SUMMARY SHEET: APPLICATIONS OF INTEGRATION
Engineering Maths II (MAE111), 2003

Reduction Formulae: When an integral contains a power of n, then it may sometimes be rewritten (using integration by parts) in terms of a similar integral containing $(n-1)$ (or $n-2$, etc.). The result is a *recurrence relation*. Eg:

$$I_n = \int x^n e^x \, \mathrm{d}x$$

produces the recurrence relation (or reduction formula)

$$I_n = x^n e^x - nI_{n-1}.$$

Hence, if I_0 can be found, then I_1 can be found, and then I_2, and so on.

Double integrals: Integral of a function of both x and y: $f(x,y)$ over some region in the x–y plane:

$$\int_{x_1}^{x_2} \int_{y_1}^{y_2} f(x,y) \mathrm{d}y \, \mathrm{d}x$$

The y integral is done first (ie work from inner integral outwards), and is done treating x as a constant. The limits y_1 and y_2 may be functions of x.

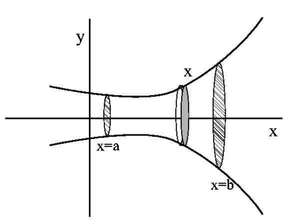
Mean of function: Mean value of function $y = f(x)$ between $x = a$ and $x = b$:

$$\bar{y} = \frac{\int_a^b y \, \mathrm{d}x}{b-a}$$

Root mean square of function: Root mean square (RMS) value y_{rms} of function $y = f(x)$ between $x = a$ and $x = b$

$$y_{\mathrm{rms}} = \sqrt{\frac{\int_a^b y^2 \, \mathrm{d}x}{b-a}}$$

Volume of Rotation – about x-axis:
$V = \int_a^b \pi y^2 \, \mathrm{d}x$



Volume of Rotation – About y-axis

$$V = \int_a^b 2\pi xy \, \mathrm{d}x$$

Centroid of plane figure: Position of centroid C of figure bounded by $y = f(x)$ and x- and y-axes is $(\bar{x}, \bar{y})$:

$$A\bar{x} = \int_a^b xy \, \mathrm{d}x$$

$$A\bar{y} = \int_a^b \frac{1}{2} y^2 \, \mathrm{d}x$$

where A is area of figure: $A = \int_a^b y \, \mathrm{d}x$.

Centre of mass of solid of revolution: By symmetry $\bar{y} = 0$, and

$$\bar{x} = \frac{\int_a^b xy^2 \, \mathrm{d}x}{\int_a^b y^2 \, \mathrm{d}x}$$

Length of curve: Length of curve given by $y = f(x)$ between $x = a$ and $x = b$:

$$S = \int_a^b \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2} \, \mathrm{d}x$$

If curve is in parametric form: $x = x(t)$, $y = y(t)$, then:

$$S = \int_{t_1}^{t_2} \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2} \, \mathrm{d}t$$

where t_1 and t_2 are limits on parameter for start and end of curve.

Surface of Rotation – about x-axis: Area of surface of rotation formes by rotating $y = f(x)$ about the x-axis:

$$A = \int_a^b 2\pi y \sqrt{1 + \left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)^2} \, \mathrm{d}x.$$

(If the surface is closed then remember to add on the area of circles at both ends.)

Approximate Integration: Split interval for integration into a number of subintervals ("strips"). For each strip there is a corresponding value of the function to be integrated. Label these F (first), L (last), R (the rest), E (even), or O (odd) depending on the method to be used. The various rules give methods to combine these values to give an approximate value for the integral. The strips are assumed constant width, equal to s.

Trapezium Rule:

$$A \approx \frac{s}{2}(F + L + 2R)$$

where the terms are: F for first value (at start of integration interval), L for last value (at end of integration interval), and R for "the rest" (i.e., the sum of the rest of values). Note that the values are those of the function to be integrated.

Simpson's Rule: The number of strips has to be even (and the number of x positions odd).

$$A \approx \frac{s}{3}\left[(F + L) + 4E + 2R\right]$$

where, the terms are: F: first, L: last, E: (sum of) even numbered values, R: (sum of) the rest of values. Note that odd/even is based on labelling the first value 1, the second 2, etc. For the same number of strips Simpson's rule is more accurate than the Trapezium rule.

Text Book References

	Mathematics for Engineers Croft & Davison	Engineering Mathematics Stroud (4th Edition)
Application of Integration	Ch 14	Programmes 17.1 – 17.30, 18.1 – 18.26, 19.2 – 19.39, 21.16 – 21.36