Answers and Hints to Exercise Questions in "Solar System Dynamics"
(Last Updated: 1 February 2000)

Chapter 1

Q1.1 Let O=octahedron, I=icosahedron, D=dodecahedron, T=tetrahedron, C=cube. (b) The largest rms is 3.139 AU for O/C, T, I/D, D/I, C/O; the smallest rms is 0.248 AU for I/D, D/I, O/C, T, C/O. By using the semi-major axes in Table A.2 the rms in (a) increases to 1.367 AU; in (b) the largest rms is 3.156 AU and the smallest is 0.258 AU but the orderings are unchanged.

Q1.2 Use the same notation as in Q1.1. (c) The largest rms is 2.484 AU for O/C, T, I/D, D/I, C/O. By using the semi-major axes in Table A.2 the rms in (b) increases to 0.159 AU; in (c) the largest rms is 2.502 AU but the ordering is unchanged. Kepler produced astrological reasons why his ordering of the solids had to be correct but he also managed to find the best possible ordering to fit his model.

Q1.4 Note that there is only one additional pair with $|c| < 0.15$. See error listings.

Q1.5 Planets 3 and 4 are close to a 7:6 (i.e. $p = 6$) commensurability and satisfy the condition.

Q1.6 (a) There is no *simple* relationship between N_r and i_max. Although there are at most $i_\mathrm{max} - q$ rationals of the form $p/(p+q)$ for a given q with $p + q < i_\mathrm{max}$, some of these can be reduced to rationals of lower order (e.g. $\frac{2}{6} = \frac{1}{3}$). For $i_\mathrm{max} = 2, 3, 4, 5, 6, 7, 8, 9, 10$ we have $N_\mathrm{r} = 1, 3, 5, 9, 11, 17, 21, 27, 31$ respectively. (b) $\epsilon_\mathrm{max} = \frac{1}{2}\left(\frac{i_\mathrm{max}-1}{i_\mathrm{max}} - \frac{i_\mathrm{max}-2}{i_\mathrm{max}-1}\right)$. (c) You need to make some assumptions to get this expression for p. (d) Think binomial distribution. (e) Note that the satellites of Neptune should be included and the upper limit of the satellite eccentricity should be 0.15 and not 0.1. See error listings. Using the J2000 values for the planets from Table A.2 as well as the satellite data gives $N_\mathrm{p} = 44$ and $N_\mathrm{obs} = 30$. With $p = 17/31$ the formula gives $P = 0.025$.

Chapter 2

Q2.1 Many of the techniques of Sects. 2.2 and 2.3 are still applicable with minor modifications. Once you get the modified version of Eq. (2.13) you could just show that the polar equation of a centred ellipse satisfies the equation.

Q2.2 Using the semi-major axes the time interval between conjunctions is 2.135 years. Given that $e_\mathrm{M} \gg e_\mathrm{E}$ calculate the separation for conjunctions at Mars' perihelion and aphelion. The ratio should be 1.75 and so the minimum distance varies by a factor ~ 2. A "very close" opposition of Mars occurs when Mars is at perihelion. Imagine Earth and Mars in this configuration with the Sun, Earth and Mars defining the reference line. Now think what the geometry will be at the next conjunction, 2.135 years later. This gives the interval between close approaches as 15.8 years. In order to see what is happening plot the *separation* of Mars and Earth as a function of time for the years 1985–2002 using the data from Table A.2. The minimum separation at the closest opposition is 0.392 AU on 22 September 1988. The minimum separation at the furtherest opposition is 0.674 AU on 12 February 1995. The next close opposition occurs on 27 August 2003.

Q2.3 For a hyperbolic orbit $a < 0$ and $e > 1$ but Eq.(2.20) still holds and the pericentre distance is still $a(1-e)$. Using data from Tables A.4 and A.9 the maximum deflections are (i) 142.4° for Jupiter and (ii) 14.06° for Titan.

Q2.4 To show why $\mathbf{e}$ has to lie in the orbital plane consider the directions of the vectors $\mathbf{r}$, $\mathbf{v}$, $\mathbf{h}$ and $\mathbf{h} \times \mathbf{v}$. To show that $\mathbf{e}$ is a constant you need to show that $\dot{\mathbf{e}} = 0$.

Q2.5 $E = 132.14°$, $f = 158.47°$, $r = 4.5892$ AU, $\theta = 240.67°$. You can see the images taken by the *Galileo* spacecraft at `http://www.jpl.nasa.gov/galileo/sepo/cruise/sl9/wfrag3.html`

Q2.6 At noon on 14 August 2126 the Earth has position vector $\mathbf{r}_\mathrm{E} = (0.773762, -0.654228, 0)$ AU while Swift-Tuttle has position vector $\mathbf{r}_\mathrm{ST} = (-4.48642, 4.73128, 1.56172)$ AU. Hence the separation is 7.68844 AU. In fact, the minimum separation is as low as 1.238 AU in mid-2128 but still nothing to worry about. However, it should be noted that these answers are based on a number of assumptions.

Chapter 3

Q3.1 Note that the angular separation should be 23.9° and not 23.5°. See error listings.

