II - Lasers

(*) Key idea: stimulated emission + population inversion in order to amplify light

Light Amplification by Stimulated Emission of Radiation

MASER ⇒ the same, but using microwave radiation

(*) First MASER: C. H. Townes et al, Phys. Rev. 95, 282 (1954)

(*) First Lasers: Proposed by A. L. Schawlow + C. H. Townes, Phys Rev. 99, 1264 (1955)

1. Lasers and Masers

(*) We will now show that:

- in order for light amplification to occur we need population inversion
- population inversion can not be achieved if the system is in thermal equilibrium

For simplicity, we will consider a two-level atom and rate equations (useful in the study of global phenomena, but insufficient for the treatment of processes involving phase differences).

Level 2: energy E_2 $\qquad E_2 > E_1$ $\qquad$ ——— E_2

Level 1: Energy E_1 $\qquad\qquad\qquad$ ——— E_1

Let us consider a beam of radiation of frequency $\omega = \frac{E_2 - E_1}{\hbar}$ and intensity I passing through the material.

- Rate of change in the average energy density (absorption from the beam):

$$\frac{d\overset{\nearrow \text{absorption - energy density}}{e_a}}{dt} = - \underbrace{N_1}_{\substack{\downarrow \\ \text{number} \\ \text{of atoms/} \\ \text{volume in} \\ \text{level 1}}} \hbar\omega \underbrace{W_{21}}_{\substack{\downarrow \\ \text{transition rate/atom} \\ \text{(absorption)}}}$$

- Rate of change in the average energy density (stimulated emission)

$$\frac{d\overset{\rightarrow \text{energy density (stimulated emission)}}{e_s}}{dt} = \underbrace{N_2}_{\substack{\downarrow \\ \text{number} \\ \text{of atoms/volume in level 2.}}} \hbar\omega \underbrace{W_{12}}_{\substack{\downarrow \\ \text{transition rate/atom}}}$$

- we know that $W_{12} = W_{21}$ and that both are proportional to I.

Let us consider the cross section $\sigma = \frac{\hbar\omega W_{12}}{I}$.

This quantity is characteristic of the pair of levels, but independent of I.

- Net rate of change (average energy density):

$$\frac{de}{dt} = \frac{de_a}{dt} + \frac{de_s}{dt} = \sigma I (N_2 - N_1)$$

Assumption: beam has a cross-sectional area A and travels along z.

Using $I(\omega) = \rho(\omega)\, c$

$$\frac{dI}{dz} = \frac{dI}{dt}\frac{dt}{dz} = \frac{1}{c} \cdot c \frac{d\rho}{dt}$$

