

✓ - Laser cooling and trapping of neutral atoms

⊛ Key idea : Light can be used to manipulate matter, due to the fact that it carries momentum. In fact, it exerts radiation pressure on material particles.

⊛ Early examples (radiation pressure):

- 17th century: Kepler proposed that the repulsion of comet tails from the sun was due to radiation pressure
- 1873: Maxwell showed that an electromagnetic field exerts a pressure $P = \frac{I}{c}$
- 1917: Einstein predicted the existence of the recoil velocity

If an atom of mass M absorbs a photon of momentum $\vec{p} = \hbar \vec{k}$, this photon causes the atom to recoil in the direction of the incident light and to change its velocity by $v_R = \frac{\hbar k}{M}$

Here we assumed that the photon is resonant with the atomic transition $a \rightarrow b$.

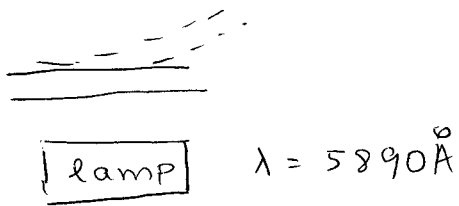
The atom also recoils when emitting a photon and returning from the state b to a state a (spontaneous emission of a fluorescence photon)

In this case, however, the direction of the photon, and therefore of the recoil velocity, is random.

- \Rightarrow Absorption : v is increased in the direction of the beam
- Emission : v is decreased in a random direction

⇒ Net result: an acceleration of the atom in the direction of the laser beam

• 1933: R. Frish observed a deflection in a beam of sodium atoms caused by a resonant sodium lamp. Due to the lamp's low intensity, however, only $\frac{1}{3}$ of the sodium atoms were excited



• 1970s: narrow band, tunable lasers ⇒ higher brightness, directionality, near monochromaticity

⇒ Substantial increase in the radiation-pressure force

$$F = \frac{\Delta p}{\Delta t} = \frac{h\nu}{cz} = \frac{h\nu}{cz}$$

Example: Na, resonance transition $3^2S_{1/2} \rightarrow 3^2P_{3/2}$
 $\tau = 16 \text{ ns}$

Radiation pressure force: $F = 2.2 \times 10^{-19} \text{ N}$

For comparison: Gravitational force

$$F = mg = 9.81 \times M_{\text{ma}} = 3.5 \times 10^{-25} \text{ N}$$

$\frac{h\nu}{cz} \approx 10^6 \Rightarrow$ the radiation pressure force is around 10^6 times larger than the gravitational force on the surface of the earth.

• Further example of manipulation of matter with

light: Optical pumping

⇒ useful for confining atoms and explaining certain cooling mechanisms

1950s, Alfred Kastler: one can use the resonant exchange of angular momentum between atoms and polarized photons to align / orient atomic spins, or heat/cool internal degrees of freedom.

1 - Slowing of atomic beams

Questions: how many photons does one need to stop a beam of sodium atoms?

Average velocity : 900 m/s
(oven at $T=600\text{K}$)

Recoil velocity (Na resonance line at $\lambda = 5890\text{\AA}$) ⇒ 3 cm/s

⇒ One needs $\approx 30\,000$ counter-propagating photons

Problem: Zeeman effect

If an atom has a velocity v ,

$$\vec{R} = \vec{v}t + \vec{R}_0$$

$$e^{i(\vec{k} \cdot \vec{R} - \omega t)} = e^{i\vec{k} \cdot \vec{R}_0} e^{-i(\omega - \vec{k} \cdot \vec{v})t}$$

It is as if $\omega \rightarrow \omega - \vec{k} \cdot \vec{v}$

As the atom beam is decelerated, \vec{v} changes. This implies that the atom becomes non-resonant and is no longer decelerated

Solutions:

• Chirping \Rightarrow the frequency of the laser field is modified to account for the Doppler shift

(V. S. Letokhov, V. G. Minogin and D. Pavlik, Opt. Comm. 19, 72 (1976))

• Zeeman tuning technique \Rightarrow the Doppler shift is compensated by the Zeeman shift introduced by an inhomogeneous magnetic field

(W. D. Phillips and H. J. Metcalf, Phys. Rev. Lett. 48, 596 (1982))

$$\omega_0 \rightarrow \omega_0 + \gamma B(z)$$

* Examples

Doppler and recoil effects (Quantum mechanical discussion).

