Given $\mu_1 + \mu_2 = 1$

$r_1^2 = (x + \mu_2)^2 + y^2$

$r_2^2 = (x - \mu_1)^2 + y^2$

Show that $\mu_1 r_1^2 + \mu_2 r_2^2 = x^2 + y^2 + \mu_1\mu_2$

$$\mu_1 r_1^2 = \mu_1 \left[(x+\mu_2)^2 + y^2 \right]$$
$$= \mu_1 \left[x^2 + 2x\mu_2 + \mu_2^2 + y^2 \right]$$
$$= \mu_1 x^2 + 2\mu_1\mu_2 x + \mu_1\mu_2^2 + \mu_1 y^2 \quad \text{——— (1)}$$

$$\mu_2 r_2^2 = \mu_2 \left[(x-\mu_1)^2 + y^2 \right]$$
$$= \mu_2 \left[x^2 - 2x\mu_1 + \mu_1^2 + y^2 \right]$$
$$= \mu_2 x^2 - 2\mu_1\mu_2 x + \mu_2\mu_1^2 + \mu_2 y^2 \quad \text{——— (2)}$$

(1) + (2) :

$$\mu_1 r_1^2 + \mu_2 r_2^2 = \mu_1 x^2 + \mu_2 x^2 + \mu_1\mu_2^2 + \mu_2\mu_1^2 + \mu_1 y^2 + \mu_2 y^2$$
$$= x^2(\mu_1 + \mu_2) + y^2(\mu_1 + \mu_2) + \mu_1\mu_2(\mu_1 + \mu_2)$$

but $\mu_1 + \mu_2 = 1$

$$\therefore \mu_1 r_1^2 + \mu_2 r_2^2 = x^2 + y^2 + \mu_1\mu_2$$

Given

$$U = \frac{n^2}{2}(x^2+y^2) + \frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}$$

$$\mu_1 r_1^2 + \mu_2 r_2^2 = x^2 + y^2 + \mu_1\mu_2$$

derive an expression for U that is an explicit function of r_1 and r_2 only.

Re-arranging the second equation, above:

$$x^2+y^2 = \mu_1 r_1^2 + \mu_2 r_2^2 - \mu_1\mu_2$$

and given the expression for U, with $n = 1$:

$$U = \frac{1}{2}\left(\mu_1 r_1^2 + \mu_2 r_2^2 - \mu_1\mu_2\right) + \frac{\mu_1}{r_1} + \frac{\mu_2}{r_2}$$

$$= \mu_1\left(\frac{1}{r_1} + \frac{r_1^2}{2}\right) + \mu_2\left(\frac{1}{r_2} + \frac{r_2^2}{2}\right) - \frac{1}{2}\mu_1\mu_2$$