$$\Rightarrow \boxed{\frac{dI}{dz} = \sigma I (N_2 - N_1)}$$

$N_1 > N_2 \Rightarrow$ radiation is absorbed

$N_1 < N_2 \Rightarrow$ radiation is amplified

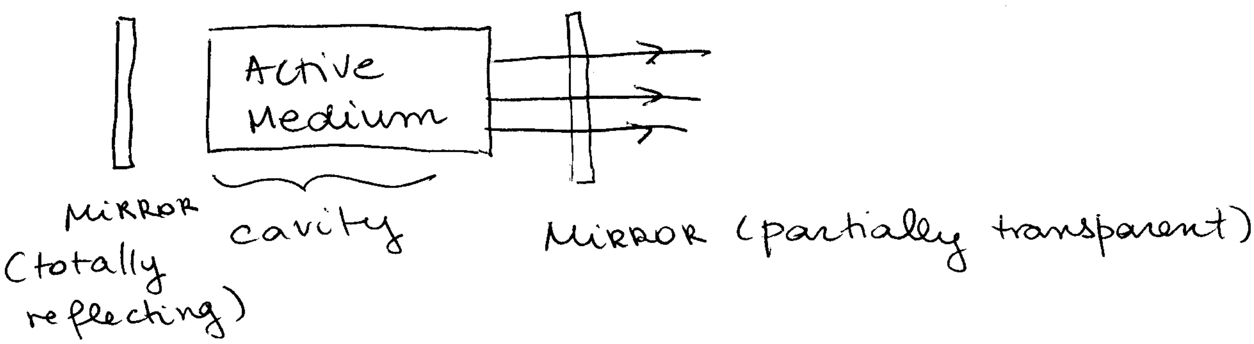
• Thermal equilibrium

$$\frac{N_2}{N_1} = \exp\left[-(E_2 - E_1)/(k_b T)\right]$$

$E_2 > E_1 \Rightarrow$ material acts as an absorber

$\Rightarrow$ A population inversion is needed so that the medium can act as an amplifier (active medium)

A positive feedback is also needed in order to make an oscillator from an amplifier - the medium is placed inside a cavity

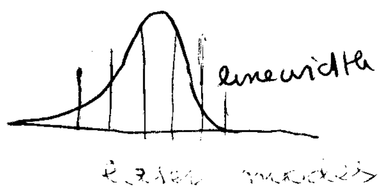


An e.m. wave will bounce back and forth between the mirrors - amplification takes place every time it passes through (stationary wave)

- Laser / Maser principle: excited atoms are injected in a cavity, to excite its modes

- Necessary condition for efficient excitation: frequency of atomic / molecular transition = frequency of the mode to be excited

- Problem in optical frequencies: $\lambda < \lambda_{microwave} \Rightarrow$ many modes can be excited.



(*) Problems to be solved in laser theory

- $\sim 10^{14}$ atoms interacting with many laser modes $\Rightarrow$ $\Rightarrow$ many-particle problem
- Laser is an open system (interaction with the surround
- Atomic system is far from thermal equilibrium

(*) Approaches

- Rate equations: global phenomena can be studied; insufficient for the treatment of phenomena involving phase differences

- Semiclassical theory

$\underbrace{\text{e.m. field of the cavity modes}}_{\text{classical}} + \underbrace{\text{laser-active atoms}}_{\text{motion of } e^-\text{'s is treated quantum mechanically}}$

Explains in which condition a single mode can be selected, or when several modes can coexist

- Quantum theory
  - Problems with the semiclassical laser theory:
    in this theory, above a critical pump power, light is created as a completely coherent wave.
    However, below this power, no light emission takes place.

    ⟹ The semiclassical theory fails to explain spontaneous emission / incoherent light

  ⟹ One should develop a theory which is QUANTUM MECHANICAL, but uses the NON-LINEARITIES of the classical theory.

(*) Structure

Full QM treatment

↓ Average over heat bath fluctuations

Semi-classical equations

↓ Average over phase

Rate equations

⊛ Properties :

- Monochromaticity : comes from the fact that only light from a transition between a single pair of levels is amplifie[d]

- Directionality : parallel beam which emerges perpendicular to the plane of the mirrors.

- Brightness : the power output is concentrated in a narrow directional beam

  Example : for an intensity of 10^{10} W one would need 10^8 100 W - light bulbs

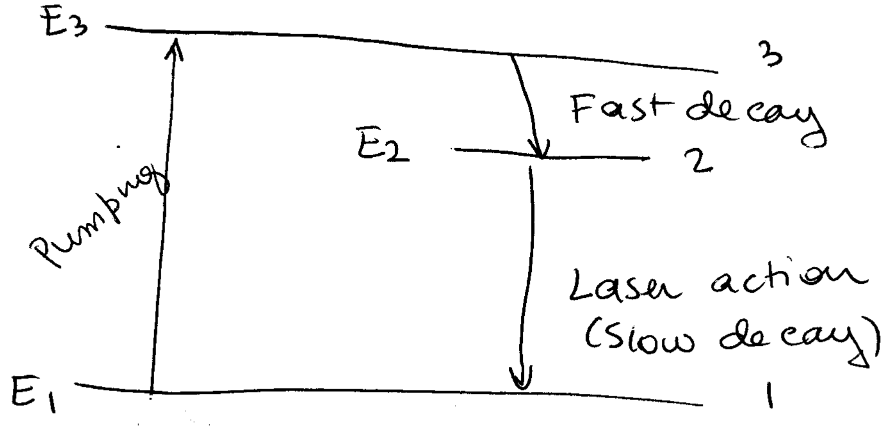
- Coherence : comes from the fact that stimulated emission is the physical mechanism behind it.