Let us consider the Hamiltonian

$$H = \underbrace{\sum_{i=1}^n \frac{p_i^2}{2m}}_{\text{internal degrees of freedom (atom)}} + V(r_1, r_2, \dots, r_n) + W_{rel} + \underbrace{\frac{P^2}{2M}}_{\text{center of mass motion}} + \underbrace{H_{int}(t, \mathbf{R})}_{\text{interaction Hamiltonian}}$$

Initial energy

Eigenstates

$$E_i = \underbrace{\frac{\hbar^2 k_i^2}{2M}}_{\text{kinetic energy (center of mass)}} + \underbrace{E_{d_i}}_{\text{internal energy}} \Rightarrow |k_i, d_i\rangle \Rightarrow \text{center of mass with momentum } k_i; \text{ atom in the state } d_i$$

Final energy

$$E_f = \frac{\hbar^2 k_f^2}{2M} + E_{\alpha f}$$

$|k_f\rangle |\alpha_f\rangle \Rightarrow$ center of mass with momentum k_f , atom in the state α_f

The transition amplitude is proportional to

$$\langle \varphi_f | H_{\text{int}} | \varphi_i \rangle = \langle k_f | e^{i\vec{k} \cdot \vec{R}} | k_i \rangle \underbrace{\langle \alpha_f | H_{\text{int}}(t) | \alpha_i \rangle}_{\text{time dependent term}}$$

(see Part I of the course for details)

$$\langle k_f | e^{i\vec{k} \cdot \vec{R}} | k_i \rangle = \int \underbrace{\langle \vec{k}_f | \vec{R} \rangle}_{\propto e^{-i\vec{k}_f \cdot \vec{R}}} e^{i\vec{k} \cdot \vec{R}} \underbrace{\langle \vec{R} | k_i \rangle}_{\propto e^{i\vec{k}_i \cdot \vec{R}}} d^3R$$

$$\int e^{i(\vec{k}_i - \vec{k} - \vec{k}_f) \cdot \vec{R}} d^3R = \delta(\vec{k}_i + \vec{k} - \vec{k}_f) \quad \text{Conservation of momentum}$$

- $\vec{k}_i \pm \vec{k} = \vec{k}_f$
- $\oplus \Rightarrow$ absorption
- $\ominus \Rightarrow$ emission

- $\vec{k}_i \Rightarrow$ initial center-of-mass momentum
- $\vec{k}_f \Rightarrow$ final center-of-mass momentum

i) Conservation of energy:

$$E_f - E_i = \pm \hbar \omega \quad \begin{matrix} + \text{ absorption} \\ - \text{ emission} \end{matrix}$$

$$\frac{\hbar^2}{2M} (k_f^2 - k_i^2) + E_{\alpha f} - E_{\alpha i} = \pm \hbar \omega$$

$$\underbrace{(k_f^2 - k_i^2)}_{(\vec{k}_f - \vec{k}_i) \cdot (\vec{k}_f + \vec{k}_i)}$$

ii) Conservation of momentum:

$$\vec{k}_f - \vec{k}_i = \pm \vec{k}$$

From this expression, $\vec{k}_f + \vec{k}_i = \pm \vec{k} + 2\vec{k}_i$

Inserting in i)

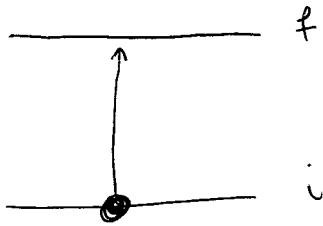
$$\pm \hbar \omega = \pm \frac{\hbar^2}{2M} \vec{k} \cdot (2\vec{k}_i \pm \vec{k}) \pm \hbar \omega_{\alpha f} = \frac{E_{\alpha f} - E_{\alpha i}}{\hbar}$$

Since $\frac{\hbar \vec{k}_i}{M} = \vec{v}_i$

$\omega = \vec{k} \cdot \vec{v}_i \pm \frac{\hbar k^2}{2m} \pm \omega_{fi}$

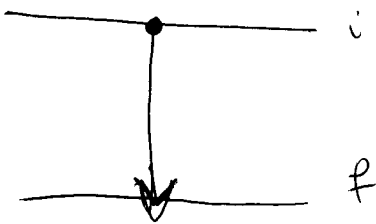
+ absorption
- emission

Absorption



$\omega = \omega_{fi} + \vec{k} \cdot \vec{v}_i + \frac{\hbar k^2}{2M}$
Labels: Doppler term, Recoil effect, transition freq.

