PHASM/G442 Particle Physics
Prof. Mark Lancaster

http://moodle.ucl.ac.uk/course/view.php?id=2589

Enrollment is automatic if you are registered on the course via (i)Portico. **BUT** moodle is mirrored at:

http://www.hep.ucl.ac.uk/~markl/teaching/4442

Exercise solutions are password protected.
Username is 442
Password is ……

Contact Details

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http://www.hep.ucl.ac.uk/~markl/teaching/4442

Office Hours

Drop by anytime - in D18.

Books

W. Cottingham, D. Greenwood : “An Introduction to the Standard Model of Particle Physics" (2nd edition)
D. Griffiths : “Introduction to Elementary Particles”

also:

D. Gingrich : "Practical Quantum Electrodynamics"
F. Halzen, A. Martin : “Quarks and Leptons”
M. Bowler : “Femtophysics”
D. Perkins : “Introduction to High Energy Physics” (2nd or 4th edition)

Assessment

90% 2.5 hr exam (3 questions from 5) + 10% problem sheets

Module incomplete unless mark > 1.5/10 achieved on 4-problem sheets for MSci & MSc.

4 Problem Sheets

- posted on web. It’s up to you to check the course web-page.

Lecture & Course Notes

- lecture slides will be on www.
- these are incomplete. Working / examples and additional material at lectures should be added in gaps on handouts and on own paper.
- Gaps are marked with **
- lecture slides + annotations = course notes.

Lecture Breaks

- at least one - 30 min in middle of 3 hr spot.

Course Outline

- BSc recap, formalism, reaction rates, Feynman Rules (w1,2)
- Symmetries and conservation laws (w2)
- The Dirac equation (w3)
- Electromagnetic interactions (w4-5)
- Strong interactions (w6-7)
- Weak interactions (w8-9)
- The electroweak theory and beyond (w10-11)
- Revision (w12 – term 3)

Week-1/2 : Outline

- BSc recap : particles & forces
- Natural units
- Four Vectors
- Fermi’s Golden Rule : Rate of reactions
- Feynman diagrams recap
- Feynman rules
- A first calculation : phase space, density of states, Matrix Element
- Renormalisation / Running Coupling constants

Prerequisites

- 3rd year/BSc Quantum Mechanics
- Special Relativity (4-vector notation)
- 3rd year/BSc Electromagnetism
- 3rd year/BSc Particle Physics

Without BSc Particle Physics – you may struggle (please discuss with me) - it's certainly possible to catch up quickly by reading a BSc Particle Physics textbook eg

"Nuclear and Particle Physics - An Introduction" : Brian R. Martin

Elementary (= **) Matter Particles

	Family 1	2	3	*Charge(e)*	*Interactions* Strong	EM	Weak	
Quarks								**
								**
Leptons								**
								**

All matter particles(**) :
1)
2)
3)

- Why 3 familes ?
- Why mass hierarchy ?
- Where does mass come from ?
- Neutrino may not be a Dirac particle but a Majorana particle ?

Particles of same type but different families are identical except for mass.

Force Particles

Force	*Name*	*Symbol*	*#*	*Mass (GeV)*	*Coupling*	**

All bosons with spin=1 (except graviton : spin = ?)

Photon massless & no-charge : so doesn't self-interact

Strong/Weak "mediators" carry their own "charge" and so do self-interact (they are NON-ABELIAN) - this has important ramifications.

SM provides a unified treatment of EM & Weak forces (and implies unification of electroweak with strong force), but needs the Higgs boson…

Natural Units

SI units not used in particle physics
More practical to use a "natural" system where: $\hbar = c = 1$

Energy, Mass, Momentum all have units of energy (eV, GeV)
Time, length have units of inverse energy (eV^{-1}, GeV^{-1})

Examples **

Why time, length are inverse energy **

The conversion factors are:
$1\ GeV^{-1} = 0.1973$ fm $= 1.973 \times 10^{-16}$ m $= 6.582 \times 10^{-25}$ sec

Cross sections : What is $1\ GeV^{-2}$ in mb ? **

What are dimensions of angular momentum (L) or spin (S) in natural units ? **

4-Vector Notation

** 4-vector definition :

** Invariant definition :

When considering a single 4-vector - we will mostly use the *Einstein/contravariant (index superscript)* form of a 4-vector x^μ

** Definition β and γ and Lorentz transformation

But products of 4 vectors are formed by introducing a *covariant (index subscript)* form of a 4-vector : x_μ

** The covariant metric tensor : $g_{\mu\nu}$

** Scalar products of 4 vectors

** The "four-derivative" 4-vector: ∂^μ

In particle physics - what do we actually measure ?