Q3.2 (i) $\bar{A} = 4 + 3\alpha$. (ii) $\bar{A} = 4 - 3\alpha$. (iii) $\bar{A} = 1 - 3\beta - (7/8)\mu_2 \approx 1 - (3/2)\beta \approx 1 + (7/8)\mu_2$.

Q3.3 You need to make use of the masses of Jupiter and Io to calculate the distance of the Io–Jupiter L_1 point from Io in units of the semi-major axis of Io's orbit. Remember that in our system of units Io has unit mean motion and therefore 2π time units correspond to one orbital period of Io. Remember that the growth away from an unstable point is exponential. The answer is 1.794 days.

Q3.4 Be warned that there are several incorrect ways to get the right answer and so the detail is important!

Q3.5 (i) $x_0 = 1.92$. (ii) $x_0 = 2.47$. (iii) $x_0 = 2.49$ gives $x_{\rm max} = 12.3256$ following encounter. Because some of these trajectories are in chaotic regions it is possible that your answers could be slightly different from these.

Q3.6 Note that $\mathbf{r}$ and $\mathbf{v}$ are the position and velocity vectors in the inertial frame and that the necessary theory is given in Sect. (3.14.2), not Sect. (3.15.2). See error listings. Remember that in the approximation we only need to include terms of $\mathcal{O}(k)$ and we can neglect terms of $\mathcal{O}(k\mu_2)$ and higher. Also, the unshifted L_4 and L_5 points are located at $r = 1$.

Chapter 4

Q4.1 You should be able to show that there is no contribution to J_2 from a sphere of uniform density and so only the thin shell contributes.

Q4.2 (a) $x = [(5\alpha\rho/2 - \rho_{\rm m})/(\rho - \rho_{\rm m})]^{1/2}$; $\rho_{\rm c} = \rho_{\rm m} + (\rho - \rho_{\rm m})^{5/2}/(5\alpha\rho/2 - \rho_{\rm m})^{3/2}$. (b) For Earth the relationships give $\rho_{\rm m} \leq 4.58$ g cm^{-3}, $\rho_{\rm c} \geq 7.30$ g cm^{-3}, $x \leq 0.91$. When we use $R_{\rm c}$ =3,480 km we have $\rho_{\rm m} = 4.18$ g cm^{-3} and $\rho_{\rm c} = 12.3$ g cm^{-3}.

Q4.3 (a) $\tan\epsilon = \omega\beta/(\omega_0^2 - \omega^2)$; $A = F/\left[(\omega_0^2 - \omega^2)^2 + \omega^2\beta^2\right]^{1/2}$. (b) To do the integral you must set $x = A\cos(\omega t - \epsilon)$ and $\dot{x} = -\omega A \sin(\omega t - \epsilon)$. (c) $E_{\rm max} = \frac{1}{2}\omega_0^2 A^2$ and hence $Q = \omega_0^2/(\beta\omega)$. This is often referred to as a "frequency-dependent Q" in geophysics, and implies $\epsilon \propto \omega$ for small phase lags and slow forcing.

Q4.4 The numerically-derived answers for the Moon's initial and final synchronous states under tidal evolution are: (a) [initial state] $a = 2.26R$, $P_{\rm orb} = P = 4.8$ h at time $\tau = -1.65 \times 10^9$y (for $Q = 12$). (b) [final state] $a = 77.5R$, $P_{\rm orb} = P = 39.7$ d at time $\tau = +6.7 \times 10^9$y. (c) [final state, with no solar torques] $a = 86.9R$, $P_{\rm orb} = P = 47.0$ d at time $\tau = +16.1 \times 10^9$y. Note that the above are averages of ~ 12 independent calculations, and may be accurate to $\pm 0.5\%$. The results for (a) and (c) may also be derived approximately from angular momentum conservation arguments.

Q4.5 For the first part you should make use of the fact that satellite tides cannot appreciably alter the orbital angular momentum, L, but only the z-component is conserved if $I \neq 0$ because the orbit will precess. The inclination damping timescale is $\tau_I = (2/3)\,(m_{\rm s}/m_{\rm p})\,(a/R_{\rm s})^5\,(Q/k_2)_{\rm s}\,(\sin I/\sin\epsilon)^2\,(n\cos I)^{-1}$. Make use of Eq. (4.198) and Eq. (4.156) to show that $\tau_I/\tau_e = 7\,(\sin I/\sin\epsilon)^2\,(\cos I)^{-1}$.

Q4.6 The theory behind this method is covered in Sect. 4.13 with the initial semi-major axis set to the synchronous value. The resulting lower limits for Mars, Jupiter, Saturn, Uranus and Neptune are 42, 1.12×10^6, 8.1×10^4, 7.9×10^4 and 5.4×10^4 where we have used a synchronous semi-major axis of two planetary radii for the case of Proteus.

