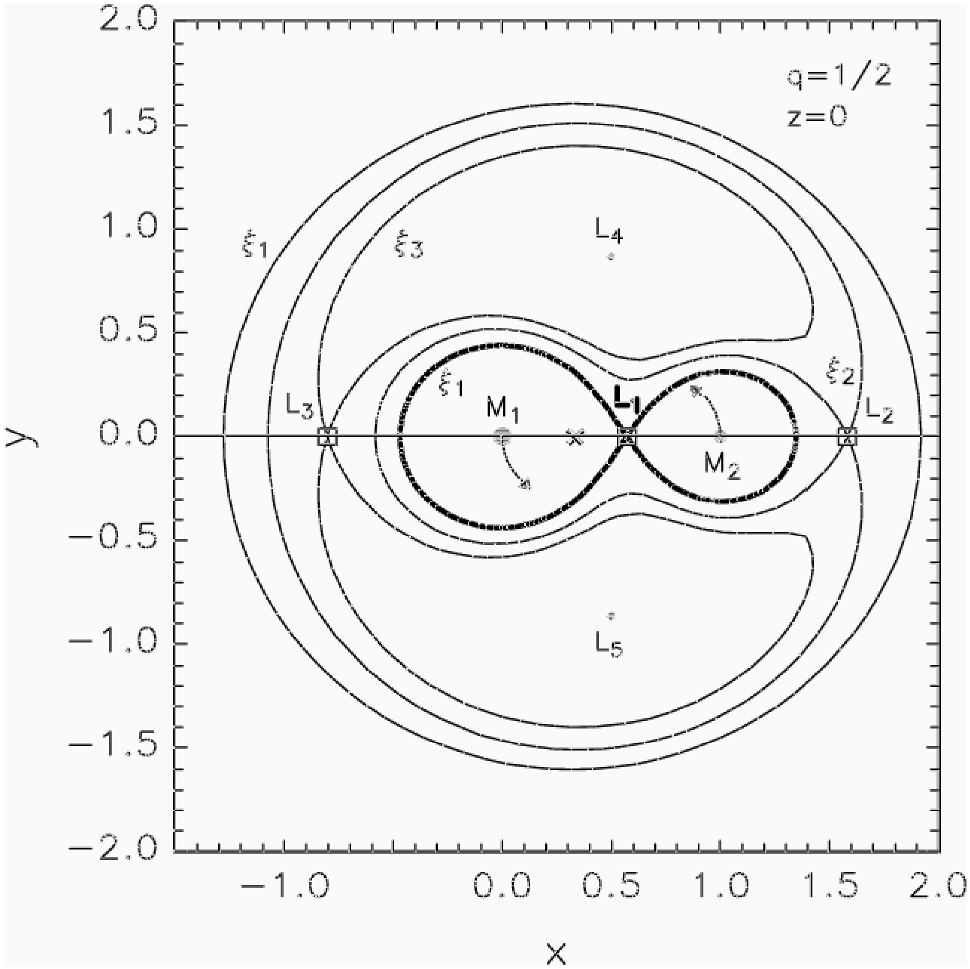
where P is the gas pressure. Note that,

- ϕ is the Roche potential, which includes centrifugal effects.
- The equation of motion includes Coriolis forces, ie the gas in the rotating frame does *not* just accelerate under the influence of gradients of potential and pressure.

For a fluid element in one of the stars, corotation implies that $\mathbf{v} = \mathbf{0}$.

Since P is a constant (≈ 0) at the stellar surfaces, we require that in corotation $\nabla\phi$ vanishes. Stellar shape is defined by the condition that $\phi =$ constant – Roche equipotentials.