- Particle decays : A → B + C + …
  - **
  - **

- Reactions : A + B → C + D + …
  - **
  - **

- Bound states
  - **

We'll start by considering particle decays

- Decay Rate, Γ : "Probability per unit time that a particle decays"

**

- if expressed in units of energy (since it is s^{-1}) then we call it a *Decay Width*

- Lifetime, τ : "Average time it takes to decay (in particle's rest-frame)"

- Γ and τ are simply related by: **

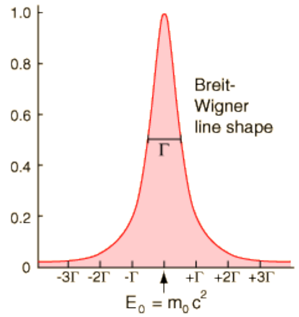
- Generally a particle can have many decay modes : concept of *partial widths*, Γ_i

**

- Branching Ratio (BR) defined as : **

- We tend to measure : BRs and Γ_{TOT} or τ and calculate Γ_i

Γ: Decay Width



Time of a particle's decay has uncertainty : $\Delta t = \tau$
Uncertainty Principle then predicts

$\Delta E.\tau = 1/2$ and hence $\Gamma = 2\ \Delta E$

If measure invariant mass of a state then Uncertainty principle gives it a "width" due to particle having a finite lifetime.

Distribution of mass follows Breit-Wigner form:
**

We can only ever measure either lifetime or width due to measuring capabilities of particle detectors (why ? - see problem sheet)

Reactions : A + B → C + D

- Rate/Probability of a reaction often expressed in terms of *cross section (σ)*
  - *it is the effective cross-sectional area that A sees of B (or B of A).*

- Often measure "differential" cross sections e.g. $d\sigma/d\Omega$ or $d\sigma/d(\cos\theta)$ **

- Luminosity definition : **

  - typical values for accelerator : 10^{30}-10^{34} $cm^{-2}s^{-1}$

- Event rates and "integrated luminosity" : **

How we calculate Reaction Rates (σ) or Decay Widths (Γ)

- Draw *Feynman diagrams* for the process
  - decide to which *"order"* we want to perform the calculation.
  - invoke *Feynman rules* to calculate a *"Matrix Element (M)"*

- Calculate the *"phase-space"* and *"flux"* for the process

- Combine $|M|^2$ with phase-space using *Fermi's Golden Rule (FGR))*

$$\mathrm{FGR}:\mathrm{Rate} = |\mathrm{M}|^2 \rho \prod_{in} \frac{1}{2E_{in}}; \sigma = \frac{\mathrm{Rate}}{\mathrm{Flux}}$$

** discussion of terms

Feynman Diagrams, Order, Feynman Rules, Phase Space, Flux
- need to be understood before we can complete a simple calculation

Feynman Diagrams (My rules)

1. Time from left to right (except in Griffiths where it's from bottom to top)
2. Draw initial particle lines on left and final to right - there will be a boson in middle
3. Based on information about reaction (initial & final state, rate) determine the type of interaction : EM(γ), Weak (W,Z), Strong (g)
4. Draw interaction vertices - make sure that charge, lepton # etc are conserved
5. Draw arrow (L→R for particles) and (R→L : backward in time for anti-particles)

** examples : muon decay (W vertices)
: top quark production and decay

** definition of s-channel, t-channel and u-channel diagrams

"Order"

- determined by number of vertices / complexity of Feynman diagrams : we speak of the lowest order process/diagram and "higher order" processes.

- ** example from electron-quark scattering

- Occasionally the lowest order permissable process is quite complex e.g. $K \rightarrow \mu\mu$

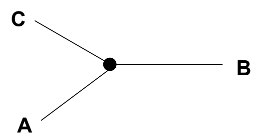
Feynman Rules for calculation |M| from diagram

1. Label all incoming/outgoing 4-momenta - p_1, p_2 … p_n (these are 4 vectors)
2. Label internal momenta - q_1,q_2, …
3. Coupling constant at each vertex : $-ig$
4. Propagator for each internal line: $i / (q^2 - m^2)$
5. Energy & momentum conservation factor at each vertex: $(2\pi)^4\ \delta^4(k_1+k_2+k_3)$
   ks are 4 momenta at each vertex and signed (+ : incoming, - : outgoing)
6. Internal momenta integration factors: $(1/(2\pi)^4)\ d^4q$: for each internal line
7. Factor to remove implicit overall E & p conservation: $1/\{(2\pi)^4\ \delta^4(p_1+p_2-p_3-..p_n)$
8. Form product: this = $-iM$