Emission



$\omega = \omega_{if} + \vec{k} \cdot \vec{v}_i - \frac{\hbar k^2}{2M}$
Labels: Doppler term, recoil effect

Orders of magnitude

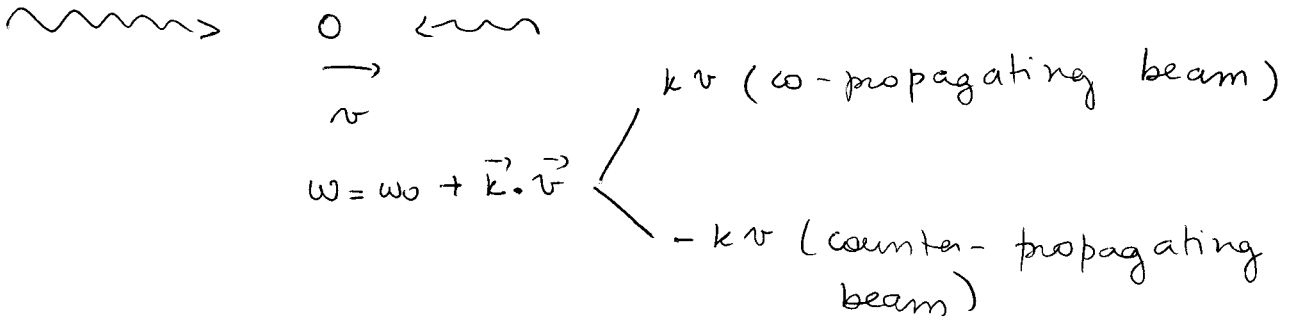
Doppler term: $\frac{\omega v}{c} \approx \frac{10^3}{3 \times 10^8} \times 10^{-6}$

Recoil effect: $\frac{\hbar^2 \omega^2}{2Mc^2} = \frac{\omega \hbar \omega}{2Mc^2} \approx 10^{-10}$ (very small)

2. - Doppler Cooling

- T. W. Hänsch and A. L. Schawlow, Opt. Comm. 13, 68 (1975)
- D. Wineland and H. Dehmelt, Bull. Am. Phys. Soc. 20, 637 (1975)

Key idea: If an atom is between two laser beams with frequency $\omega < \omega_0$, then it will absorb more photons from the counter-propagating beam

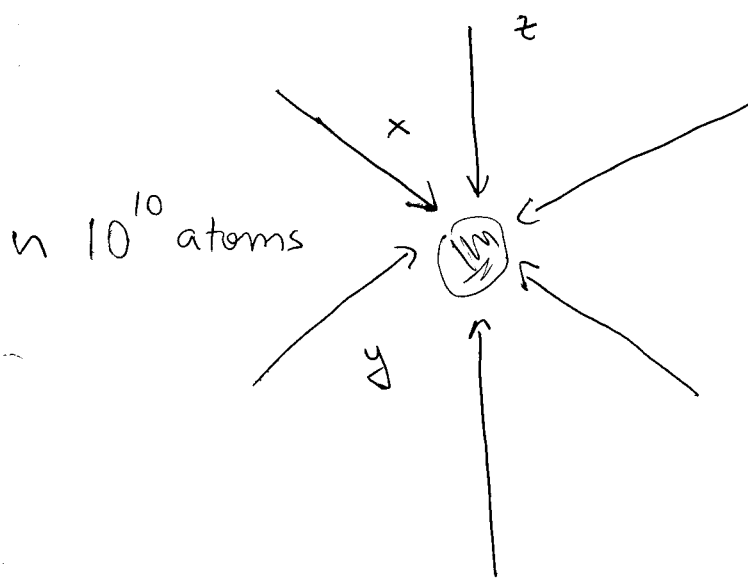


* Optical molasses

(S. Chu, J. E. Bjorkholm, A. Cable and A. Ashkin, Phys. Rev.