Equipotentials in the orbital plane,



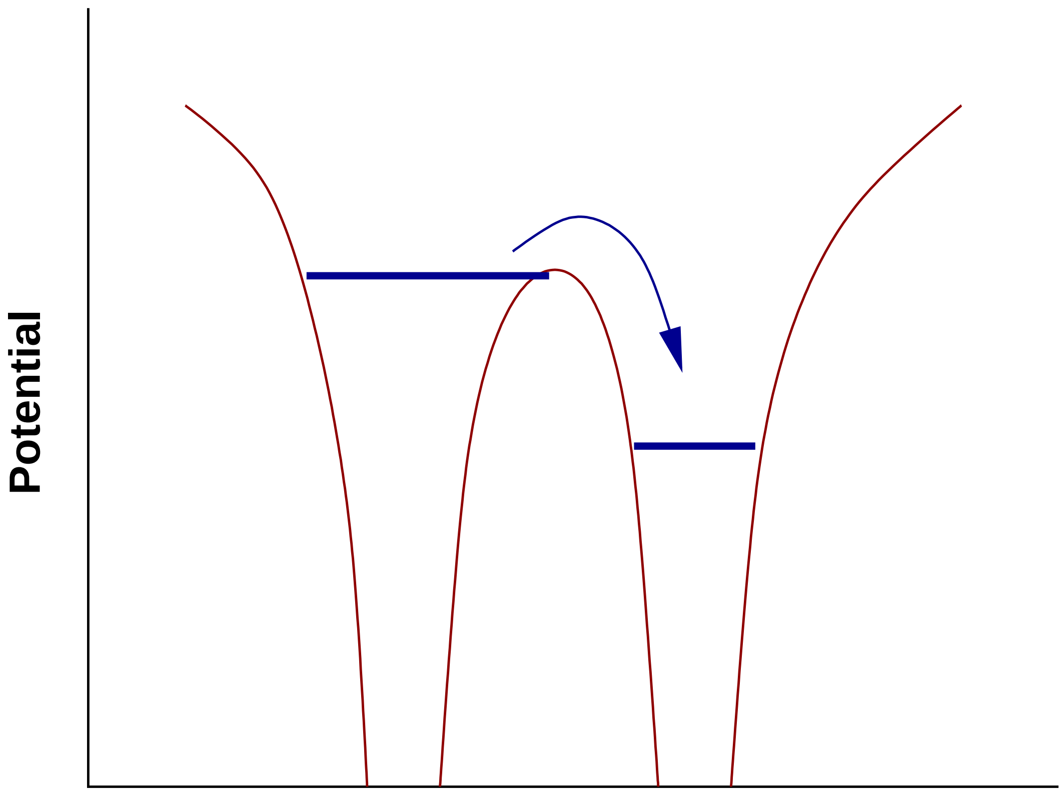
Critical equipotential which passes through the inner Lagrangian point L_1 defines the Roche lobes of the two stars. L_1 is a *saddle point* of the potential – ie gas bound to one star in the vicinity of L_1 finds it easier to pass through L_1 onto the other star than to escape completely.

Note: the Roche potential assumes corotation. This will not be valid for gas trying to leave the system entirely.

Terminology. A binary system is,

- **Detached** if both components are smaller than their Roche lobes.
- **Semi-detached** if one component fills its Roche lobe.
- A **contact** binary if both components fill their Roche lobes.

Schematically, for a semi-detached system:

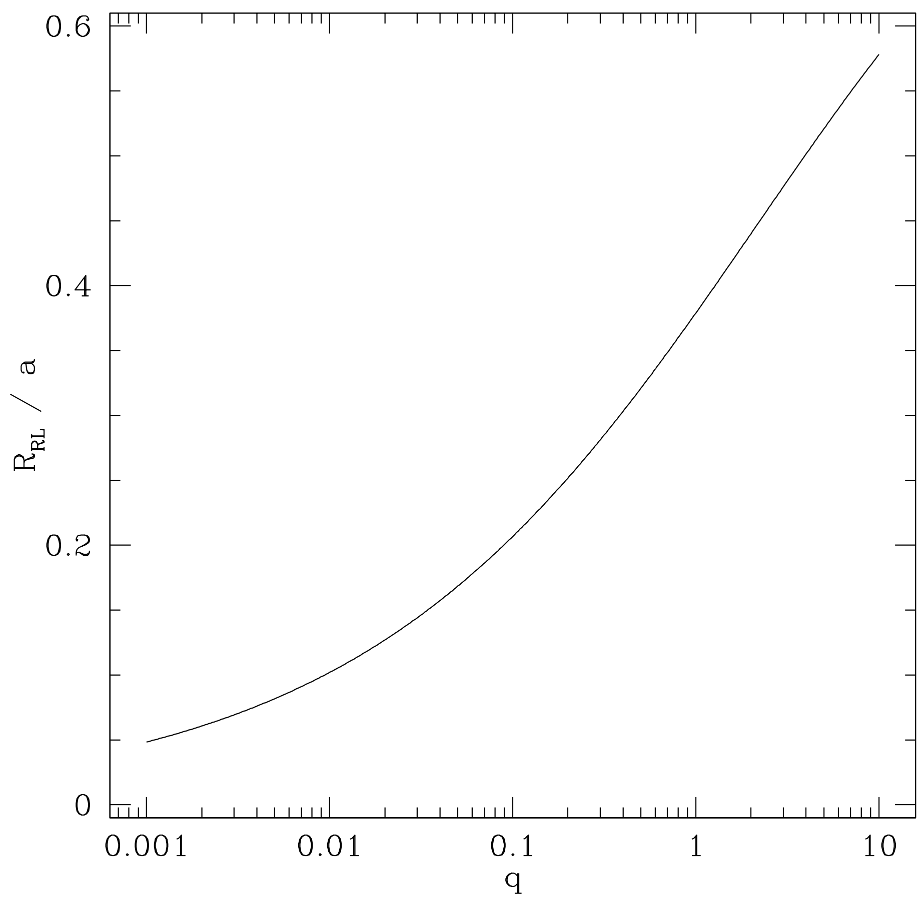


Semi-detached systems have one highly distorted component → strong tidal effects → assumption of corotation is justified.