** : an aside on delta (δ) functions

Calculation of |M| for Toy Model using Feynman Rules

- see Griffiths sec 6.3
- ignore spin (spin = 0) + anti-particle complications (Majorana particles)
- only one interaction vertex
- $m_A > (m_B + m_C)$



- consider A + A → B + B via C exchange
- what are the diagrams ? Why no s-channel ? **
- calculation **

$$M_{t-diag} = \frac{g^2}{(p_1 - p_3)^2 - m_c^2}$$

Evaluation of |M| in the CM frame

- CM frame : one in which there is no net $\vec{p}$ in initial (or final state)
- ** Some properties in the CM frame for 2 → 2 scattering

$$s = (p_1 + p_2)^2 = (E_1^{CM} + E_2^{CM})^2$$
$$E_1 + E_2 = E_{CM}$$

- ** Some properties / simplifications in E >> m limit

$$M = \frac{-4g^2}{s} \cdot \frac{1}{\sin^2\theta}$$

Mandelstam Variables

- Recall: s,t,u diagrams and $M = \frac{g^2}{(p_1 - p_3)^2 - m_c^2} + \frac{g^2}{(p_1 - p_4)^2 - m_c^2}$

- Propagators depending on whether s,t, or u process have factors of:

$$(p_1 + p_2)^2 = s$$
$$(p_1 - p_3)^2 = t$$
$$(p_1 - p_4)^2 = u$$

Use these variables as convenient short-hand and from formula we have some insight of the type of process

$$s + t + u = m_1^2 + m_2^2 + m_3^2 + m_4^2$$: ** proof : problem sheet

Phase Space (ρ)

$$\text{FGR : Rate} = |\mathrm{M}|^2 \rho \prod_{in} \frac{1}{2E_{in}}$$

- Lorentz invariant - crudely it is the energy available to distribute to final state
- It can have a large impact on the rate of processes e.g. $\rho \to \pi\pi$ $\varphi \to$ KK (**)

$$d\rho = (2\pi)^4 \int \delta^4 (p_{in} - p_{out}) \prod_{out} \frac{d^3 p_{out}}{(2\pi)^3} \frac{1}{2E_{out}}$$

- Calculation of phase space for our A+A → B + B process in CM **

$$d\rho = \frac{1}{16\pi^2} \frac{p_F}{\sqrt{s}} d\Omega = \frac{d\Omega}{32\pi^2} \text{ for E >> m}$$

Flux

$$\sigma = \frac{\text{Rate}}{\text{Flux}}$$; Flux for 2-particles = relative velocity = $|\beta_1 - \beta_2|$

- ** Calculation in CM : $= \frac{p_1 E_{CM}}{E_1 E_2} = 2(\text{for E >> m})$

Finally bringing together : |M|, phase space & flux; we get :

$$\frac{d\sigma}{d\Omega} = |M|^2 \cdot \frac{1}{(8\pi)^2} \cdot \frac{p_F}{p_{IN}} \cdot \frac{1}{s} ; |M|^2 \sim \frac{g^4}{\sin^4 \theta}$$

- ** Observations

Going beyond lowest order : higher orders & renormalisation

- Lowest order A + A $\rightarrow$ B +B had $d\sigma/d\Omega \sim g^4$
- First assumption is that higher orders are suppressed since involve g^n (n > 4) but it is instructive to try the calculation in our “toy model”

** calculation

- the calculation gives a divergent result at high energies !!
- this was a killer problem for 40 years and often plagues any new theories

- the fix is to ask the question - what is g (or equivalent “e” for QED processes) in the Feynman diagrams / rules

** explanation / illustration

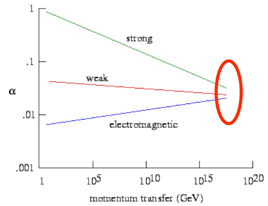
- if we use a “renormalised” value for “e” which actually corresponds to the one measured at a given momentum transfer (q) in the |M| calculation then this cancels the divergences. But it means our couplings are not fixed but “run”

Renormalisable theories & Running couplings

- A renormalisable theory is one in which the “trick” of using renormalised quantities (masses, couplings) remove all infinities to all orders.

- It was shown that the class of theories known as gauge theories (of which QED and QCD are examples) are all renormalisable and so this is the type of theory people always start with, (Nobel Prize 1999).

- EM (QED) coupling constant increases with energy
- Strong (QCD) coupling constant decreases with energy (Nobel Prize 2004)



** : explanation

Don’t actually meet or unify unless new particles !