Lecture 11. *Last updated 14.04.10*

XI. EXPERIMENTAL CONFIRMATION OF GR AND GRAVITATIONAL WAVES (GWS)

A. Relativistic experiments in the Solar system and Binary pulsar

General relativity is currently the most successful gravitational theory, being almost universally accepted and well-supported by observations. General relativity's first success was in explaining the anomalous perihelion precession of Mercury, then observations of stars near the eclipsed Sun quantitatively confirmed general relativity's prediction that massive objects bend light. Other observations and experiments have since confirmed many of the predictions of general relativity, including the gravitational redshift of light and the gravitational time dilation. All these effects in the Solar System were then observed in tremendously magnified version in binary pulsars.
In 1916 Einstein proposed three famous tests of general relativity, subsequently called the classical tests of general relativity.

1. *The perihelion precession of Mercury's orbit.*

In Newtonian physics, an object orbiting a spherical mass would trace out an ellipse with the spherical mass at a focus.There are a number of solar system effects that cause the perihelion of a planet to precess, or rotate around the sun. These are mainly because of the presence of other planets, which perturb orbits. Another effect is solar oblateness, which produces only a minor contribution. The precession of the perihelion of Mercury was a longstanding problem in celestial mechanics. Careful observations of Mercury showed that the actual value of the precession disagreed with that calculated from Newton's theory by 43 seconds of arc per century, which was much larger than the experimental error at the time. In general relativity, this orbit will precess, or change orientation within its plane, due to the curvature of spacetime.

2. *Deflection of Light by the Sun.*

The first observation of light deflection was performed by noting the change in position of stars as they passed near the Sun on the celestial sphere. The observations were performed by Sir Arthur Eddington who traveled to the island of Principe near Africa to watch the solar eclipse of May 29, 1919.According to GR, stars near the Sun would appear to have been slightly shifted because their light had been curved by its gravitational field. This effect is noticeable only during an eclipse, since otherwise the Sun's brightness obscures the stars.Prediction of Newtonian theory is exactly two times smaller than predicted by GR. Eddington's 1919 measurements of the bending of star-light by the Sun's gravity confirmed GR.

3. *Gravitational Redshift.*

Einstein predicted the gravitational redshift of light in 1907.
This prediction was confirmed by Pound and Rebka in 1959. They measured the relative redshift of two sources situated at the top and bottom of Harvard University's Jefferson tower. The result was in excellent agreement with GR.

4. *Examples of other experiments in Solar System.*

There were a lot of other precision tests of general relativity, which are not discussed here. I will give you just two examples:
Example 1. Gravity Probe A was launched in 1976. This experiment showed that gravity and velocity affect the ability to synchronize the rates of clocks orbiting a central mass.
Example 2. Current experiment Gravity Probe B. is testing the prediction of GR which says that rotating bodies drag spacetime around themselves in a phenomenon referred to as frame-dragging (or gravimagnetism). This is the same effect as in the vicinity of rotating black holes (see notes to Lecture 9), but extremely small about one part in a few trillion.
The Gravity Probe B satellite, launched in 2004, is currently attempting to detect frame dragging.

5. *Binary Pulsar.*

General relativity has been extremely well tested after 1974, when Hulse and Taylor discovered the first binary pulsar. Pulsar is a highly magnetized rotating neutron star. A neutron star is formed from the collapsed remnant of a massive star and consists mostly of neutrons. A typical neutron star has a mass between 1.35 and about 2.1 solar masses, with a corresponding radius of order 10 km. The density of a neutron star, ρ_{ns}, is comparable with the density of an atomic nucleus, i.e. $\rho_{ns} \sim 10^{17} \div 10^{18}$ kgm^{-3}. Pulsars emit a beam of radio waves. Their observed periods range from 1 ms to 10 s. The radiation can only be observed when the beam of emission is pointing towards the Earth. Because neutron stars are very dense objects, the rotation period and thus the interval between observed pulses are very regular. For some pulsars, the regularity of pulsation is as precise as an atomic clock.
A binary pulsar is a pulsar with a binary companion, often another pulsar, white dwarf or neutron star. The first binary pulsar, PSR 1913+16 or the "Hulse-Taylor binary pulsar" was discovered in 1974 at Arecibo by Joseph Taylor, Jr. and Russell Hulse, for which they won the 1993 Nobel Prize in Physics.
The binary pulsars allow astrophysicists to test general relativity in the case of a strong gravitational field. The timing of the pulses from the pulsar can be measured with an extraordinary accuracy.
The orbit of the pulsar in binary system experiences periastron advance, the radiation is gravitationally redshifted and the orbital period decreases with time due to gravitational radiation.Binary pulsar timing has thus indirectly confirmed the existence of gravitational radiation and verified GR.
The rotation of the pulsar's periastron is analogous to the advance of the perihelion of Mercury in its orbit. The observed advance for PSR 1913+16 is about 4.2 degrees per year: the pulsar's periastron advances in a single day by the same amount as Mercury's perihelion advances in a century!

