Engineering Mathemathics III Summary Sheet 8

Laplace Transforms

1 Definition and Table

Laplace Transforms			
$f(t)$	$F(s) \equiv \mathcal{L}\{f(t)\}$	$f(t)$	$F(s) \equiv \mathcal{L}\{f(t)\}$
$f(t)$	$\int_0^\infty e^{-st} f(t)dt$		
1	$\frac{1}{s}$	e^{at}	$\frac{1}{s-a}$
$\sin at$	$\frac{a}{s^2+a^2}$	$\cos at$	$\frac{s}{s^2+a^2}$
$\sinh at$	$\frac{a}{s^2-a^2}$	$\cosh at$	$\frac{s}{s^2-a^2}$
t^n	$\frac{n!}{s^{n+1}}$	$e^{-at} f(t)$	$F(s+a)$
$t^n f(t)$	$(-1)^n \frac{d^n F(s)}{ds^n}$	$\frac{f(t)}{t}$	$\int_s^\infty F(s)\,ds$
$u(t)$ unit step $\mathcal{H}(t)$ Heaviside	$\frac{1}{s}$	$u(t-d)$	$\frac{e^{-sd}}{s}$
$\delta(t)$	1	$\delta(t-d)$	e^{-sd}
$\mathcal{L}\{f+g\}$	$\mathcal{L}\{f\}+\mathcal{L}\{g\}$	$\mathcal{L}\{kf\}$, k const	$k\mathcal{L}\{f\}$
$\frac{dx}{dt}$	$s\mathcal{L}\{x\}-x(0)$	$\frac{d^2x}{dt^2}$	$s^2\mathcal{L}\{x\}-sx(0)-x'(0)$

2 Inverse Laplace Transforms

2.1 Definition and Properties

1. $\mathcal{L}\{f(t)\} = F(s) \Rightarrow \mathcal{L}^{-1}\{F(s)\} = f(t)$
2. $\mathcal{L}^{-1}\{F+G\} = \mathcal{L}^{-1}\{F\} + \mathcal{L}^{-1}\{G\}$
3. $\mathcal{L}^{-1}\{kF\} = k\mathcal{L}^{-1}\{F\}$

2.2 Evaluating

Given $F(s)$ use the properties and tables to find $f(t)$ such that $\mathcal{L}\{f(t)\} = F$. Often this involves algebraic manipulation including completing squares and partial fraction techniques.

3 Solving Ordinary Differential Equations

1. Take $\mathcal{L}$ of the equation. For differential operators, you need to know the initial conditions.
2. The result is an algebraic equation for the Laplace transform of the dependent variable. Solve for this transform.
3. Take the inverse Laplace transform

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