Let. 55, 48 (1985) $\rightarrow 10^5$ Na atoms were cooled to $T \approx 240 \mu K$

3 sets of counter-propagating laser beams with ~~red~~ detuned frequency $\omega_L < \omega_0$



- The atoms will absorb more photons from the counter-propagating beam
 - For small velocities
- $$F = -\alpha v$$
- (A damping force)
- \Rightarrow The atoms move in a "viscous" medium created by the laser fields (optical molasses)

This can be seen if we consider the average radiation pressure force in a two-level atom, given by

$$\vec{F} = \frac{\hbar \Gamma \Omega^2 \vec{k}}{\Gamma^2 + 2\Omega^2 + 4[\Delta - \vec{k} \cdot \vec{v}]^2}$$

(R. J. Cook, Phys Rev A 20, 224 (1979))

- $\Delta \equiv \omega_L - \omega_0$ (detuning) excited state
- $\Omega \equiv$ Rabi frequency $= \frac{e E_0 \langle e | r | g \rangle}{\hbar}$
- $\Gamma \equiv$ linewidth of the $g \rightarrow e$ transition in question state
- $\vec{v} \equiv$ atom velocity
- $\vec{k} \equiv$ wave vector of the laser beam

$$F_x = F_{1x} + F_{2x} = \hbar \Gamma \Omega^2 k \left[\frac{1}{\Gamma^2 + 2\Omega^2 + 4(\Delta - kv_x)^2} - \frac{1}{\Gamma^2 + 2\Omega^2 + 4(\Delta + kv_x)^2} \right]$$

Similar forces along the y and z axis

Expanding F_x in series,

$$F_x = \frac{\hbar \Gamma \Omega^2 k^2 \cdot 16 \Delta v_x}{[\Gamma^2 + 4\Delta^2 + 2\Omega^2]^2} + O(v^3) = -\beta v_x$$

\uparrow
 negative as Δ
 is negative

$S_0 = \frac{2\Omega^2}{\Gamma^2}$ is known as the saturation term.

If the driving-field intensity is low, $S_0 \ll 1$
 For high intensities, it causes F to saturate.

* The Doppler cooling limit

Ideally, all atoms would cool to $v=0 \rightarrow T=0K$. However, the light beam also causes heating. This occurs due to the discrete steps in momentum experienced by the atom in absorption/re-emission \Rightarrow this is a random walk process.

The kinetic energy of the atom will change by

$$E_r = \frac{\hbar^2 k^2}{2m} = \hbar \omega_r \quad \text{at each absorption/emission}$$

\uparrow
 Freq. associated with recoil

The average frequency:

- (i) at each absorption: $\omega_{ab} = \omega_0 + \omega_r \Rightarrow$ the light field loses $2\hbar\omega_r$ of energy in each absorption/emission cycle.
- (ii) at each emission: $\omega_{eb} = \omega_0 - \omega_r$

9
⇒ The atoms heat up by $2\hbar\omega_R$ at each cycle.

• Steady-state situation : cooling rate = heating rate

$$\text{cooling rate} : Fv = \beta v^2$$

$$\text{heating rate} : 2\hbar\omega_R\Gamma \cdot 2$$

↑
to account for absorption and emission

$$\beta v^2 = 4\hbar\omega_R\Gamma$$

It is possible to show that the kinetic energy of the atom depends on the detuning and that it will have a minimum for $\Delta = -\frac{1}{2}\gamma$

$$\Rightarrow T_0 = \frac{\hbar\Gamma}{2k_B} \quad (k_B \equiv \text{Boltzmann const.}) \quad \text{Na: } T_0 \approx 240\mu\text{K}$$

T_0 is known as the Doppler cooling limit

Details: D. Wineland and W. Itano, Phys Rev A 20, 1521 (1979);
V.S. Letokhov and V.G. Minogin, Phys Rev A 73, 1 (1981)

3 - Trapping of Atoms - the Magneto-Optical Trap

• In the optical molasses scheme, one cannot say that the atoms are trapped, as there is no potential minimum confining the atoms.