B. Propagation of GWs

A weak gravitational field is a small perturbation of the Galilean metric:

$$g_{ik} = \eta_{ik} + h_{ik}. \tag{XI.1}$$

It is easy to show that

$$g^{ik} = \eta^{ik} - \eta^{in}\eta^{km}h_{nk}. \tag{XI.2}$$

The gravitational wave is a transverse and traceless part of these perturbations and the plane wave has two independent states of linear polarization. Using a linear coordinate transformation

$$x^{'i} = x^i + \xi^i, \tag{XI.3}$$

where ξ^i are small functions of x^i, we can impose on h_{ik} the following four supplementary conditions:

$$\eta^{km} h_{mi,k} - \frac{1}{2}\delta_i^k \eta^{nm} h_{nm,k} = 0. \tag{XI.4}$$

After such transformation the Ricci tensor is reduced to

$$R_{ik} = -\frac{1}{2}\eta^{lm} \frac{\partial^2 h_{ik}}{\partial x^l \partial x^m}. \tag{XI.5}$$

According to the Einstein equations in empty space-time $R_{ik} = 0$, hence gravitational waves satisfy the wave equation

$$(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}) h_{ik} = 0, \tag{XI.6}$$

where ∇^2 is the 3-dimensional Laplacian operator.

C. Detection of GWs

Let us consider a ring of test particles initially at rest in the $(y - z)$ plane, perturbed by a plane monochromatic gravitational wave propagating in x-direction with frequency ω and amplitude h_0. Then it is possible to show that all components of h_{ik} can be eliminated by the transformation of coordinates except

$$h_{22} = -h_{33} \equiv h_+ \tag{XI.7}$$

and

$$h_{23} = h_{32} \equiv h_\times, \tag{XI.8}$$

corresponding to "+" and "×" polarizations. By calculating the physical distances between the test particles on the ring and its center we can determine distortions in shape and in orientation of the ring produced by a gravitational wave at different moments of time and for different polarizations of the gravitational wave:

$$(i)\;\; h_+ = h_0 \sin\omega(t - x/c), \;\;\; h_\times = 0 \tag{XI.9}$$

and

$$(ii)\;\; h_+ = 0, \;\;\; h_\times = h_0 \sin\omega(t - x/c). \tag{XI.10}$$

The distortions of the originally circular ring for these two states of polarization of the wave at $t = 0$, $t = T/4$, $t = T/2$, $t = 3T/4$ and $t = T$, where T is the period of the wave, are shown below. Without loss of generality we can assume that the ring is located at $x = 0$. If

$$h_+ = h_0 \sin\omega t \tag{XI.11}$$

and

$$h_\times = 0, \tag{XI.12}$$

we have

$$\delta l(\theta) = -\frac{1}{2} l_0 h_0 \sin\omega t \cos 2\theta. \tag{XI.13}$$

ωt	$\delta l(\theta)$
0	0
$\frac{\pi}{2}$	$-\frac{1}{2} l_0 h_0 \cos 2\theta$
π	0
$\frac{3\pi}{2}$	$\frac{1}{2} l_0 h_0 \sin\omega t \cos 2\theta$
2π	0

If

$$h_+ = 0, \tag{XI.14}$$

and

$$h_\times = h_0 \sin\omega t, \tag{XI.15}$$

we have

$$\delta l(\theta) = -\frac{1}{2} l_0 h_0 \sin\omega t \sin 2\theta. \tag{XI.16}$$

