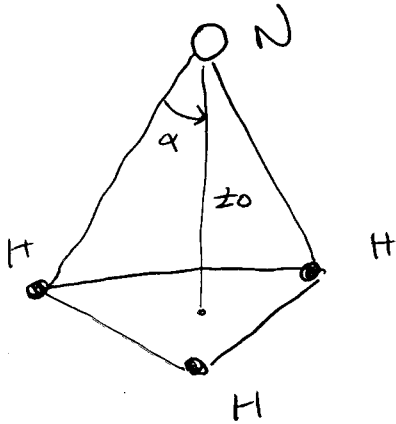


3. (a) - Ammonia Maser

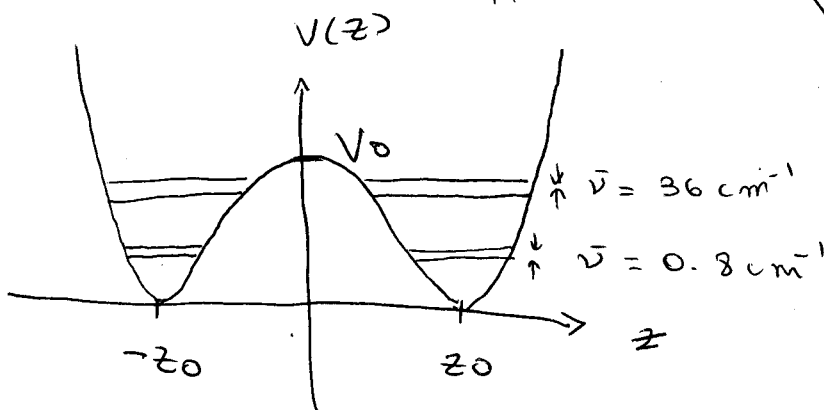
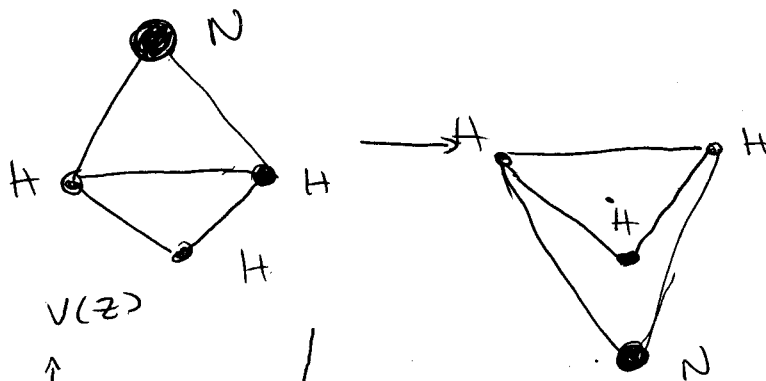
Let us consider the Ammonia molecule (NH_3):

\Rightarrow 3 H atoms in the xy plane +

a N atom at an equilibrium distance z_0 ,



and the specific vibrational mode in which the N atom inverts its position from $z_0 \rightarrow -z_0$



V_0 is a potential barrier with a FINITE height

- \Rightarrow tunneling can occur due to the inversion of the molecule) and the degenerate levels split.

Evolution: Schrödinger equation

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi(z)}{dz^2} + V(z) \psi(z) = E \psi(z)$$

Let us concentrate on the Lower doublet.
Energy eigenfunctions:

(*) $\Psi_1 = \frac{1}{\sqrt{2}} (\Psi_u + \Psi_d)$ $\Psi_u \equiv \Psi$ "up" \rightarrow concentrated near $z = z_0$

(**) $\Psi_2 = \frac{1}{\sqrt{2}} (\Psi_u - \Psi_d)$ $\Psi_d \equiv \Psi$ "down" \rightarrow concentrated at $z = -z_0$

$\Rightarrow \Psi(z,t) = c_1 \Psi_1(z) \exp(-iE_1 t/\hbar) + c_2 \Psi_2(z) \exp(-iE_2 t/\hbar)$
 $E_2 = E_1 + \Delta E$

Assumption: $\Psi(0) = \Psi_u \Rightarrow c_1 = c_2 = \frac{1}{\sqrt{2}}$

Add (*) + (**) to convince yourself.