Chapter 5

Q5.1 (a) For Io: $a - c = 15.6\text{km} = 0.00856 R_{\text{Io}}$. (b) For the Moon in its current orbit: $a - c = 0.0658\text{km} = 0.0000379 R_{\text{Moon}}$; this is undetectable. (c) For the Moon at 10 Earth radii: $a - c = 14.4\text{km} = 0.00830 R_{\text{Moon}}$; for the Earth with a 10 h rotation period and $h_2 = 1.94$: $\epsilon = 0.0192$ and $J_2 = 0.00619$. (d) Taking the spin period to be 58.65 days (see error listing) you should get $\epsilon = 1.3 \times 10^{-6}$; this corresponds to $a - c = 0.003$ km. (e) In this part you could use the mean density for Pluto given in Table A.4. For Pluto $a - c = 1.3$ km; for Charon $a - c = 0.7$ km.

Q5.2 (a) The total mass is 1.47×10^{22} kg giving a mean density of $\langle \rho \rangle = 2.08$ g cm^{-3}. See Sect. 2.7 for a clue as to how to measure the individual masses and densities. (b) $\tau_{\text{P}} \approx 7 \times 10^6$ y. (c) Note that for smaller bodies $k_2 \propto R^2$ and so $\tau_{\text{C}}/\tau_{\text{P}} \approx (R_{\text{C}}/R_{\text{P}})^4$ is more realistic. (d) The intial semi-major axis and orbital period would have been $a = 14620$ km and $P = 4.103$ d, respectively.

Q5.3 This is a fairly simple task using the outline in Sect. 5.3.

Q5.4 The critical eccentricity is $e_{\text{crit}} = 0.2793$.

Q5.5 Measuring along the $\dot{\theta}/n = 1$ line, the maximum variation is reduced from 180° to 76°.

Q5.6 For $p = -\frac{1}{2}$, $\left(\dot{\theta}/n\right)_{\text{max/min}} = -\frac{1}{2} \pm \sqrt{e^3/48}$. For $p = -1$, $\left(\dot{\theta}/n\right)_{\text{max/min}} = -1 \pm \sqrt{e^4/24}$. For fixed e intersection of the "islands" of these resonances occurs for $\alpha = \frac{1}{2}\left\{\sqrt{e^3/48} + \sqrt{e^4/24}\right\}^{-1}$. The equation for e as a function of α is transcendental in e and so has to be solved numerically. Based on the equation for α as a function of e, are the islands ever likely to intersect?

Chapter 6

Q6.1 $8n' - 3n = -0.00789237\,^\circ\text{d}^{-1}$. If we write a general argument as $\varphi = 8\lambda' - 3\lambda + \varphi_i$ then the φ_i are $\varphi_1 = -5\varpi$, $\varphi_2 = -\varpi' - 4\varpi$, $\varphi_3 = -2\varpi' - 3\varpi$, $\varphi_4 = -3\varpi' - 2\varpi$, $\varphi_5 = -4\varpi' - \varpi$, $\varphi_6 = -5\varpi'$, $\varphi_7 = -3\varpi - 2\Omega$, $\varphi_8 = -\varpi' - 2\varpi - 2\Omega$, $\varphi_9 = -2\varpi' - \varpi - 2\Omega$, $\varphi_{10} = -3\varpi' - 2\Omega$, $\varphi_{11} = -\varpi - 4\Omega$, $\varphi_{12} = -\varpi' - 4\Omega$, $\varphi_{13} = -3\varpi - \Omega' - \Omega$, $\varphi_{14} = -\varpi' - 2\varpi - \Omega' - \Omega$, $\varphi_{15} = -2\varpi' - \varpi - \Omega' - \Omega$, $\varphi_{16} = -3\varpi' - \Omega' - \Omega$, $\varphi_{17} = -\varpi - \Omega' - 3\Omega$, $\varphi_{18} = -\varpi' - \Omega' - 3\Omega$, $\varphi_{19} = -3\varpi - 2\Omega'$, $\varphi_{20} = -\varpi' - 2\varpi - 2\Omega'$, $\varphi_{21} = -2\varpi' - \varpi - 2\Omega'$, $\varphi_{22} = -3\varpi' - 2\Omega'$, $\varphi_{23} = -\varpi - 2\Omega' - 2\Omega$, $\varphi_{24} = -\varpi' - 2\Omega' - 2\Omega$, $\varphi_{25} = -\varpi - 3\Omega' - \Omega$, $\varphi_{26} = -\varpi' - 3\Omega' - \Omega$, $\varphi_{27} = -\varpi - 4\Omega'$, $\varphi_{28} = -\varpi' - 4\Omega'$. The term associated with the argument $\varphi = 8\lambda' - 3\lambda - \varpi' - 2\varpi - \Omega' - \Omega$ is $\mathcal{G}(m'/a')e^2 e' s s' \frac{1}{16}\left\{-1488\alpha b_{3/2}^{(6)} - 433\alpha^2 \text{d}b_{3/2}^{(6)}/\text{d}\alpha - 38\alpha^3 \text{d}^2 b_{3/2}^{(6)}/\text{d}\alpha^2 - \alpha^4 \text{d}^3 b_{3/2}^{(6)}/\text{d}\alpha^3\right\}$. The Laplace coefficients are $\alpha b_{3/2}^{(6)} = 0.0942442$, $\alpha^2 \text{d}b_{3/2}^{(6)}/\text{d}\alpha = 0.677156$, $\alpha^3 \text{d}^2 b_{3/2}^{(6)}/\text{d}\alpha^2 = 4.49721$ and $\alpha^4 \text{d}^3 b_{3/2}^{(6)}/\text{d}\alpha^3 = 28.5378$. The smallest integers for which the condition is satisfied are $p = 10$ and $q = 17$.

