$$4\pi R_{\odot}^2 E_{\odot} = 4\pi R_{\odot}^2 \sigma T_{\odot}^4 = 3.8 \times 10^{26}\ \mathrm{W}$$

which is the Sun's luminosity ($L_{\odot}$)—its total radiative power. This same energy flows through a sphere of area $4\pi r_p^2$ at the Sun–planet distance r_p, so that the energy flux there is

$$E_p = 4\pi R_{\odot}^2 E_{\odot}/4\pi r_p^2 = (R_{\odot}/r_p)^2 E_{\odot}$$

If 1 m^2 of blackbody material intercepts this energy head-on with the Sun overhead, it will be raised to the **subsolar temperature** (in kelvins):

$$T_{ss} = (R_{\odot}/r_p)^{1/2} T_{\odot} \approx 394(r_p)^{-1/2} \qquad \textbf{(2–4)}$$

since $E_p = \sigma T_{ss}^4$. In the last equality, we have inserted the appropriate solar values and expressed r_p in AU. The subsolar temperatures are essentially the equilibrium **noontime** temperatures.

Although subsolar temperatures apply to the local noon on the surfaces of very slowly rotating planets (Mercury, the Moon, and Pluto), they are not appropriate for planets with atmospheres or for planets in rapid rotation. For these, assume that the effective absorbing area is the cross section πR_p^2 and that the effective radiating area is the total surface area $4\pi R_p^2$, where R_p is the radius of the planet. The albedo A is the fraction of incident solar radiation that is reflected, so that only the fraction $1 - A$ is absorbed. Therefore, energy absorbed per second is

$$(1 - A)\pi R_p^2 E_p = (1 - A)\pi R_p^2 (R_{\odot}/r_p)^2 \sigma T_{\odot}^4$$

while the planet radiates the power $4\pi R_p^2 \sigma T_p^4$. We again use the concept of equilibrium: that the rate of absorption of energy equals the rate at which it is radiated away fixes an equilibrium temperature. (Otherwise the temperature would either rise or fall.) Equating these and solving for the temperature, we have (in kelvins)

$$T_p = (1 - A)^{1/4}(R_{\odot}/2r_p)^{1/2} T_{\odot} \approx 279(1 - A)^{1/4}(r_p)^{-1/2} \qquad \textbf{(2–5a)}$$

with r_p expressed in AU. The equilibrium blackbody temperatures (Table A3–3) follow when $A = 0$. For the subsolar temperature, we get

$$T_{ss} \approx 394(1 - A)^{1/4}(r_p)^{-1/2} \qquad \textbf{(2–5b)}$$

By including the albedo effect, we may more closely approximate the observed planetary temperatures, but remember that these estimates neglect important complications, such as the circulation of planetary atmospheres, their heat retention, internal heat sources, and the variation of A with wavelength.

(D) ATMOSPHERES

Of the terrestrial planets, Mercury and the Moon have essentially no atmosphere, Venus and Mars possess a carbon dioxide (CO_2) atmosphere, and the Earth's atmosphere is primarily molecular nitrogen (N_2) and oxygen (O_2). The principal constituents of the atmospheres of the Jovian planets are molecular hydrogen (H_2) and helium (He). In general, planetary atmospheres are densest near the planet's surface and thin rapidly with increasing altitude. The composition of an atmosphere may be stratified, with the densest gases residing closest to the surface of the planet, but turbulent mixing and winds can lead to regions of homogeneous composition. Far from the planet's surface, incoming solar ultraviolet rays and X-rays usually ionize atmospheric atoms or dissociate molecules to form the layered **ionosphere** (Chapter 4).

To gain some understanding of planetary atmospheres, consider a simple model for their retention. To a first approximation, an atmosphere behaves like a **perfect gas,** that is, as particles that interact only through elastic collisions. Such a gas obeys a special relationship between pressure, temperature, and density:

$$P = nkT$$

where P is the pressure (the rate of change of the particles' momenta from collisions) in units of force per unit area (N/m^2), n is the number density of particles ($\#/m^3$), T is the absolute temperature (K), and k is Boltzmann's constant, equal to 1.38×10^{-23} J/K. Now, from the continuous collisions, the particles of the gas achieve, at a given temperature, an equilibrium distribution of velocities so that

$$F(v)\,dv \propto \exp\left(-\tfrac{1}{2}mv^2/kT\right) v^2\,dv$$

known as the **Maxwellian distribution** of velocities in a gas (Figure 2–7). Note that, because of the exponential decrease, the distribution has a long tail at large velocities—a few of the particles have been boosted to high speeds by the collisions. The peak of this distribution defines a **most probable speed:**