You will find $\Psi_u = \frac{1}{\sqrt{2}} [\Psi_1(z) + \Psi_2(z)]$

$\Rightarrow \Psi(z,t) = \frac{1}{\sqrt{2}} [\Psi_1(z) \exp(-i\frac{E_1 t}{\hbar}) + \Psi_2(z) \exp(-i\frac{E_2 t}{\hbar})]$
 $\times \exp(-i(\Delta E t)/\hbar)$

$= \frac{1}{\sqrt{2}} [\Psi_1(z) + \Psi_2(z) \exp[-i\omega t]] \exp(-i\frac{E_1 t}{\hbar})$

\Rightarrow The wave function oscillates between the 2 energy eigenfunctions.

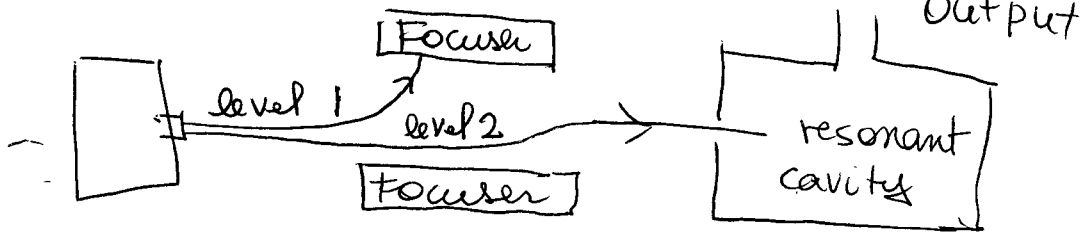
- C. E. Cleeton and N. W. Williams, \Rightarrow observed a 1934 peak in the NH_3 absorption spectrum corresponding

to this doublet

• Thermal equilibrium: population in E_1 > population in E_2

• The molecules exhibit a permanent dipole moment: D .

• Set up: beam of ammonia molecules passing through an inhomogeneous electric field \mathcal{E} .



\Rightarrow Field separates the molecules in a specific energy level

$$E'_1 = E_1 - \frac{(D\mathcal{E})^2}{\Delta E}$$

Details: Cohen Tannoudji, QM, Complement 5, IV

$$\langle F_z \rangle_{1/2} = -\frac{\partial}{\partial z} (E'_{1,2})$$

$$E'_2 = E_2 + \frac{(D\mathcal{E})^2}{\Delta E}$$

" energy difference between two levels "

$$= \pm \left(\frac{D^2}{\Delta E} \right) \mathcal{E} \frac{\partial \mathcal{E}}{\partial z}$$

MASER action from 2 to 1 is achieved by passing the beam through a cavity tuned to the required frequency.

In the cavity

$$\mathcal{E}(t) = \mathcal{E}_0 \cos(\omega t)$$

$$\Psi(t) = A_1(t) \Psi_1 \exp(-iE_1 t / \hbar) + A_2(t) \Psi_2 \exp(-iE_2 t / \hbar)$$

Transition amplitudes $A_1(t)$, $A_2(t)$ must satisfy

$$i\hbar \dot{A}_1(t) = \langle \Psi_1 | H' | \Psi_2 \rangle \exp(-i\omega_0 t) A_2(t)$$

$$i\hbar \dot{A}_2(t) = \langle \psi_2 | H' | \psi_1 \rangle \exp(i\omega_0 t) A_1(t)$$

(15)

with

$$\langle \psi_1 | H' | \psi_2 \rangle = \langle \psi_2 | H' | \psi_1 \rangle = -D\epsilon(t) = -D\epsilon_0 \cos \omega t$$

$$\hbar\omega_0 = E_2 - E_1 = \Delta E$$

$$i\hbar \dot{A}_1(t) = -\frac{D\epsilon}{2} [e^{-i\omega t} + e^{i\omega t}] e^{i\omega_0 t} A_2(t)$$

$$= -\frac{D\epsilon}{2} \left\{ \exp[-i(\omega + \omega_0)t] + \exp[-i(\omega - \omega_0)t] \right\} \times$$

$$\times A_2(t)$$

$$i\hbar \dot{A}_2(t) = -\frac{D\epsilon}{2} \left\{ \exp[i(\omega - \omega_0)t] + \exp[i(\omega + \omega_0)t] \right\} A_1(t)$$