The conventional measure of the 'size' of the Roche lobe is the radius of a sphere which has the same volume as the lobe. This radius R_{RL} can be approximated as (Eggleton 1983),

$$\frac{R_{RL}}{a} = \frac{0.49q^{2/3}}{0.6q^{2/3} + \ln(1 + q^{1/3})}$$

where $q = M_2/M_1$. This formula is accurate for all q, and has the form:



For $q < 0.5$ or so, a simple alternative approximation is,

$$\frac{R_{RL}}{a} \simeq 0.462 \left(\frac{q}{1+q}\right)^{1/3}.$$

Written in this form, easy to see that the mean density of a secondary that fills its Roche lobe is directly related to the period of the binary orbit,

$$\bar{\rho} = \frac{M_2}{\frac{4}{3}\pi R_{RL}^3} \propto \frac{(M_1 + M_2)}{a^3} \propto P^{-2}$$

where $P = 2\pi/\Omega_B$ is the orbital period.

In many mass transfer binaries, the secondaries appear to be unevolved low mass stars. Such stars have an approximate mass-radius relation,

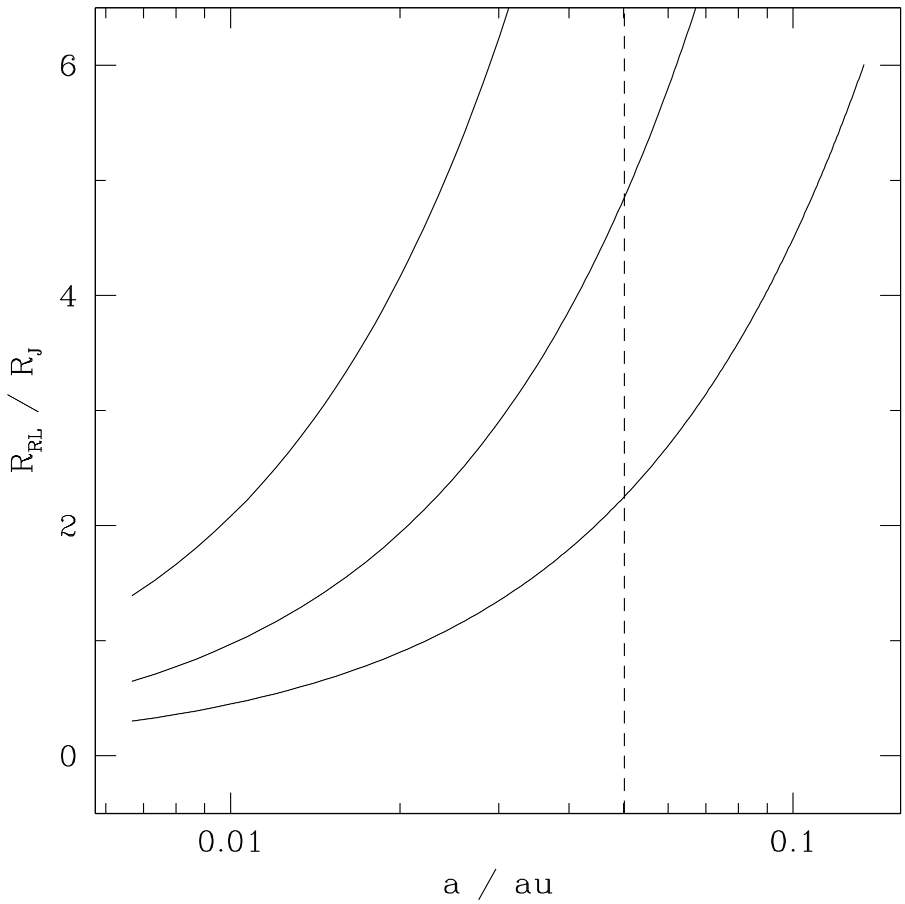
$$\frac{R}{R_\odot} = \frac{M}{M_\odot},$$

ie their mean density scales as $\bar{\rho} \propto M^{-2}$. If they fill their Roche lobes, the binary geometry then requires that $P \propto M$, specifically,

$$\frac{M_2}{M_\odot} \simeq 0.11 \left(\frac{P}{1 \text{ hour}}\right).$$

Implies that for cataclysmic variables with periods less than about 7 hours we can obtain an estimate of the secondary mass using only knowledge of the orbital period.