• One can build a Magneto-Optical trap by inserting an inhomogeneous magnetic field in the previous scheme

Proposed: D.E. Pritchard, E.L. Raab, V. Bagnato, C.E. Wieman, R.N. Watts, PRL 57, 310 (1986)

Built: E. L. Raab, M. Prentiss, A. Cable, S. Chu and D. E. Pritchard, Phys Rev Lett. 59, 2631 (1987)

Principle

Two-level atom:

Ground state: $J=0 (m_J=0)$

Excited state: $J=1 (m_J=-1, 0, 1)$

Inhomogeneous

magnetic field

$B(z) = b z$

$\Rightarrow m_J = 1$ and $m_J = -1$ sublevels experience Zeeman shifts which are linear in z .

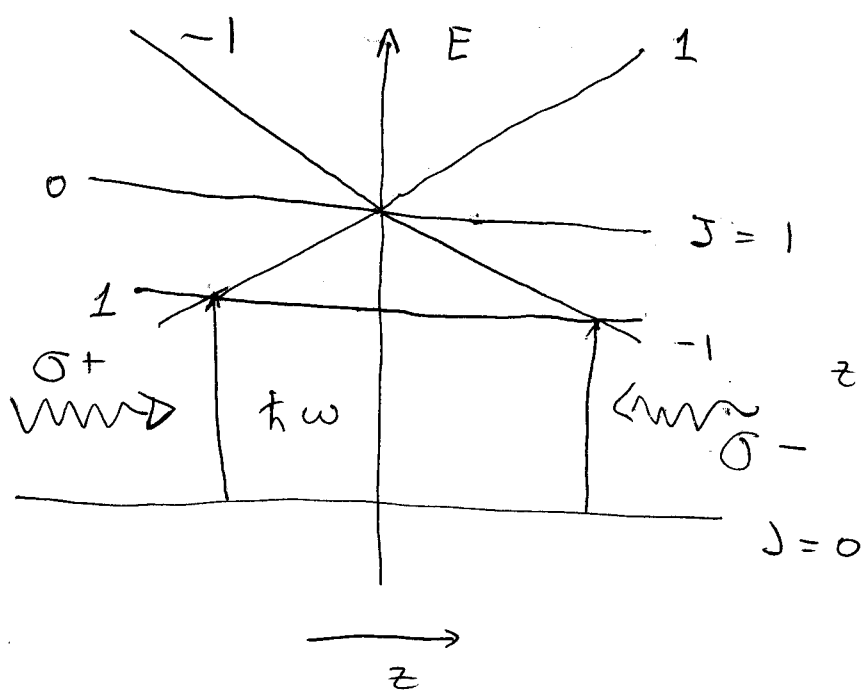
$\Delta E = \mu m_J B(z)$

Let us now consider two red detuned circularly polarized fields:

