4442 Particle Physics
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Week 2 - Part 2

http://www.hep.ucl.ac.uk/~markl/teaching/4442

Week 2

- finish treatment of calculating decay rates & concept of renormalisation.
- transformations and invariance
- symmetries and conservation laws
- groups in particle physics
- discrete transformations: parity(P), charge conjugation(C) and CP

Symmetries are very powerful - often the basis of a theory or invoked when theory is incomplete and they're intimately connected with conservation laws.

Connection between symmetry, invariance and transformations **

Maths of describing symmetries is "group theory" e.g. the set of rotations we can perform on a system forms a group - each rotation is an element of the group

Mathematical properties of a group **

Some QM Revision

Probability that system described by $|\psi\rangle$ in state $|\phi\rangle = |\langle\phi|\psi\rangle|^2$ **eq(1)**

Hamiltonian, H, is the observable describing energy of system & it generates the time evolution of the quantum state ie $i\frac{\partial}{\partial t}|\langle\psi(t)\rangle = H|\psi(t)\rangle$

Expectation value of operator X : $\langle\phi(t)|X|\phi(t)\rangle$ **eq(2)**

If X corresponds to an observable then and eigenvalues are real $X^\dagger = X$ **eq(3)**

$X|\psi\rangle = \langle\psi|X^\dagger$ **eq(4)**

$|\psi'\rangle = U|\psi\rangle \text{ then } \langle\psi'| = \langle\psi'|U^\dagger$ **eq(5)**

$\frac{d\langle Q\rangle}{dt} = -i\langle[Q,H]\rangle$ **eq(6)**

$\text{Momentum operator } : \hat{p_x} = -i\frac{d}{dx}$ **eq(7)**

QM Operator of a transformation is a "constant of motion"

We require physics to be unchanged by a symmetry operation, U

If $|\psi\rangle \rightarrow |\psi'\rangle = U|\psi\rangle \text{ then } \frac{d\langle U\rangle}{dt} = 0$ **

Emmy Noether's Theorem

"Every invariant symmetry transformation has an associated conservation law"

"the most significant creative mathematical genius thus far produced since the higher education of women began" : Albert Einstein.

Demonstration of theorem by considering a transformation as the sum of infinitesimal transformations **

Relation between transformation operator and the group "generator" operator ,G. G is conserved ** $U(\theta) = e^{-i\theta G}$

Typical groups found in particle physics

Unitary groups : U(n)
Special Unitary Groups : SU(n) : determinant = 1
Special Real Orthogonal Groups : SO(n) : determinant = 1 & all elements real

Non-continuous (discrete) transformations: P and C

Parity (P) operator : space inversion : $P(\vec{r}) = -\vec{r}$

For more complex vectors other than the position vector the parity operator can flip or retain the sign e.g. $P(\vec{L}) = \vec{L}; \quad \vec{L} = \vec{r} \times \vec{p}$ has eigenvalue +1 ie is "even" under the parity operator

Definition of axial vector, pseudoscalars **

Intrinsic parity of particles **

Parity is conserved in EM and strong interactions but violated maximally in weak interactions : example pion decay **

Charge Conjugation Operator (C) : more than just charge, actually flips all non momenta (spin, L) values : charge, color, lepton-# etc and so converts a particle to anti-particle. $C|X\rangle = c|\overline{X}\rangle; \; c^2 = 1$

But there aren't so many particles where particle = anti-particle except e.g. γ, π^0 and so concept of limited use.

Again it is conserved in EM & strong interactions but not weak (pion decay **)

The more interesting operator is the combined "CP" operator.
It is a more relevant matter to anti-matter operator.

Given we know matter predominates in our universe then we know CP cannot be conserved in all weak interactions (although it is in pion decay **)

Sakharov conditions for matter preponderance (life…) ***

CP violation has been observed in a handful of weak interaction decays **

Feynman Diagrams inducing CP violation are subtle **

To discuss CP violation concept of expressing $|\Psi>$ in terms of eigenstates of the measurement is important. The nomenclature and what actually “is the particle” can get confusing **

We will consider the CP violation in Kaon decays ***

The Mythical Axion

Finally, while it is observed that strong interactions do not violate CP (e.g. no discernible neutron electric dipole moment has been measured) there is no a priori reason from the symmetries/structure of QCD why this should be so (unlike QED) and indeed the QCD Lagrangian has been “fiddled” such that CP violation is zero by adding a new particle (named after a brand of detergent) - the axion - that cleans up QCD.

This particle, has yet to be observed, although evidence for it was presented by the PVLAS collaboration in Dec 2006 by using an axion property that it should change into a photon in the presence of a large magnetic field….