Rotating wave approximation: terms in $\omega + \omega_0$ neglected

$$i\hbar \dot{A}_1(t) = -\frac{D\epsilon}{2} \exp[-i\Delta\omega t] A_2(t)$$

$$i\hbar \dot{A}_2(t) = -\frac{D\epsilon}{2} \exp[i\Delta\omega t] A_1(t)$$

$$i\hbar \ddot{A}_1(t) = -\frac{D\epsilon}{2} \left[-i\Delta\omega t \exp[-i\Delta\omega t] A_2(t) + \exp[i\Delta\omega t] \dot{A}_2(t) \right]$$

Resonance: $\Delta\omega = 0$

$$\hbar \ddot{A}_1(t) = -\frac{D\mathcal{E}}{2} \left(-\frac{D\mathcal{E}}{2} \right) \frac{1}{\hbar} A_2(t)$$

$$\Rightarrow \ddot{A}_1(t) + \frac{D^2 \mathcal{E}^2}{4\hbar^2} A_1(t) = 0$$

Initially, all molecules are in the level 2

$$A_1(0) = 0$$

$$\Rightarrow A_1(t) = C_1 \sin\left(\frac{D\mathcal{E}}{2\hbar} t\right)$$

$$|A_1|^2 = \sin^2\left(\frac{D\mathcal{E}}{2\hbar} t\right) \Rightarrow |A_2|^2 = 1 - |A_1|^2 = \cos^2\left(\frac{D\mathcal{E}}{2\hbar} t\right)$$

Laser transition:

$$|A_1(T)|^2 = 1 \Rightarrow \sin\left(\frac{D\mathcal{E}}{2\hbar} T\right) = 1$$

$$\frac{\bar{\omega}_0 T}{4} = \frac{D\mathcal{E} T}{2\hbar} = \frac{\pi}{2}$$

Let us consider $v \equiv$ the most probable velocity in the direction of motion

$L \equiv$ length of the cavity

$$L = v T = \frac{2\pi v}{\bar{\omega}_0}$$

More generally, if $P_{2 \rightarrow 1}(T)$ is the probability that the transition $2 \rightarrow 1$ occurs when a molecule crosses the cavity n times

→ Power supplied by the microwave field

$$P = N \Delta E P_{2,1}(T)$$

↳ number of molecules / second

Power lost from the field to the cavity:

$$P_e = P_{out} + P_{abs}$$

\downarrow \downarrow
 Power output Power absorbed by the walls

$$P_{abs} = \frac{\omega_0 E}{Q}$$

\swarrow total energy of the field
 \nwarrow cavity quality factor

$$P_{out} = \frac{\omega_0 E}{Q_1}$$

\rightarrow quality factor determined by the coupling between the cavity and the output wave guide

$$E = \frac{1}{8} \epsilon_0 (\mathcal{E})^2 V$$

If $P_{out} \ll P_{abs}$, the system acts as an oscillator of

$$P \approx P_e = P_{abs} = N \Delta E P_{2 \rightarrow 1}(T) = \frac{\omega_0}{Q} \frac{1}{8} \epsilon_0 (\mathcal{E})^2 V$$

$$\Rightarrow N = \frac{\epsilon_0 \omega_0 (\mathcal{E})^2 V}{8 Q \Delta E P_{2 \rightarrow 1}(T)}$$

If P_{21} small, $P_{2 \rightarrow 1}(T) \approx \left(\frac{D \epsilon_0 T}{2 \hbar} \right)^2$

$$= \left(\frac{D \epsilon_0 L}{2 \hbar \nu} \right)^2$$

We also know that $\Delta E = \hbar \omega_0$

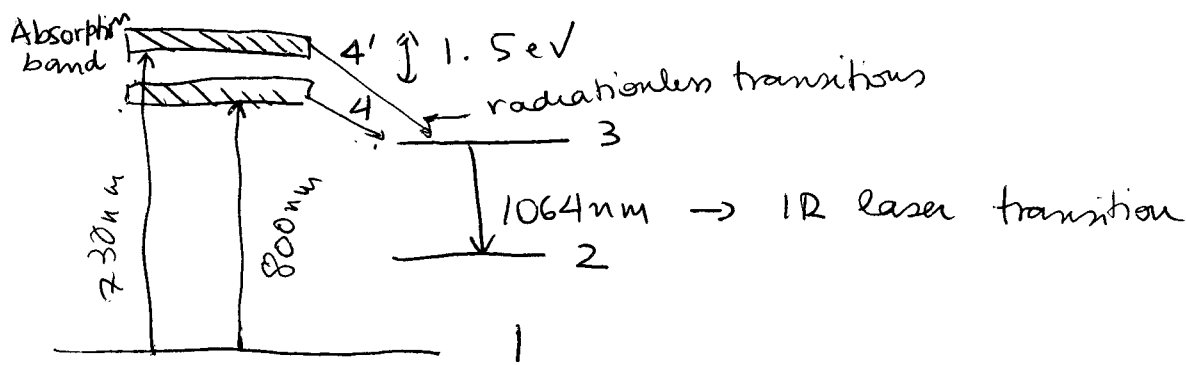
$$\Rightarrow N = \frac{\epsilon_0 \hbar \nu^2}{2 Q D^2 L^2}$$