  ⟹ at each transition, one photon is added to a mode of the resonator, which is completely in phase with the other photons and has the same polarization.

2 - Methods for obtaining a population inversion

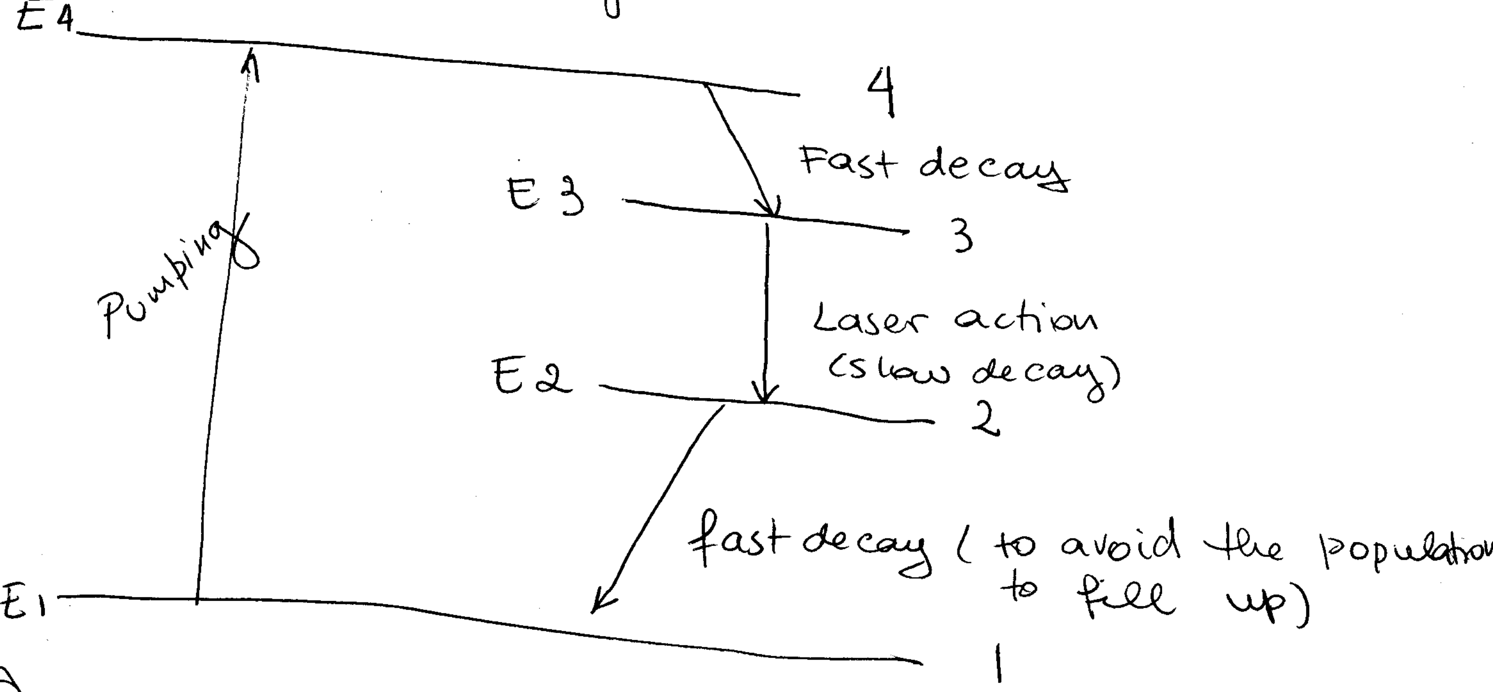
- Quantum-state selection (ammonia MASER, Hydrogen maser)
- Pumping techniques

