

A. G. Polnarev. *Mathematical aspects of cosmology (MTH6123)*, 2009. VI. **Gravitational Instability and Formation of Structure in the Universe.**

Part VI. Gravitational Instability and Formation of Structure in the Universe.

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Chapter 25. Formation of structure from Density fluctuations.

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25.1. Gravitational instability.

Astronomers observe considerable structure in the universe, from stars to galaxies to clusters and superclusters of galaxies. The famous "Deep Field Image" taken by the Hubble Space Telescope provides a stunning view of such structure. How did these structures form? The Big Bang theory is widely considered to be a successful theory of cosmology, but the theory is incomplete. It does not account for the needed fluctuations to produce the structure we see. Most cosmologists believe that the galaxies that we observe today grew from the gravitational pull of small fluctuations in the nearly-uniform density of the early universe. These fluctuations leave an imprint in the CMB (cosmic microwave background radiation) in the form of temperature fluctuations from point to point across the sky. The WMAP satellite measures these small fluctuations in the temperature of the cosmic microwave background radiation and in turn probe the early stages of structure formation (see Fig. 25.1).

Although the Universe is well described on large scales by the isotropic, homogeneous Friedman model, the existence of galaxies and clusters implies that it must have contained small density fluctuations at early times. Small density fluctuations evolve with time due to the process of "gravitational instability": Regions which were more dense than average would then have expanded more slowly than the background, so that the overdensity would have increased. The fluctuations evolve predominantly in the period after decoupling, when photons can not stop the process of gravitational instability. Such fluctuations develop into the objects we see today. Eventually the region would have stopped expanding altogether, at which point it would have separated out from the background and formed a gravitationally bound system.

25.2. The origin of density fluctuations.

There are several options:

i.[unlikely] The Universe started out chaotic, with large inhomogeneities and anisotropies on all scales, but that various dissipative processes at early times reduced it to nearly Friedman form. In this picture the inhomogeneities which develop into large-scale structure would represent the small residual imprint of the initial chaos. It now seems unlikely that this is true. It is difficult to smooth out a chaotic Universe anyway and, even if it were possible, the dissipation involved would tend to produce far more entropy than is observed in the CMB.

ii.[most cosmologists believe] The deviations from the isotropic and homogeneous Friedman model always being small. In this case, the inhomogeneities required for the structure formation can be:

a) Primordial, in the sense that they can be considered as the initial conditions of the Universe.

b) Spontaneous in the sense that they arose naturally in a Universe which started off perfectly smooth. The latter possibility is obviously more appealing since, if one has to claim that the inhomogeneities exist from the beginning, one is not really explaining anything.

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Several schemes have been proposed for the spontaneous generation of inhomogeneities: 1) the Universe undergoes some sort of phase transition (for example, a spontaneously broken symmetry epoch - monopoles, strings, domain walls or textures), 2) quantum effects associated with an early inflationary phase, 3) quantum gravity effects at the Planck epoch (our understanding is still is poor).

25.3. Power spectrum of fluctuations.

Whatever the origin of the fluctuations, we need to know how to characterize their form. If we consider an ensemble of regions with fixed mass M , some will have higher density (or smaller volume) than average, while others will have lower density (or larger volume). One usually has Gaussian fluctuations, in which case the average density over the ensemble will give the mean cosmological value while the variance will specify the amplitude of the fractional density perturbation as a function of M :

$$\Delta(z, M) = \sqrt{\left\langle \left(\frac{\rho'(z, \vec{r}) - \rho(z)}{\rho(z)} \right)^2 \right\rangle}, \quad (1)$$

where $\rho(R)$ is the average density of the Universe and $\langle \rangle$ means the average over volumes containing mass M . Assume that

$$\Delta(z, M) = \delta(z)F(M),$$

where $F(M)$ is determined by the power spectrum of primordial fluctuations. If $\delta(z) \ll 1$, we can apply so called linear approximation. In this case one can obtain rather simple analytical solution and in the next Chapter we will find $\delta(z)$ as a function of redshift z in the linear approximation.

25.4. Simulating the large-scale formation.

If $\delta(z) \geq 1$ the linear approximation is not applicable any more, analytical approach to the problem is impossible and rather complicated numerical simulations are required.

The cold dark matter model has become the leading theoretical paradigm for the formation of structure in the Universe. Together with the theory of cosmic inflation, this model makes a clear prediction for the initial conditions for structure formation and predicts that structures grow hierarchically through gravitational instability. Testing this model requires that the precise measurements delivered by galaxy surveys can be compared to robust and equally precise theoretical calculations. A novel framework for the quantitative physical interpretation of such surveys and the largest simulation of the growth of dark matter structure ever carried out with new techniques, you can find, if interested, [here](#).