σ^- propagating along $-\hat{z}$

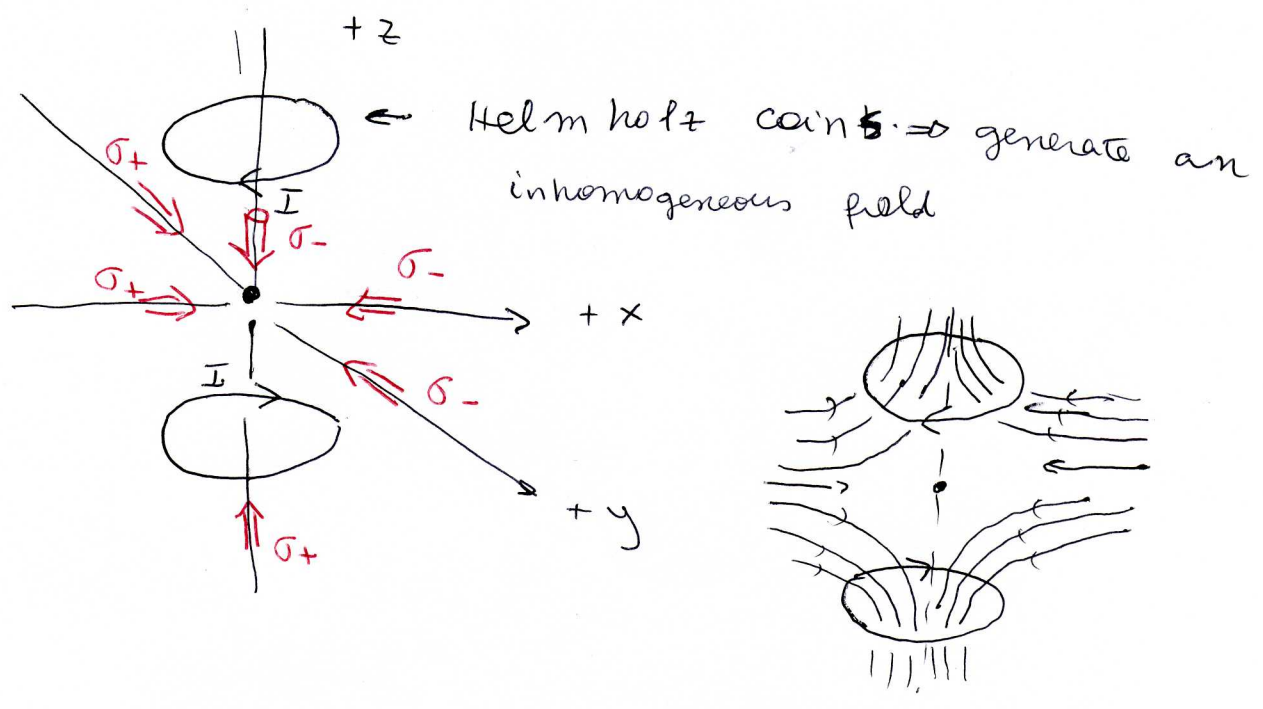
σ^+ propagating along $+\hat{z}$

The atom will absorb more photons from the counter-propagating beam



$z > 0$: The atom will absorb more photons from the σ^- beam

$z < 0$: The atom will absorb more photons from the σ^+ beam



4 - Sisyphus cooling

(J. Dalibard and C. Cohen-Tannoudji, J. Opt. Soc. Am. B 6, 2023 (1989) ; see C. Cohen-Tannoudji and W. D. Phillips, Physics Today, Oct. 1990 for a more accessible discussion).

⊛ Problems with Doppler cooling / two level picture:

- The confinement times in an optical molasses were optimized for much larger detunings than those predicted by the theory
- Sodium atoms could be cooled far below the Doppler limit ($T \approx 40 \mu K$ instead of $T \approx 240 \mu K$)

⊛ Key issues:

- The two-level picture breaks down: Alkali atoms have several ^{Zeeman} sublevels in the g state
 => Optical pumping can transfer atoms from g_m to $g_{m'}$
- The optical interaction induces energy shifts in g (light shifts)

⇒ The light shifts depend on the laser polarization, strength and vary for each Zeeman sublevel

• Within the molasses, there exist polarization gradients :
The population of the sublevels and the light shifts depend on the position of the atom in the laser wave

⊛ Examples - polarization gradients

Let us consider two plane waves propagating along the Oz axis.

$\epsilon_0, \epsilon'_0 \equiv$ amplitudes (real)

$\hat{\epsilon}, \hat{\epsilon}' \equiv$ polarizations

$\omega_L \equiv$ frequency (same for both)

• Total electric field : $\vec{E}(z,t) = E^+(z) \exp(-i\omega_L t) + c.c.$

$$E^+(z) = \epsilon_0 \hat{\epsilon} e^{ikz} + \epsilon'_0 \hat{\epsilon}' e^{-ikz}$$

(a) - The $\sigma^+ - \sigma^-$ configuration

$$\hat{\epsilon} = \hat{\epsilon}_+ = \frac{1}{\sqrt{2}} (\hat{\epsilon}_x + i\hat{\epsilon}_y)$$

$$\hat{\epsilon}' = \hat{\epsilon}_- = \frac{1}{\sqrt{2}} (\hat{\epsilon}_x - i\hat{\epsilon}_y)$$

$$E^+(z) = \epsilon_0 \left(\frac{1}{\sqrt{2}} (\hat{\epsilon}_x + i\hat{\epsilon}_y) \right) e^{ikz} + \epsilon'_0 \left(\frac{1}{\sqrt{2}} (\hat{\epsilon}_x - i\hat{\epsilon}_y) \right) e^{-ikz}$$