Q6.2 The differential equation for G is $(y-1)y\,\text{d}^2G/\text{d}y^2 + [(2s+j+1)y - 2s]\,\text{d}G/\text{d}y + s(s+j)G = 0$. Taking $s = \frac{3}{2}$ (see error listing), substituting the solution for G in this differential equation and equating coefficients of y^{-2} and y^{-1} gives $A_1 = \frac{1}{4}(2j-1)A_0$ and $B_0 = \frac{1}{8}(2j+1)A_1$ respectively. Taking $l = k$ and $l = k+1$ and equating coefficients of $y^k \ln y$ gives $B_{k+1} = \frac{1}{4}B_{\text{k}}(2k+3)(2k+2j+3)/[(k+1)(k+3)]$. Taking $l = k-2$, $k-1$, k and $k+1$ and equating coefficients of y^{k-2} gives $A_{k+1} = [-2kB_{k-1} + (2k+j-1)B_{k-2} + \frac{1}{4}(1-2k)(1-2k-2j)A_k]/[(k+1)(k-1)]$. Although A_0 and A_2, on which all the remaining A_l and B_l depend, are not defined, they can be calculated numerically. Consider G as a function of y, A_0 and A_2. We can isolate A_0 and A_2 as factors by noting that $G(y; A_0, A_2) = A_0\,G(y; 1, 0) + A_2\,G(y; 0, 1)$. Now, since $F(x) = G(y)$ where $y = 1 - x$, we can evaluate $F(x)$ at two arbitrary values of $x = 1 - y$ giving the two simultaneous equations $F_1 = F(1 - y_1) = G(y_1) = A_0\,G(y_1; 1, 0) + A_2\,G(y_1; 0, 1)$ and $F_2 = F(1 - y_2) = G(y_2) = A_0\,G(y_2; 1, 0) + A_2\,G(y_2; 0, 1)$. Because we know the form of F we are left with two linear equations in two unknowns, A_0 and A_2. The choice of y_1 and y_2 is arbitrary provided they are different positive values less than 1. The value of $b_{3/2}^{(2)}(0.999)$ is 636930.0087516. This sort of precision can be achieved with $l < 10$ in the series for G.

Q6.3 $\dot{\varpi}_{\text{S}} = -\dot{\Omega}_{\text{S}} \approx (3/4)\,(m_{\text{J}}/M)\,(a_{\text{J}}/a_{\text{S}})^2\,n_{\text{s}}$. This gives a pericentre precession period of 137000 y. Note that this is off by a factor 2.5 but the right order of magnitude. It would have been a better approximation if $a_{\text{J}} \ll a_{\text{S}}$.

Q6.4 $\mathrm{d}I/\mathrm{d}t = -\left(3nJ_3R^3e\right)\cos I\left((5/4)\sin^2 I - 1\right)\cos\omega/\left(2a^3(1-e^2)^3\right)$. The expression for $\mathrm{d}\omega/\mathrm{d}t$ is not given in Sect. 6.11 (see error listing) so you should use: $\mathrm{d}\omega/\mathrm{d}t = \left(3nJ_2R^2\right)\left(1-(5/4)\sin^2 I\right)/\left(a^2(1-e^2)^2\right)$. The approximate variation in I is $\pm\left(J_3Re\right)/\left(2J_2a(1-e^2)\right)$.

Q6.5 (a) $n = \left(\mathcal{G}M/a^3\right)^{1/2}\left\{1+3h^2/(2c^2a^2)\right\}^{1/2}$ and $\kappa = \left(\mathcal{G}M/a^3\right)^{1/2}\left\{1-3h^2/(2c^2a^2)\right\}^{1/2}$. (b) For Earth $\dot{\varpi}_{\mathrm{GR}} = 0.038$ arcsec y^{-1}. For Mercury $\dot{\varpi}_{\mathrm{GR}} = 0.41$ arcsec y^{-1}.

Q6.6 (a) $a_{\mathrm{c}} = \left\{2J_2\left(M_{\mathrm{p}}/M_{\mathrm{Sun}}\right)R^2a_{\mathrm{p}}^3\cos I/\cos\beta\right\}^{1/5}$. (b) For Earth, Saturn and Uranus $a_{\mathrm{c}}/R = 9.82$, 42.5 and 77.6, respectively. (c) Think about the important plane in each case. (d) Here we are calculating the precession due to J_2 alone. Moon: $\dot{\Omega} = -5.9\times10^{-6}\,^\circ\mathrm{d}^{-1}$; $T = 6.1\times10^7$ d. Mimas: $\dot{\Omega} = -0.99^\circ\mathrm{d}^{-1}$; $T = 365$ d. Titan: $\dot{\Omega} = -0.0013^\circ\mathrm{d}^{-1}$; $T = 2.7\times10^5$ d. Miranda: $\dot{\Omega} = -0.052^\circ\mathrm{d}^{-1}$; $T = 6.9\times10^3$ d. Oberon: $\dot{\Omega} = -2.7\times10^{-4}\,^\circ\mathrm{d}^{-1}$; $T = 1.3\times10^6$ d.

