4442 Particle Physics
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Week 3

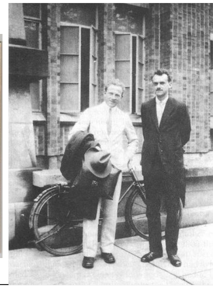
http://www.hep.ucl.ac.uk/~markl/teaching/4442

Week 3

- finish symmetries / transformations :
    - groups in particle physics
    - discrete transformations: parity(P), charge conjugation(C) and CP

- Problem sheet 1 : discussion & answers

- The DIRAC equation



The Schrödinger equation**

is a non relativistic : E = KE (1/2 mv^2) + PE wave eqn

The Klein-Gordon equation**

is a relativistic : $E^2 = p^2 + m^2$: wave eqn but there is a troublesome negative probability density solution (arising from $E = -\sqrt{p^2+m^2}$).

Dirac managed to produce an equation valid in both the relativistic and non-relativistic regimes which also has a “natural” interpretation of the negative energy solutions - he invented the notion of anti-particles (with positive probability density solutions) which were then experimentally verified a year later.

Derivation of Dirac equation rests on the introduction of 2x2 (Dirac)-matrices since matrices can have the property that AB + BA = 0 whereas numbers are commutative and this allowed : positive density solutions since the equation contained no E^2 ($-\partial^2\psi/\partial t^2$) terms <u>and</u> for the solutions to satisfy $E^2 = p^2 + m^2$.

Derivation of the Dirac Equation and Dirac gamma Matrices **

Dirac gamma Matrices satisfy :

$$(\gamma^0)^2 = 1 \;\; ; \;\; (\gamma^i)^2 = -1 \; , \; i = 1,2,3 \qquad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

And $(\gamma^5)^2 = I \;\; : \;\; \{\gamma^5, \gamma^\mu\} = 0 \;\; , \;\; \mu = 0,1,2,3$

The Dirac equation is written as :

$(i\gamma^\mu \partial_\mu - m)\psi = 0$ or $(i\not{\partial} - m)\psi = 0$

The Dirac Hamiltonian is (derivation) **

$$\hat{H} = -i\gamma^0 \vec{\gamma} \cdot \vec{\nabla} + m\gamma^0$$

The solutions of the Dirac equation : $(\gamma^\mu p_\mu - m)u(\vec{p}) = 0$

$\psi = u(\vec{p})e^{-ip_\mu x^\mu}$ are known as Dirac spinors.

And can be expressed in terms of 2-spinors: χ_1, χ_2 $\chi_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\chi_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$$u^{(1)}(\vec{p}) = \sqrt{|E|+m}\begin{bmatrix} \chi^{(1)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(1)} \end{bmatrix} \quad u^{(2)}(\vec{p}) = \sqrt{|E|+m}\begin{bmatrix} \chi^{(2)} \\ \frac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi^{(2)} \end{bmatrix} \quad E = +\sqrt{(\vec{p})^2+m^2}$$

The "anti-particle" solutions : $\psi = v(\vec{p})e^{-ip_\mu x^\mu}$

$$v^{(1)}(\vec{p}) = \sqrt{|E|+m}\begin{bmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E-m}\chi^{(1)} \\ \chi^{(1)} \end{bmatrix} \quad v^{(2)}(\vec{p}) = \sqrt{|E|+m}\begin{bmatrix} \frac{\vec{\sigma}\cdot\vec{p}}{E-m}\chi^{(2)} \\ \chi^{(2)} \end{bmatrix} \quad E = -\sqrt{(\vec{p})^2+m^2}$$

There are four independent solutions to the Dirac equation: two "particle" and two "anti-particle" solutions. Each describes (anti-)particles with mass and spin ½ e.g. quarks and electrons, in two helicity states. Derivation of solutions **

Dirac's Interpretation of negative energy solutions

If the -ve energy solution were an electron then we could get the situation where a +ve energy electron could interact with a photon to reduce it's energy to be the more naturally preferred (and allowable by the Dirac eqn) -ve energy and this is not observed i.e. the negative energy solutions cannot be electrons.

Vacuum is a sea of -ve energy electrons. If we create a "hole" in the sea e.g. by hitting one of the -ve E electrons with a photon of $E > 2m_e$ then the sea effectively increases in charge by +1, and becomes more positive in energy - our "hole" thus looks like a +ve E particle of +ve charge with the mass of an electron. This hole can be identified as a real particle - the positron with opposite charge, momentum, spin and energy to the original -ve E electron eigenstate but the same mass.

Shares many similarities with the discussion of holes in "Fermi seas" of semiconductors in condensed matter physics.

Discussion is actually more complex since we need to introduce Quantum Field Theory operators for particle creation/annihilation……

Next week:

- the continuity equation
- “interaction currents” in terms of gamma matrices
- the Lagrangian formalism
- the addition of EM interactions (Maxwell’s equations) into the Dirac equation ie a real theory : QED