ωt	$\delta l(\theta)$
0	0
$\frac{\pi}{2}$	$-\frac{1}{2}l_0h_0\sin 2\theta$
π	0
$\frac{3\pi}{2}$	$\frac{1}{2}l_0h_0\sin 2\theta$
2π	0

If

$$h_+ = h_0 \sin\omega t \tag{XI.17}$$

and

$$h_\times = h_0 \cos\omega t \tag{XI.18}$$

we have

$$\delta l(\theta) = -\frac{1}{2} l_0 h_0 (\sin\omega t \cos 2\theta + \cos\omega t \sin 2\theta) = -\frac{1}{2} l_0 h_0 (\sin\omega t + 2\theta) = -\frac{1}{2} l_0 h_0 \sin 2(\theta - \theta_0(t)), \tag{XI.19}$$

where

$$\theta_0(t) = -\frac{1}{2}\omega t. \tag{XI.20}$$

ωt	$\theta_0(t)$	$\delta l(\theta)$
0	0	$-\frac{1}{2}l_0h_0\sin 2\theta$
$\frac{\pi}{2}$	$-\frac{\pi}{4}$	$-\frac{1}{2}l_0h_0\sin 2(\theta+\frac{\pi}{4}) = -\frac{1}{2}l_0h_0\cos 2\theta$
π	$-\frac{\pi}{2}$	$-\frac{1}{2}l_0h_0\sin 2(\theta+\frac{\pi}{2}) = \frac{1}{2}l_0h_0\sin 2\theta$
$\frac{3\pi}{2}$	$-\frac{3\pi}{4}$	$\frac{1}{2}l_0h_0\cos 2\theta$
2π	$-\pi$	$-\frac{1}{2}l_0h_0\sin(2\theta+2\pi) = -\frac{1}{2}l_0h_0\sin 2\theta$

This polarization can be called circular polarization.

D. Generation of GWs

Starting from the Einstein equations we can linearize them by taking into account that gravitational waves are characterized by small amplitudes. Then in approximation of slow motions and small separations we can use the Quadrupole formula for gravitational waves:

$$h_{\alpha\beta} = -\frac{2G}{3c^4 R}\frac{d^2 D_{\alpha\beta}}{dt^2}(t - R/c), \tag{XI.21}$$

where R is the distance to the source of gravitational radiation and

$$D_{\alpha\beta} = \int (3x_\alpha x_\beta - r^2\delta_{\alpha\beta})dM \tag{XI.22}$$

is the quadrupole tensor.

E. Examples, problems and summary

Example: A white dwarf (or netron star, or black hole) of mass m moves around a black hole of mass $M \gg m$ on a circular orbit with radius r. Find the frequency of gravitational radiation if T is the orbital period.
Taking into account that

$$x_\alpha = \delta_{\alpha\beta} x^\beta = e^\alpha \cos\omega_0 t, \tag{XI.23}$$

where e^α is some constant vector, we have

$$h_{\alpha\beta} \sim \ddot{D}_{\alpha\beta} \sim \left(3x_\alpha x_\beta - r^2 \delta_{\alpha\beta}\right)^{\cdot\cdot}$$

$$\sim \left(x_\alpha x_\beta\right)^{\cdot\cdot}$$

$$\sim e^\alpha e^\beta \left(\cos^2 \omega_0 t\right)^{\cdot\cdot}$$

$$\sim \frac{1}{2} e^\alpha e^\beta \left(1 + \cos 2\omega_0 t\right)^{\cdot\cdot} \sim \cos\omega, \tag{XI.24}$$

where

$$\omega = 2\omega_0 = 4\pi/T. \tag{XI.25}$$

Estimate to an order of magnitude, h_0, the amplitude of the gravitational wave.

Solution: To an order of magnitude and omitting indices we have

$$h \sim \frac{2G}{3c^4 R}\ddot{D} \sim \frac{2G}{3c^4 R}\frac{3}{2}(2\omega_0)^2 m r^2 = \frac{4Gmr^2\omega_0^2}{c^4 R} = \frac{4Gmr^2}{c^4 R}\frac{GM}{r^3} =$$

$$= \frac{m}{M}\frac{r_g^2}{rR} = \frac{m}{M}\frac{r_g}{r}\frac{r_g}{R}. \tag{XI.26}$$

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