Orbits

Review: Generalized Kepler's Laws

1. Gravitating objects orbit one another in an ellipse, of eccentricity e and semimajor axis length a.

$$r = \frac{a\left(1-e^2\right)}{1+e\cos\theta}$$

4. Conservation of angular momentum means that objects move faster when they are closest to one focus (e.g. the Sun)

$$L = 2m\frac{\pi a^2\sqrt{1-e^2}}{P} = mrv_\theta$$

7. The orbital period P increases with the size of the semimajor axis, a.

$$P^2 = \frac{4\pi^2 a^3}{G(M+m)}$$

Circular Velocity

- A body in circular motion will have a constant velocity determined by the force it must "balance" to stay in orbit.
- By equating the circular acceleration and the acceleration of a mass due to gravity:

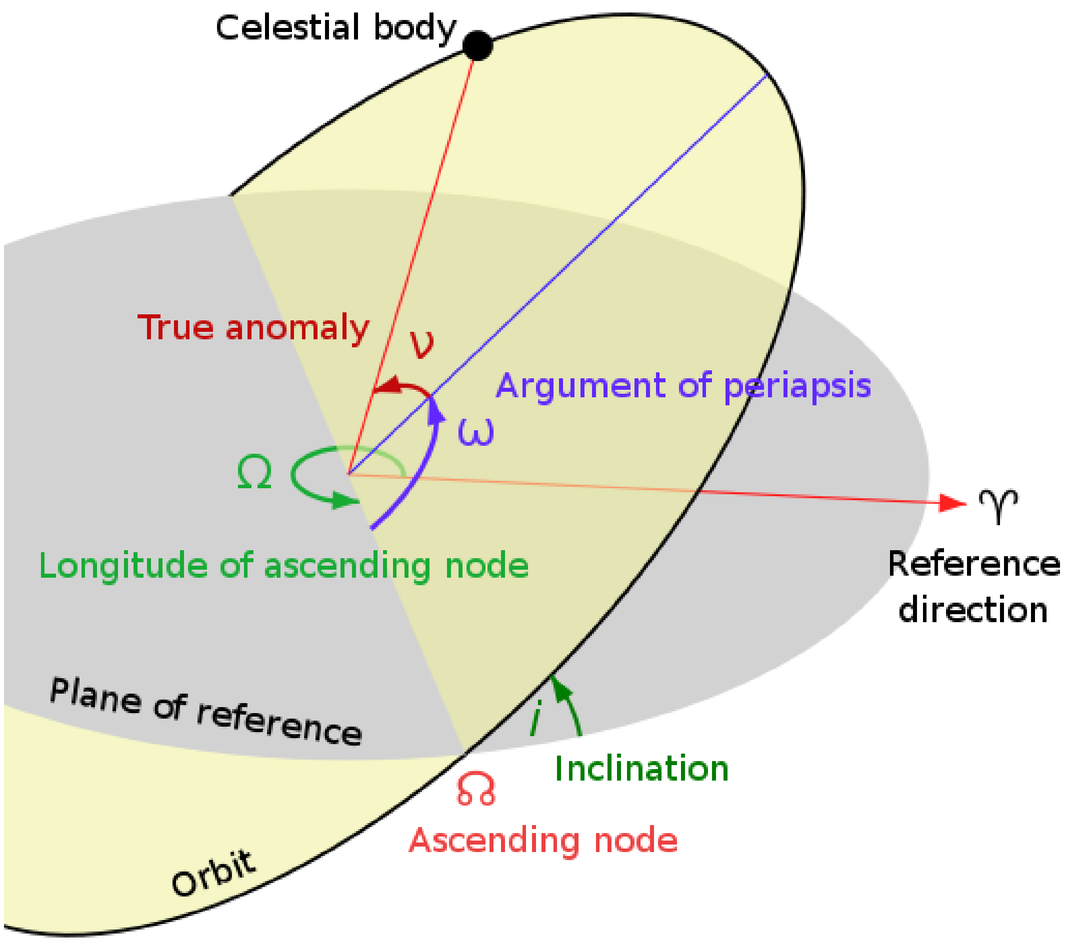
$$v_{circ} = \sqrt{\frac{GM}{r}}$$

where M is the mass of the central body and r is the separation between the orbiting body and the central mass.

- This is convenient because most planet and moon orbits are close to circular.

Orbital Elements

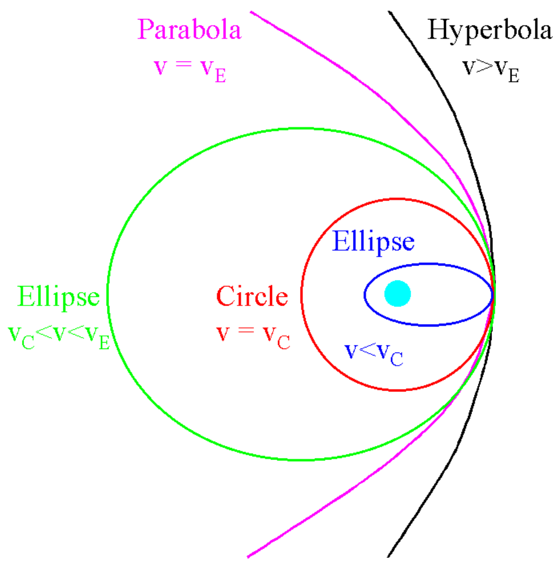
- Semi-major axis (a)
- Eccentricity(e)
- Inclination (i)
- Argument of Pericenter (omega)
- Longitude of ascending node (OMEGA)
- True anomaly (nu)