Chapter 7

Q7.1 Note that $\mu_1 = m_1/(m_{\mathrm{s}}+m_2)$ and $\mu_2 = m_2/(m_{\mathrm{s}}+m_1)$ where m_{s} is the mass of the star. See error listing.

Q7.2 The precession rate of Saturn due to Jupiter is $2.6\times10^{-3}\,^\circ\mathrm{y}^{-1}$ using this method. The formula for g_- given in Q7.1 is identical to that derived in Q6.3. The precession rate of Venus due to Earth is $6.9\times10^{-4}\,^\circ\mathrm{y}^{-1}$ usingthis method. The precession rate of the lunar orbit due to the Sun is $5.53\times10^{-2}\,^\circ\mathrm{d}^{-1}$ using this method; this gives a precessional period of 17.8 y. The precession rate due to the Earth's J_2 is $5.88\times10^{-6}\,^\circ\mathrm{d}^{-1}$; this is much smaller than the solar effect.

Q7.3 The rate is $\dot{\varpi} = \frac{1}{4}(m'/M)n\alpha\left\{(2\alpha D+\alpha^2D^2)b_{1/2}^{(0)} + (e'/e)(2-2\alpha D-\alpha^2D^2)b_{1/2}^{(1)}\cos(\varpi'-\varpi)\right\}$. (i) When $\varpi' = \varpi$, $\dot{\varpi} = 1.447^\circ$/century. (ii) When $\varpi' = \varpi + 180^\circ$, $\dot{\varpi} = 2.220^\circ$/century.

Q7.4 Using the secular theory for Jupiter and Saturn alone gives $e_{\mathrm{forced}} = 0.0350$, $\varpi_{\mathrm{forced}} = 9.40^\circ$, $e_{\mathrm{free}} = 0.0452$, $\varpi_{\mathrm{forced}} = 122.54^\circ$, $I_{\mathrm{forced}} = 1.150^\circ$, $\Omega_{\mathrm{forced}} = 96.55^\circ$, $I_{\mathrm{free}} = 2.091$, $\Omega_{\mathrm{forced}} = 300.34^\circ$. Using Brouwer & van Woerkom's secular theory gives $e_{\mathrm{forced}} = 0.0371$, $\varpi_{\mathrm{forced}} = 6.28^\circ$, $e_{\mathrm{free}} = 0.0479$, $\varpi_{\mathrm{forced}} = 123.85^\circ$, $I_{\mathrm{forced}} = 1.149^\circ$, $\Omega_{\mathrm{forced}} = 96.02^\circ$, $I_{\mathrm{free}} = 2.086$, $\Omega_{\mathrm{forced}} = 300.08^\circ$.

Q7.5 For a density of 1.2 g cm^{-3} the minimum separation varies from 57.8 km for a zero mass F ring, to 162.2 km for an F ring mass equal to three times that of Prometheus. The separation is less than 70 km when $0 \le m < 0.4$ where m is measured in units of a Prometheus mass. For a density of 0.6 g cm^{-3} the minimum separation varies from 79.5 km for a zero mass F ring to 132.5 km for an F ring mass equal to three times that of Prometheus. The separation is never less than 70 km.

Q7.6 The eccentricity–pericentre eigenfrequencies, g_i (in degrees per day) and the associated locations in semi-major axes (in km) where $A = g_i$ are $g_1 = 1.00184$ (187488, 237296, 238739, 292761, 296549, 374945, 379850, 523206, 530872, 1186250, 1257440, 1480710, 1481490, 3556240, 3566360), $g_2 = 0.417727$ (241742, 290962, 298218, 373313, 381449, 520975, 533089, 1166710, 1276970, 1480490, 1481710, 3553470, 3569130), $g_3 = 0.198099$ (305715, 370258, 384213, 517838, 536160, 1141710, 1301890, 1480190, 1482010, 3549920, 3572680), $g_4 = 0.0842694$ (397518, 510745, 542585, 1098550, 1344560, 1479590, 1482600, 3543850), $g_5 = 0.0275128$ (574249, 1000210, 1436770, 1476800, 1485110, 353070, 3591890), $g_6 = 0.0188353$ (618933, 944487, 1491350, 3524270, 3598320), $g_7 = 0.00137212$ (2145640, 3414930, 3704710), $g_8 = 0.000152186$ (4161030). The inclination–node eigenfrequencies, f_i (in degrees per day) and the associated locations in semi-major axes (in km) where $B = f_i$ are $f_1 = -0.999287$ (187491, 237294, 238740, 292758, 296552, 374942, 379853, 523201, 530877, 1186210, 1257490, 1480710, 1481490, 3556240, 3566360), $f_2 = -0.417079$ (241745, 290958, 298222, 373309, 381453, 520970, 533094, 1166670, 1277010, 1480490, 1481710, 3553460, 3569140), $f_3 = -0.197899$ (305721, 370253, 384217, 517833, 536165, 1141670, 1301930, 1480190, 1482010, 3549920, 3572680), $f_4 = -0.0842182$ (397524, 510739, 542591, 1098510, 1344590, 1479590, 1482600, 3543840), $f_5 = -0.0275047$ (574257, 1000170, 1436800, 1476800, 1485110, 353070, 35918900), $f_6 = -0.0188356$ (618899, 944493, 1491350, 3524270, 3598320), $f_7 = -0.00137098$ (2146000, 3414860, 3704780), $f_8 = -0.00015199$ (4161600).