Example: How close are hot Jupiters to overflowing their Roche lobes? For $q = 10^{-4}$, $q = 10^{-3}$ and $q = 10^{-2}$ obtain,



Transiting planet HD 209458 has a radius of $1.35 R_J$, an orbital radius of 0.05 au, and $q \sim 10^{-3} \rightarrow$ very safe against Roche lobe overflow.

Possibility of mass loss at early epoch when the planets were larger.

Stability of mass transfer

Once mass starts being transferred between the stars in a semi-detached binary, both the mass of the components and the orbital separation will change. The stability of the mass transfer process is determined by:

- How the size of the Roche lobe changes in response to the change in mass, *relative to...*
- How the size of the star changes in response to the mass loss.

First part is simple to quantify. Angular momentum of the binary is,

$$J = (M_1 a_1^2 + M_2 a_2^2)\Omega_B$$

where a_1 and a_2 are the distances of the two stars from the center of mass,

$$\begin{aligned} a_1 &= \left(\frac{M_2}{M}\right) a \\ a_2 &= \left(\frac{M_1}{M}\right) a \end{aligned}$$

and $M = M_1 + M_2$ is the total mass. Substituting for Ω_B, a_1 and a_2,

$$J = M_1 M_2 \left(\frac{Ga}{M}\right)^{1/2}.$$

If all the mass lost by the secondary is accreted by the primary,

$$\dot{M}_1 + \dot{M}_2 = 0$$

and $\dot{M} = 0$. Differentiating the expression for the angular momentum gives,

$$\frac{\dot{a}}{a} = \frac{2\dot{J}}{J} - 2\frac{\dot{M}_2}{M_2}\left(1 - \frac{M_2}{M_1}\right).$$

The mass transfer is described as **conservative** if the mass and angular momentum of the binary remains fixed. Setting $\dot{J} = 0$ in this limit, and noting that $\dot{M}_2 < 0$ if we assume that the secondary is losing mass, find,

- That the binary expands ($\dot{a} > 0$) if conservative transfer occurs from the less massive star to the more massive.
- The binary shrinks if it is the more massive star which loses mass.

Obvious: if more mass ends up near the center of mass (as occurs if the mass losing star is of lower mass), then the remaining mass must move to larger radius to conserve total angular momentum.

The size of the Roche lobe also depends upon the mass ratio. For illustration, use the simple form for the size of the secondary's Roche lobe,

$$R_2 \propto M_2^{1/3} a.$$

Differentiating gives,

$$\frac{\dot{R}_2}{R_2} = \frac{\dot{a}}{a} + \frac{\dot{M}_2}{3M_2}.$$

Eliminating $\dot{a}/a$,

$$\frac{\dot{R}_2}{R_2} = \frac{2\dot{J}}{J} - 2\frac{\dot{M}_2}{M_2}\left(\frac{5}{6} - \frac{M_2}{M_1}\right).$$

Within this simple model (note: we are now cheating by using the approximation for the Roche lobe outside its domain of validity) there are two cases.

q > 5/6: Conservative mass transfer *shrinks* the Roche lobe down on the mass-losing star. Any angular momentum loss ($\dot{J} < 0$) only makes matters worse. Unless the star can contract rapidly enough, expect,

- Violent mass transfer, occurring on a dynamical or thermal timescale.
- A cessation of mass transfer once the mass ratio is reversed.

Expect this to be the initial phase of mass transfer in binaries when the more massive star expands to fill its Roche lobe after leaving the main sequence.

Very short, therefore unobservable.

q < 5/6: Conservative mass transfer expands the Roche lobe. Mass transfer will then continue if,

- The star expands, either *due* to mass loss, or due to ongoing evolution after leaving the main sequence. In the latter case, the radius will expand on the timescale set by the nuclear evolution $t_{\rm nuc}$. Then,

$$\frac{\dot{R}_2}{R_2} \propto t_{\rm nuc}^{-1}$$

  which implies a mass transfer rate,

$$-\dot{M}_2 = \frac{M_2}{(5/3 - 2q)t_{\rm nuc}}$$

  and a change in separation,

$$\frac{\dot{a}}{a} = \frac{(1-q)}{(5/3 - 2q)t_{\rm nuc}}.$$

- The binary loses angular momentum. This will be the most important case if the secondary is of low mass, when the evolutionary timescale is very long. If, on losing mass, the star remains in thermal equilibrium, then $\dot{R}_2/R_2 = \dot{M}_2/M_2$, yielding,

$$-\frac{\dot{M}_2}{M_2} = \frac{-\dot{J}/J}{4/3 - M_2/M_1}.$$

  Transfer rate and shrinkage of the orbit directly set by the angular momentum loss rate.