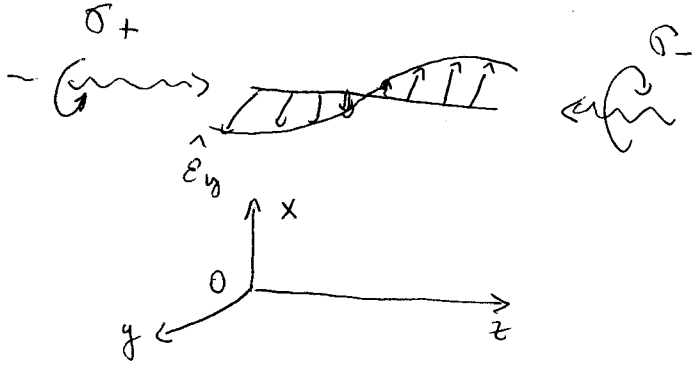
$$\Rightarrow E^+(z) = \frac{1}{\sqrt{2}} (\epsilon'_0 - \epsilon_0) \hat{\epsilon}_x - \frac{i}{\sqrt{2}} (\epsilon'_0 + \epsilon_0) \hat{\epsilon}_y$$

$$\hat{\epsilon}_x' = \hat{\epsilon}_x \cos(kz) - \hat{\epsilon}_y \sin(kz)$$

$$\hat{\epsilon}_y' = \hat{\epsilon}_x \sin(kz) + \hat{\epsilon}_y \cos(kz)$$

\hat{x}', \hat{y}' are orthogonal and deduced from x, y by a rotation around Oz ($\varphi = -kz$)

$E^+(z)$ is elliptically polarized and rotates around Oz



$E_0 = E_0'$
 $\rightarrow E^+(z)$ is linearly polarized along E_y
 (Helix with a pitch λ)

(b) The lin \perp lin configuration

$\hat{E} = \hat{E}_x$; $\hat{E}' = \hat{E}_y$ (two counter-rotating waves of orthogonal linear polarization)

$$E^+(z) = E_0 \sqrt{2} \left(\cos kz \frac{(\hat{E}_x + \hat{E}_y)}{\sqrt{2}} - i \sin kz \frac{(\hat{E}_y - \hat{E}_x)}{2} \right)$$

The ellipticity changes when one moves along Oz

$z = 0$: linear polariz. along $\frac{\hat{E}_x + \hat{E}_y}{\sqrt{2}}$ ($\cos kz = 1$; $\sin kz = 0$)

$$z = \lambda/8 : E^+(z) = E_0 \sqrt{2} \left(\underbrace{\cos\left(\frac{\pi}{4}\right)}_{\frac{1}{\sqrt{2}}} \frac{(\hat{E}_x + \hat{E}_y)}{\sqrt{2}} - i \underbrace{\sin\left(\frac{\pi}{4}\right)}_{\frac{1}{\sqrt{2}}} \frac{(\hat{E}_y - \hat{E}_x)}{\sqrt{2}} \right)$$

$$E^+(z) = E_0 \left(\frac{(\hat{E}_x + \hat{E}_y)}{\sqrt{2}} - i \frac{(\hat{E}_y - \hat{E}_x)}{\sqrt{2}} \right) \Rightarrow \sigma^- \text{ polarization}$$

$z = \frac{\lambda}{4} : E^+(z) = -i E_0 \frac{(\hat{E}_y - \hat{E}_x)}{\sqrt{2}}$ linearly polarized along $\hat{E}_x - \hat{E}_y$

$$z = \frac{3\lambda}{8} \Rightarrow E^+(z) = E_0 \sqrt{2} \left[\underbrace{\cos\left(\frac{3\pi}{4}\right)}_{-\frac{1}{\sqrt{2}}} \frac{(\hat{E}_x + \hat{E}_y)}{\sqrt{2}} - i \underbrace{\sin\left(\frac{3\pi}{4}\right)}_{\frac{1}{\sqrt{2}}} \frac{(\hat{E}_y - \hat{E}_x)}{\sqrt{2}} \right]$$

$$= -E_0 \left(\frac{(\hat{E}_x + \hat{E}_y)}{\sqrt{2}} + i \frac{(\hat{E}_y - \hat{E}_x)}{\sqrt{2}} \right) \Rightarrow \sigma^+ \text{ polarization}$$

$z = \lambda/2 \Rightarrow$ linear along $-\hat{e}_1 = -\frac{(\hat{E}_x + \hat{E}_y)}{\sqrt{2}}$