Chapter 8

Q8.1 $\bar{\delta} = 2.66\,(m_{\rm D}/m_{\rm Saturn})^{-2/3}$ and $R = 1.587 \times 10^{-3}\,(m_{\rm D}/m_{\rm Saturn})^{-1/3}\,e$. This gives $m_{\rm D}/m_{\rm Saturn} = 0.0101$; hence $\bar{\delta} = 56.8$ and $R = 0.0352$. This leads to an implausible density of 7832 g cm^{-3}.

Q8.2 Let M be the mass of the star; use primed quantities to denote the outer mass, unprimed quantities for the inner mass. The resulting amplitudes in the eccentricities are: $\Delta e = 4.87m'/M$, $\Delta e' = 3.50m/M$ for pair (1,2); $\Delta e = 7980m'/M$, $\Delta e' = 2280m/M$ for pair (1,3); $\Delta e = 4.87m'/M$, $\Delta e' = 3.62m/M$ for pair (2,3). Note the large amplitudes for the (1,3) pair and make a comparison with the LONGSTOP Uranus experience.

Q8.3 Taking the semi-major axis of Jupiter from Table A.2, the first-order resonances in the range are the 2:1 at 3.27791 AU (width 0.1568 AU), 3:2 at 3.97091 AU (width 0.2728 AU), 4:3 at 4.29528 AU (width 0.3634 AU) and 5:4 at 4.48412 AU (width 0.4394 AU). The second-order resonances in the range are the 3:1 at 2.50152 AU (width 0.02872 AU), 5:3 at 3.70156 AU (width 0.1209 AU), 7:5 at 4.15782 AU (width 0.2231 AU) and 9:7 at 4.40069 AU (width 0.3285 AU). Using only these resonances there is overlap between two or more resonances in the semi-major axis range of 4.046 AU $< a <$ 4.565 AU. However, this is a slightly artificial upper limit because including the remaining resonances leads to more overlap.

Q8.4 Taking the semi-major axes of Jupiter and Saturn from Table A.2, the jovian external first-order resonances in the range are the 1:2 at 8.2598 AU (width 0.5930 AU), 2:3 at 6.81833 AU (width 0.5938 AU), 3:4 at 6.30343 AU (width 0.6311 AU) and 4:5 at 6.03797 AU (width 0.6742 AU). The jovian external second-order resonances in the range are the 3:5 at 7.31448 AU (width 0.2044 AU), 5:7 at 6.51183 AU (width 0.2567 AU), 7:9 at 6.51244 AU (width 0.3131 AU), and 9:11 at 5.94818 AU (width 0.3710 AU). The saturnian internal first-order resonances are the 2:1 at 6.00798 AU (width 0.1574 AU), 3:2 at 7.27815 AU (width 0.2737 AU), 4:3 at 7.87268 AU (width 0.3647 AU), 5:4 at 8.2188 AU (width 0.4409 AU), 6:5 at 8.44554 AU (width 0.5075 AU), 7:6 at 8.60565 AU (width 0.5672 AU) and 8:7 at 8.72476 AU (width 0.6217 AU). The saturnian internal second-order resonances are the 5:3 at 6.78447 AU (width 0.1213 AU), 7:5 at 7.62073 AU (width 0.2239 AU), 9:7 at 8.06588 AU (width 0.3287 AU), 11:9 at 8.34286 AU (width 0.4368 AU), 13:11 at 8.53194 AU (width 0.5446 AU), 15:13 at 8.66928 AU (width 0.6529 AU) and 17:15 at 8.7736 AU (width 0.7614 AU). Using only these resonances there is overlap between two or more resonances in the semi-major axis ranges of 5.763 AU $< a <$ 6.640 AU, 6.724 AU $< a <$ 6.845 AU, 7.212 AU $< a <$ 7.415 AU, 7.690 AU $< a <$ 7.733 AU and 7.901 AU $< a <$ 9.036 AU. Again, the last upper limit is slightly misleading because including the remaining resonances leads to more overlap.