3. (b) - Neodymium lasers

- Host medium: a crystal of $Y_3Al_5O_{12}$ (YAG \equiv yttrium aluminium garnet)

Some of the Y^{3+} are replaced by Nd^{3+}

- Four-level laser



3. (c) - Argon lasers

- Excited Ar^+ ions are produced by e^- impact collisions
 - ① Neutral Ar atoms are ionized by e^- s ($E_{kin} > 15.75 eV$)
 - ② Gd state Ar^+ ion is excited by e^- impact (strong DC discharge in a low-density gas)

Gd state: $1s^2 2s^2 2p^6 3s^2 3p^5$

Excited states: $1s^2 2s^2 2p^6 3s^2 3p^4 4s$

$1s^2 2s^2 2p^6 3s^2 3p^4 4p$

$\tau(4p) \sim 10^{-8} s$ one order of magnitude longer than $\tau(4s)$

\Rightarrow Excited Ar^+ accumulate mostly in 4p

Lasing transitions: $4p \rightarrow 4s$
(blue/green lines)

Strongest transitions:

$$\lambda = 5145 \text{ \AA} \quad 4p \ 4D_{5/2} \rightarrow 4s \ 2P_{3/2} \quad (\text{green})$$

$$\lambda = 4880 \text{ \AA} \quad 4p \ 2D_{5/2} \rightarrow 4s \ 2P_{3/2} \quad (\text{blue})$$

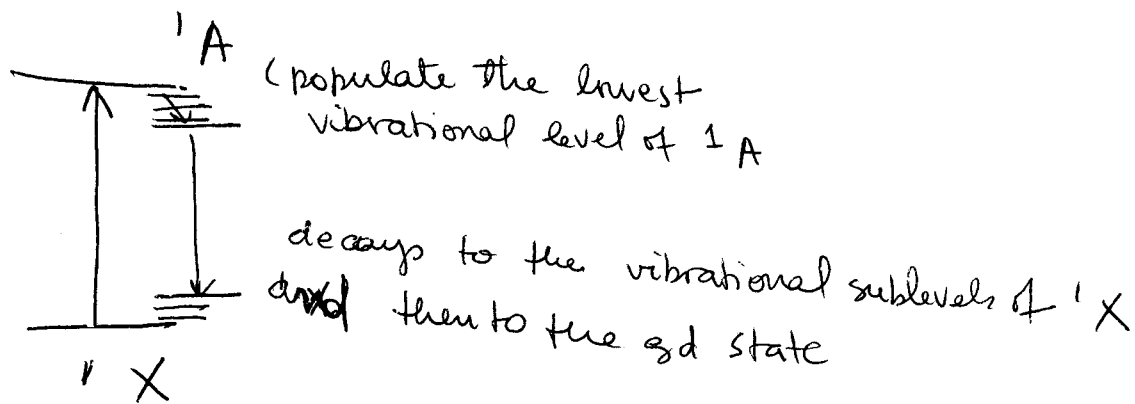
For all transitions: see page 831, Bransden + Joachain, "Physics of Atoms and Molecules".

3. (d) - Dye lasers

P. P. Sorokin + J. R. Lankard, 1966

F. P. Schäfer, W. Schmidt and J. Volze, 1966

- ⊕ Active material: organic dye in a liquid solvent (complex polyatomic molecules)
- ⊕ Operate in a four-level scheme
- ⊕ Relevant transitions: occur between the two lower singlet states



⊕ Usefulness:

- Tunability
- wide spectral range

Note that transitions such as

$$1A \rightarrow 1B, \quad 1A \rightarrow 3A \text{ can also occur.}$$