(a) Three-level system



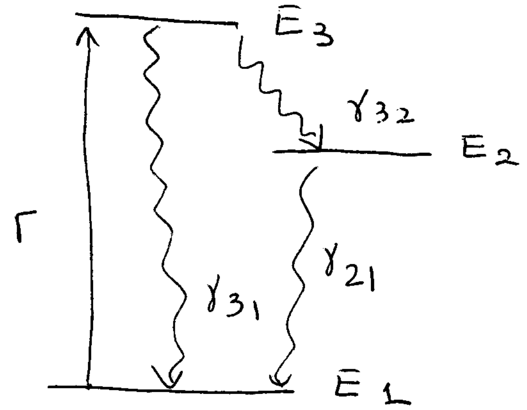
Difficulty: most of the atoms will be in the gd. state ⟹ one needs to get half of the atoms in level 2.

Solution: four level system



Please note: A population inversion cannot be obtained in a two-level system, because when the numbers of atoms in each level are equal, absorption ≡ stimulated emission.

* Population inversion - three level system



Assumptions:

$E_3 - E_2 >> k_bT$

$E_3 - E_1 >> k_bT$ $\Rightarrow$ no thermal excitation

$E_2 - E_1 >> k_bT$

$\Gamma \equiv$ pumping rate / atom

$\gamma_{31}, \gamma_{21}, \gamma_{32} \Rightarrow$ decay rates / atom

Rate equations

$$\frac{dN_1}{dt} = -\Gamma N_1 + \gamma_{21} N_2 + \gamma_{31} N_3$$

$N_i \equiv$ number of atoms in the i^{th} level

$N_1 + N_2 + N_3 = N$

$$\frac{dN_2}{dt} = -\gamma_{21} N_2 + \gamma_{32} N_3$$

$$\frac{dN_3}{dt} = \Gamma N_1 - (\gamma_{31} + \gamma_{32}) N_3$$

Stationary solution:

Physically, the system has reached equilibrium.

$$0 = -\Gamma N_1 + \gamma_{21} N_2 + \gamma_{31} N_3 \quad (*)$$

$$0 = -\gamma_{21} N_2 + \gamma_{32} N_3 \quad (**)$$

$$0 = \Gamma N_1 - (\gamma_{31} + \gamma_{32}) N_3 \quad (***)$$

Using (*) and (**) together with $N_3 = N - (N_1 + N_2)$

$$(*) \Rightarrow \quad 0 = -\Gamma N_1 + \gamma_{21} N_2 + \gamma_{31}(N - (N_1 + N_2)) =$$

$$= -(\Gamma + \gamma_{31}) N_1 + (\gamma_{21} - \gamma_{31}) N_2 + \gamma_{31} N$$

$$(**) \Rightarrow \quad 0 = -\gamma_{21} N_2 + \gamma_{32}(N - (N_1 + N_2))$$

$$= -(\gamma_{21} + \gamma_{32}) N_2 - \gamma_{32} N_1 + \gamma_{32} N = 0$$

Solving for N_1: $(*) \Rightarrow \quad N_1 = \dfrac{(\gamma_{21} - \gamma_{31}) N_2 + \gamma_{31} N}{\Gamma + \gamma_{31}}$

$$(**) \Rightarrow \quad N_1 = \frac{\gamma_{32} N - (\gamma_{21} + \gamma_{32}) N_2}{\gamma_{32}}$$

$$(*) = (**) \Rightarrow \quad N - \frac{(\gamma_{21} + \gamma_{32}) N_2}{\gamma_{32}} = \frac{\gamma_{31}}{\Gamma + \gamma_{31}} N + \frac{(\gamma_{21} - \gamma_{31}) N_2}{\gamma_3 + \Gamma}$$

$$\frac{(\gamma_{21} - \gamma_{31}) N_2}{\Gamma + \gamma_{31}} + \frac{(\gamma_{21} + \gamma_{32})}{\gamma_{32}} N_2 = \frac{\Gamma N}{\Gamma + \gamma_{31}}$$

$$\Rightarrow \left[\frac{\gamma_{21}(\gamma_{32} + \gamma_{31}) + \Gamma(\gamma_{21} + \gamma_{32})}{\gamma_{32}} \right] N_2 = \Gamma N$$

$$\boxed{N_2 = \frac{\gamma_{32} \Gamma N}{\gamma_{21}(\gamma_{32} + \gamma_{31}) + \Gamma(\gamma_{21} + \gamma_{32})}}$$