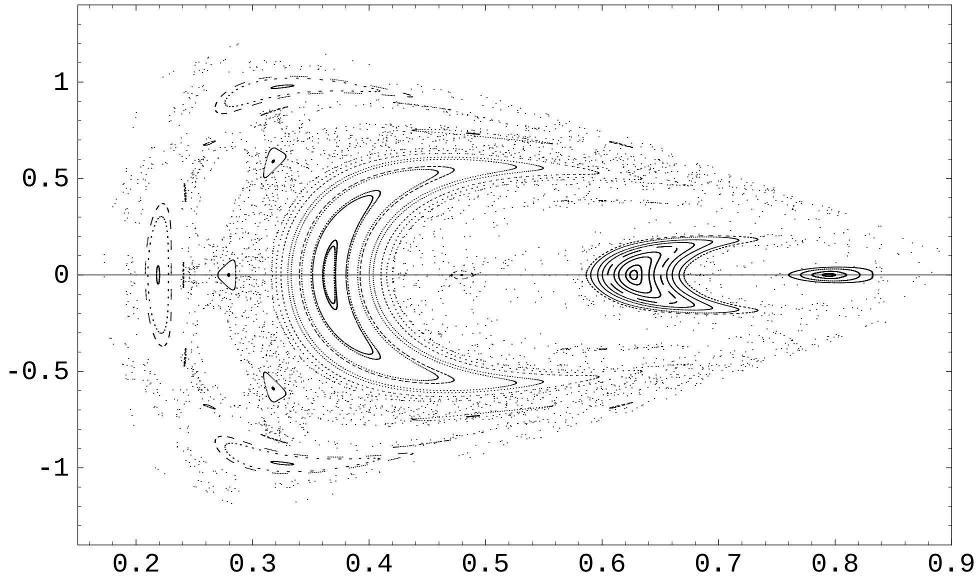
Q8.5 The particle's eccentricity at the start of the evolution should be taken to be zero (see error listing). The times of encounter and the increases in e at each possible e- and e^2-resonance are: 13:11 ($t = 3.56 \times 10^5$ y, $\Delta e = 0.00162$), 6:5 ($t = 5.40 \times 10^6$ y, $\Delta e = 0.00742$), 11:9 ($t = 1.14 \times 10^7$ y, $\Delta e = 0.000144$), 5:4 ($t = 1.85 \times 10^7$ y, $\Delta e = 0.00144$), 9:7 ($t = 2.73 \times 10^7$ y, $\Delta e = 0.000124$), 4:3 ($t = 3.82 \times 10^7$ y, $\Delta e = 0.00141$), 7:5 ($t = 5.22 \times 10^7$ y, $\Delta e = 0.000110$), 3:2 ($t = 7.10 \times 10^7$ y, $\Delta e = 0.00161$), 5:3 ($t = 9.72 \times 10^7$ y, $\Delta e = 0.0000991$), 2:1 ($t = 1.37 \times 10^8$ y, $\Delta e = 0.00236$) and 3:1 ($t = 2.02 \times 10^8$ y, $\Delta e = 0.00299$). Note that when $e_{\rm init} \ll e_{\rm crit}$ the formula $e_{\rm init}^2 + e_{\rm final}^2 = e_{\rm crit}^2$ can be used. This corresponds to putting $\delta_{\rm t} = 0$ in Eqs. (8.218) and (8.232).

Q8.6 Note that the changes in semi-major axis are not required (see error listing). The variations in e and ϖ for each planet should have a period of 5.56 y. The amplitudes are $\Delta e_1 = 0.000284$, $\Delta e_2 = 0.000369$, $\Delta\varpi_1 = 0.739°$ and $\Delta\varpi_2 = 1.06°$. Remember that these are only due to the effects of the resonance.

Chapter 9

Q9.1 For $\lambda_0 = 293°$, $\Delta a = +2.989$, $\Delta e = +0.3340$, $\Delta\varpi = -238.3°$. For $\lambda_0 = 293.3°$, $\Delta a = +4.508$, $\Delta e = +0.4111$, $\Delta\varpi = -223.9°$. By varying the initial mean longitude in steps of 0.1° from 0 to 360° we get $\Delta a_{\rm max} = +4.508$ for $\lambda_0 = 293.3°$ (one of the two starting conditions above) and $\Delta a_{\rm min} = -0.1836$ for $\lambda_0 = 334.8°$.

Q9.2 The surface of section plot should look something like the figure below. In the plot the horizontal axis is x and the vertical axis is $\dot{x}$. There are also some points with negative values of x but these have been excluded for clarity. The islands at $x = 0.218$, 0.279, 0.365, 0.479 and 0.626 are associated with the 5:2, 9:4, 2:1, 7:4 and 3:2 resonances. Note that because these are all of odd order, they have an odd number of islands and therefore, because of symmetry, they will have islands on the x-axis. The apparent island close to $x = 0.79$ is actually associated with a periodic orbit of the first kind (see Winter & Murray, 1994a).



Q9.3 Taking $\mu = 1/1048.672$ (from values of the solar and jovian masses given in Tables A.1 and A.2) and using the absolute value of the difference in the eccentricities for the first 3500 Jupiter periods gives a maximum Lyapounov characteristic exponent for the eccentricity of 0.00399 per Jupiter period. Note that the divergence is minute at first and then becomes larger. Because there is no renormalisation the later data values are not good to use in the calculation of the Lyapounov characteristic exponent.