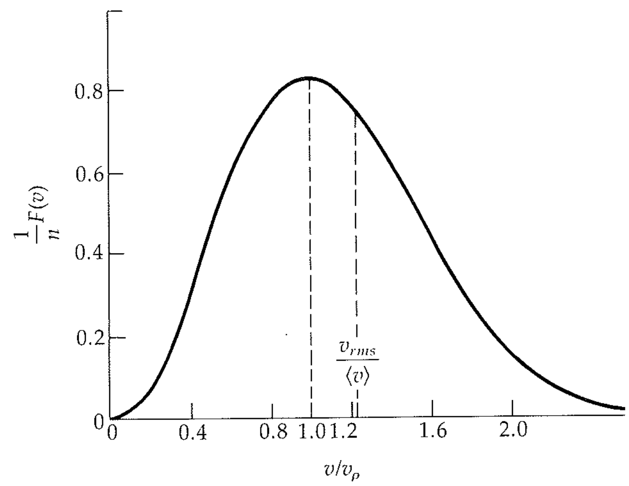

FIGURE 2–7 Maxwellian distribution of particles in a gas. The velocities of the particles spread around the most probable velocity v_p, which marks the peak of the distribution. The numbers on the vertical axis have been normalized to the total, n, in the gas.

$$v_p = (2kT/m)^{1/2}$$

where m is the mass of a gas particle. While the speeds of these particles (atoms or molecules) are distributed over a large range and change violently in each collision, the *average* kinetic energy per particle is

$$\langle KE \rangle = (m/2)\langle v^2 \rangle = 3kT/2 \qquad \textbf{(2–6)}$$

Here m is the particle's mass and T is the kinetic absolute temperature of the gas. From Equation 2–6 we obtain the **root mean square speed** v_{rms}:

$$v_{rms} = \langle v^2 \rangle^{1/2} = (3kT/m)^{1/2} \qquad \textbf{(2–7)}$$

which tells us that the mean speed of the particles increases with temperature and decreases with mass. In the very thin upper regions of atmospheres, a particle that moves outward with the *escape speed* v_e has an excellent chance of leaving the atmosphere, with v_e being

$$v_e = (2GM/R)^{1/2} \qquad \textbf{(2–8)}$$

where M is the planet's mass and R is its radius. If $v_{rms} = v_e$ for a given particle species, that gas will leave the atmosphere in only a few days. To retain an atmosphere for several billion years (approximately the age of the Solar System), a planet must have $v_e \geq 10v_{rms}$. (The factor of 10 takes into account the high-speed tail of the Maxwellian distribution of speeds.) Therefore a given type of molecule is retained indefinitely when (Equations 2–7 and 2–8)

$$T \leq GMm/150kR \qquad \textbf{(2–9)}$$

Figure 2–8 shows points corresponding to the equilibrium blackbody temperature (Table A3–3) and v_e for the planets and some moons; the dashed lines represent $10v_{rms}$ for various molecular species. In terms of this crude model, a planet retains all gases with lines passing below its point, and the other gases escape. This model reasonably explains that the Jovian planets have retained all gases and the Earth, Venus, and Mars have lost their hydrogen and helium but retained nitrogen and carbon dioxide; Mercury and the Moon have essentially no atmosphere; and the largest moons have thin atmospheres. (Titan, in fact, has a dense atmosphere composed largely of nitrogen but made smoggy because of photochemical reactions of methane and other compounds.)

2–2 ◐ MOONS, RINGS, AND DEBRIS

Natural planetary satellites, or **moons,** are many in the Solar System, but together they comprise a total mass of only $0.1M_\oplus$. Eight satellites are about the size of our Moon, while the others are much smaller and resemble large asteroids. Our Moon, the two satellites of Mars, the five inner satellites of Jupiter, the eight innermost of Saturn, and five of Uranus' moons have nearly circular orbits lying essentially in their planet's equatorial plane. Observations show that these 21 moons exhibit synchronous rotation from tidal friction (as does our Moon).

