4442 Particle Physics
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Week 5

http://www.hep.ucl.ac.uk/~markl/teaching/4442

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- Currents and the adjoint spinor + solutions to problems
- Adding interactions : covariant Maxwell's equations
- The merger : Dirac (electron equation) + covariant Maxwell (photon) = QED
- Gauge invariance and QED
- Lagrangian formalism

$\overline{\Psi}$ The Adjoint spinor

- The maths of QED (and all interactions) is based on : $\overline{\psi}.....\psi$

$$\overline{\psi} = \psi^{\dagger}\gamma^{0}; \quad \psi^{\dagger} = \left(\psi^{T}\right)^{*}$$

$\overline{\psi}\psi$ Is Lorentz invariant; but the usual $\psi^{\dagger}\psi$ is not.

$\overline{u}\left(\gamma^{\mu}p_{\mu} - m\right) = 0$: the Adjoint Dirac Equation (derivation)*

2 useful adjoint properties (derivation)*

$$\sum u_s \overline{u}_s = \gamma^{\mu}p_{\mu} + m; \quad \overline{u}_r u_s = 2m\delta_{rs}$$

The following combinations are Lorentz invariant and used to define interactions.

$$\overline{\psi}\psi; \quad \overline{\psi}\gamma^{\mu}\psi; \quad \overline{\psi}\gamma^{5}\psi; \quad \overline{\psi}\gamma^{\mu}\gamma^{5}\psi; \quad \overline{\psi}\sigma_{\mu\nu}\psi; \quad \sigma_{\mu\nu} = 1/2(\gamma^{\mu}\gamma^{\nu} - \gamma^{\nu}\gamma^{\mu})$$

The first 4 are eigenvectors of the parity operator (*)

What does "V-A" interaction mean ? (*)

Gauge Invariance

This & renormalisation are the truly great "discoveries" of theoretical physics in the past 50 years & are the cornerstone of modern physics.

Any predictive & useful theory must be renormalisable, gauge and Lorentz invariant

The imposition of gauge invariance / symmetry naturally leads to
requirement of "gauge boson" fields e.g. photon for QED and conservation laws.

The most used formalism to describe a system and demonstrate gauge invariance is the Lagrangian formalism which is derivable from the wave-equation formalism or The Hamiltonian formalism.

Gauge invariance - requirement that our physics is unaltered by changes in parameters, α, that are not measurable e.g. phase of a wave-function, electrostatic potential.

If α is fixed everywhere then we speak of <u>global gauge invariance</u> under global transformations; if α depends on position then we speak of <u>local gauge invariance</u> under local transformations - we require our theories to be locally gauge invariant

Consideration of gauge invariance of electrostatic potential and consequence for conservation of charge*

The covariant 4-vector potential, A_μ - definition and why we introduce it (*)

- Aharonov-Bohm effect (look it up on Wiki) demonstrates that in QM, A_μ is real.

- A_μ reduces the redundancy in Maxwell's equations (*)

- A Gedanken to illustrate why we need A_μ and a gauge transformation of A_μ to preserve gauge invariance in QED (*)

Imposition of local Gauge invariance in QED has 3 consequences:

1. the existence of a field (the EM field) of infinite range which consequently must be mediated by a massless particle
2. the conservation of charge
3. simplification of Maxwell's equations (*)

The addition of the photon-electron interaction via A_μ that restores gauge invariance under a local phase transformation can be added as an interaction term to the Dirac equation which becomes:

$$\left(i\gamma^\mu \partial_\mu + e\gamma^\mu A_\mu - m\right)\psi = 0 \quad or \quad \left(i\gamma^\mu D - m\right)\psi = 0$$

Lagrangian Formalism

QM and covariant analogue of Euler-Lagrange equation (ELE) is:

$$\partial_\mu \left(\frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \phi\right)} \right) - \frac{\partial \mathcal{L}}{\partial \phi} = 0$$

Where ϕ is a field amplitude

The wave equation is obtained by imposing the ELE on a QM Lagrangian
- examples (*) : Klein-Gordon, Maxwell's, Dirac equation.

$$X_\mu = \frac{\partial \mathcal{L}}{\partial \left(\partial_\mu \phi\right)} \frac{\partial \phi}{\partial \alpha}$$

Is the conserved quantity in a gauge invariant / Symmetric Lagrangian (this is Noether's theorem Expressed in the Lagrangian formalism)

Demonstration (*) from requirement that QED Lagrangian is invariant that:

1. We need a long range field that has the gauge transformation:
   $$A_\mu \rightarrow A_\mu + \partial_\mu \theta(x)$$
2. That photon must be massless

The connection between Lagrangian and Feynman rules (*)

The QCD Lagrangian (*)