Q9.4 Taking $\mu = 1/1048.672$ (from values of the solar and jovian masses given in Tables A.1 and A.2) and starting values in the stated range gives maximum eccentricities above 0.3 for the following values of a and e: (i) $a = 0.470-0.476$ with $e = 0.3$; (ii) $a = 0.478$ with $e = 0.27-0.30$; (iii) $a = 0.479$ with $e = 0.22-0.26$ and $e = 0.30$; (iv) $a = 0.480$ with $e = 0.16-0.17$ and $e = 0.30$; (v) $a = 0.481$ with $e = 0.04$, $e = 0.12-0.18$ and $e = 0.29-0.30$; (vi) $a = 0.482$ with $e = 0.16-0.18$, $e = 0.21-0.24$ and $e = 0.26-0.30$; (vii) $a = 0.483$ with $e = 0.21-0.30$; (viii) $a = 0.484$ with $e = 0.25-0.30$; (ix) $a = 0.485$ with $e = 0.28-0.30$; (x) $e = 0.29-0.30$ with $a = 0.486-0.488$; (xi) $e = 0.30$ with $a = 0.489-0.490$. Note that the chaotic nature of some of these orbits means that your results could have small differences from those quoted above.

Q9.5 The largest semi-major axis before the condition becomes satisfied is $a = 0.773$; i.e. the first value for which $e > \Delta a/a'$ is $a = 0.774$. Using the overlap criterion given in Eq. (9.148), the predicted value is $a = 0.822$. Note that the value of $\Delta a/a'$ from the encounter map is larger than that predicted; the same over-estimate is seen in Fig. 9.23 for a range of mass ratios.

Q9.6 The maximum absolute differences in longitude for each planet for $\Delta t = 1$d and $\Delta t = 10$d, respectively, are as follows. Mercury: 0.0754°, 0.0730°. Venus: 0.191°, 0.193°. Earth: 0.0358°, 0.0331°. Mars: 0.183°, 0.183°. Jupiter: 0.377°, 0.377°. Saturn: 4.41°, 4.41°. Uranus: 0.511°, 0.511°. Neptune: 0.537°, 0.537°. Pluto: 1.022°, 1.022°.

Chapter 10

Q10.1 The exact resonance is located at 136792 km giving a wave of amplitude of 2.0 km.

Q10.2 The Prometheus 13:12 ILR is at 132196 km; the first observed feature is just exterior to this location.

Q10.3 Under this model the mass of Pan is 1.96×10^{17} kg and its density is an unbelievable 46.9 g cm^{-3}! (This paradaox can be resolved by realising that the width of the cleared gap is determined by the shepherding mechanism; see Cuzzi & Scargle 1985.) Using this large mass gives $p = 258$ for overlap and this occurs at a separation in semi-major axis of 345 km. The maximum radial width of a horseshoe is twice the distance to the L_1 point or ~ 130 km; the observed ring is much narrower than this (see Fig. 10.21a). For the last part of the question think about resonance overlap.

Q10.4 The ratio of the widths is $W_{\rm cr}/W_{\rm Lr} = \sqrt{2}\left(e'(j-1)[-1+2j+\alpha D]b_{1/2}^{(j-1)}/[2j+\alpha D]b_{1/2}^{(j)}\right)^{1/2}$.

Q10.5 The locations and widths of the Cordelia OLRs in the stated range are 26:27 at 51014.8 km (width 1.689 km), 25:26 at 51065.1 km (width 1.691 km), 24:25 at 51119.5 km (width 1.693 km), 23:24 at 51178.6 km (width 1.695 km) and 22:23 at 51243.0 km (width 1.697 km). There is only one Ophelia ILR in the range; it is the 15:14 at 51176.6 km (width 2.300 km). The 24:25 Cordelia OLR produces a wave of amplitude 0.34 km. The 15:14 Ophelia ILR produces a wave of amplitude 0.24 km. *Note*: When dealing with the OLRs in this question you cannot make use of the formulae given in Eqs. (10.21)–(10.23) because these are only valid for ILRs. However, similar formulae can be derived. These give $e'_{\rm f} = |n' f_{\rm d}\,(m/m_{\rm c})\,/\,[(j-1)n - jn']|$, $a'e'_{\rm f} = 2a'^2\,|f_{\rm d}|\,(m/m_{\rm c})\,/\,(3j\,|a' - a'_{\rm res}|)$ and $W' = 4a'\,\{2\,|f_{\rm d}|\,(m/m_{\rm c})\,/(3j)\}^{1/2}$ where $f_{\rm d} = \frac{1}{2}\,[-1+2j+\alpha D]\,b_{1/2}^{(j-1)}$.

Q10.6 Here we have assumed that the density of Galatea is 1.2 g cm^{-3}. (a) The 42:43 CIR is at 62929.5 km. (b) The width of the 42:43 CIR is 0.500 km. (c) This has to be done very carefully! See section II.A of the paper by Horanyi & Porco (1993) for a complete explanation and numerical evaluation. By taking of account of the variation of the mean longitude at epoch, Horanyi & Porco found that the location of the resonances shifted outwards by 0.2 km. (d) The 42:43 OLR is at 62927.8 km. (e) Using the formulae given in the answer to Q10.5 above, the radial amplitude of the wave should be 31 km. Compare this with the full width of the OLR (29 km) and the width of the CIR (0.5 km)! (f) Check out fig. 2 in the paper by Goldreich *et al.* (1986) for an idea of how this might look. *Note*: The numbers used in this question (apart from the assumed density of Galatea) were taken from Appendix A and are not identical to those used by Porco (1991). Furthermore, the mean motion of Galatea and hence the location of the resonances have been revised by Sicardy *et al.* in *Nature* **400**, 731–733 (1999).