(A) MOONS

Only three moons circle the terrestrial planets, whereas the Jovian planets possess at least 51. The larger the mass of a planet, the greater the range of its gravitational influence. This coupled with the Jovian planet's proximity to the asteroid belt means that they can capture asteroids gravitationally. The Jovian satellites with small masses,

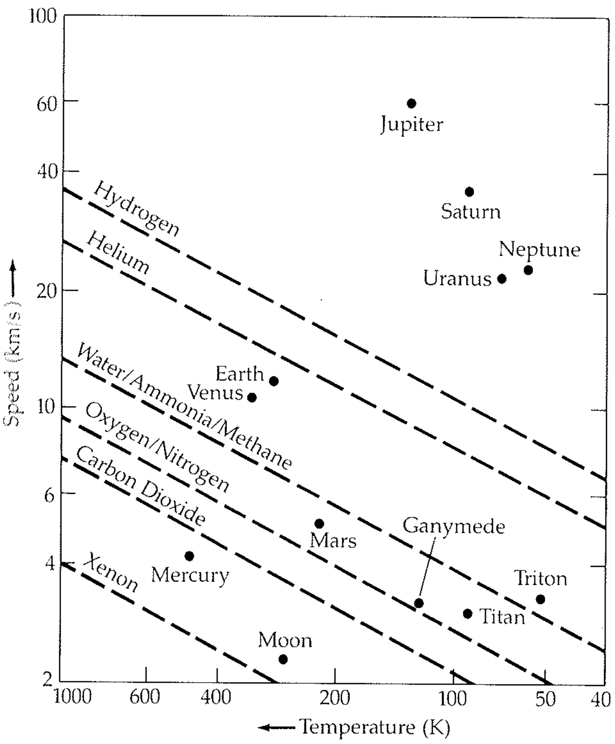

FIGURE 2–8 Retention of atmospheric gases. Mean molecular speeds are given as a function of temperature, along with the escape speeds for the indicated bodies. The dashed lines show ten times the mean molecular speeds, which defines an essentially infinite lifetime for that component in the atmosphere.

highly eccentric and inclined orbits, and retrograde motion are probably captured asteroids, as are the Martian moons.

(B) RINGS

Four of the Jovian planets are known to have rings. Of these ring systems, Saturn's is by far the most spectacular. In each case, the ring system is within the Roche limit [Section 3–4(d)]. Some rings appear to be kept in line by small satellites just outside and inside their boundaries.

(C) ASTEROIDS

Bode's law "predicted" the existence of a planet at 2.8 AU, between Mars and Jupiter, but it was not until 1801 that Giuseppe Piazzi (1746–1826) discovered the **minor planet** Ceres in this region. By 1980, more than 300 **asteroids** had been found with semimajor orbital axes between 2.3 and 3.3 AU—they made up the **asteroid belt.** Today, the orbital elements of more than 3000 asteroids are known, and each such body is numbered in order of orbit determination and given a name, such as asteroid *1000 Piazza*.

Asteroids are too small to retain atmospheres, for their observed diameters range from relatively large (Ceres, 1020 km; Pallas, 538 km; Vesta, 549 km) to the more abundant smaller ones (about 1 km); certainly a multitude of small rocks also swarm in the asteroid belt. The total mass in asteroids is probably about a few percent of our Moon's mass. The largest asteroids tend to reside farthest from the Sun, and the smallest closest; they are all in direct orbit about the Sun. Asteroids exhibit orbital eccentricities up to 0.83, with most in the 0.1 to 0.3 range, and orbital inclinations as large as 68° but more typically less than 30°.

The asteroid belt shows distinctly depleted regions, called **Kirkwood gaps,** at semimajor axes where the orbital period is a simple fraction (such as 1/2, 1/3, 1/4, 2/5, 3/7) of Jupiter's orbital period. Periodic gravitational perturbations from Jupiter have removed all asteroids from these gaps. Where the ratio of periods is 2/3 and 1/1, asteroids accumulate in **groups,** or **families;** the 15-member Trojan groups are situated at Jupiter's orbit on the vertices of equilateral triangles with the Sun and Jupiter at the other two vertices. The Trojans are bound here where they are stable against perturbations about their equilibrium positions. Some asteroids, which are known as the Apollo group and include Daedalus, Icarus, and Geographos, have perihelia within the Earth's orbit and may pass near the Earth at times. On June 15, 1968, Icarus careened past the Earth at a distance of a mere 6.4×10^6 km. Some people feared a collision and predicted the world would end. It did not.

Just as with Pluto, the rotation periods of asteroids are determined from fluctuations in their reflected sunlight. Some observed light curves exhibit two maxima and two minima per cycle, corresponding to oblong bodies that tumble every 3 to 20 h (the average rotation period for asteroids is 7 h). So we picture irregular fragments of rock, several kilometers in diameter, spinning in a few