Inserting into $(**)$

$$\Rightarrow \boxed{N_1 = \frac{\gamma_{21}(\gamma_{32} + \gamma_{31}) N}{\gamma_{21}(\gamma_{32} + \gamma_{31}) + \Gamma(\gamma_{21} + \gamma_{32})}}$$

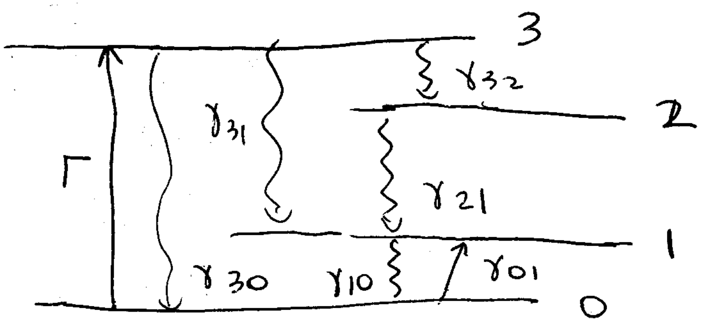
Population inversion:

$$\frac{N_2}{N_1} > 1 \Rightarrow \frac{\gamma_{32}\Gamma}{\gamma_{21}(\gamma_{32}+\gamma_{31})} > 1 \Rightarrow \Gamma > \gamma_{21}\left(1+\frac{\gamma_{31}}{\gamma_{32}}\right)$$

Goal:
- γ_{21} small (2 metastable)
- $\gamma_{31} \ll \gamma_{32}$ (fast decay from 3 to 2)

$$\lim_{\Gamma\to\infty} \Delta N = \frac{\gamma_{32}}{\gamma_{21}+\gamma_{32}} N$$

(*) Population inversion - 4-level system



We now assume that, for the ground state, thermal excitation can occur

- Total relaxation rate from 2: $\gamma_2 = \gamma_{20} + \gamma_{21}$
  from 3: $\gamma_3 = \gamma_{30} + \gamma_{31} + \gamma_{32}$

Rate equations

$$\frac{dN_1}{dt} = \gamma_{01}N_0 - \gamma_{10}N_1 + \gamma_{21}N_2 + \gamma_{31}N_3$$

$$\frac{dN_2}{dt} = -\gamma_2 N_2 + \gamma_{32}N_3$$

$$\frac{dN_3}{dt} = \Gamma N_0 - \gamma_3 N_3$$

$$N_0 = N - N_1 - N_2 - N_3$$

Stationary solution:

$$N_3 = \frac{\Gamma}{\gamma_3} N_0$$

$$N_2 = \frac{\gamma_{32}\Gamma}{\gamma_2 \gamma_3} N_0$$

$$N_1 = \left(\frac{\gamma_{01}}{\gamma_{10}} + \frac{\gamma_{21}\gamma_{32} + \gamma_2 \gamma_{31}}{\gamma_{10}\gamma_2\gamma_3} \Gamma \right) N_0$$

We are interested in $N_2 - N_1 > 0$ (population inversion)

$$\Gamma > \gamma_{01} \frac{\gamma_2 \gamma_3}{\gamma_{32}\gamma_{10} - \gamma_{21}\gamma_{32} - \gamma_2\gamma_{31}}$$

Note that $\gamma_{01} = \gamma_{10}\, e^{-\frac{(E_1 - E_0)}{k_B T}} \ll 1$

$\Rightarrow$ Population inversion can in principle be obtained with relatively weak pumping.