Orbital Energy

$$E = -\frac{GMm}{2a} = \frac{m}{2}v^2 - \frac{GMm}{r}$$

- In the solar system we observe bodies of all orbital types:
  - planets etc. = elliptical, some nearly circular;
  - comets = elliptical, parabolic, hyperbolic;
  - some like comets or miscellaneous debris have low energy orbits and we see them plunging into the Sun or other bodies



orbit type	v	E_{tot}	e
circular	$v=v_{circ}$	E<0	e=0
elliptical	$v<v_{esc}$	E<0	0<e<1
parabolic	$v=v_{esc}$	E=0	e=1
hyperbolic	$v>v_{esc}$	E>0	e>1

Escape velocity

- Escape velocity is the velocity a mass must have to escape the gravitational pull of the mass to which it is "attracted".
- We define a mass as being able to *escape* if it can move to an infinite distance just when its velocity reaches zero. At this point its *net* energy is zero and so we have:

$$\frac{GMm}{r} = \frac{1}{2} m v_{esc}{}^{2}$$

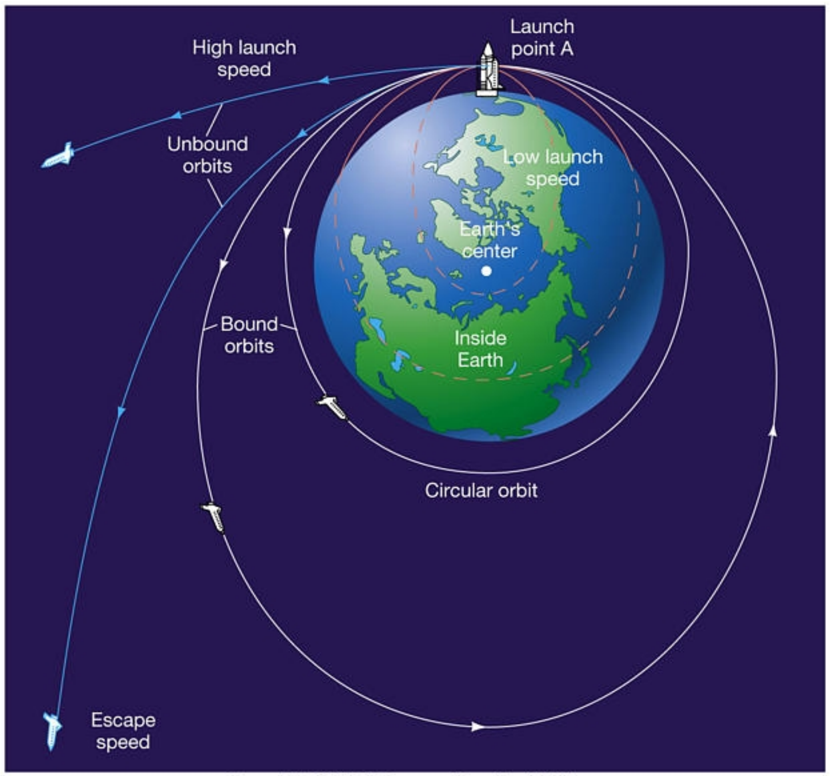
$$v_{esc} = \sqrt{\frac{2GM}{r}}$$

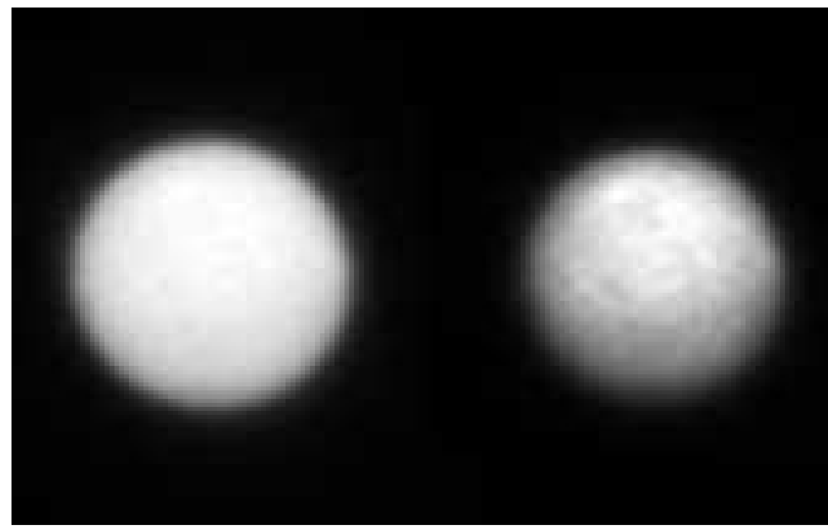
Escape velocity

What is the escape velocity at

a) the surface of the Earth?

b) the surface of the asteroid Ceres?





Vis-Viva Equation

- Since we know the relation between orbital energy, distance, and velocity we can find a general formula which relates them all – the *Vis Viva* equation

$$v^2(r) = 2GM\left(\frac{1}{r} - \frac{1}{2a}\right)$$

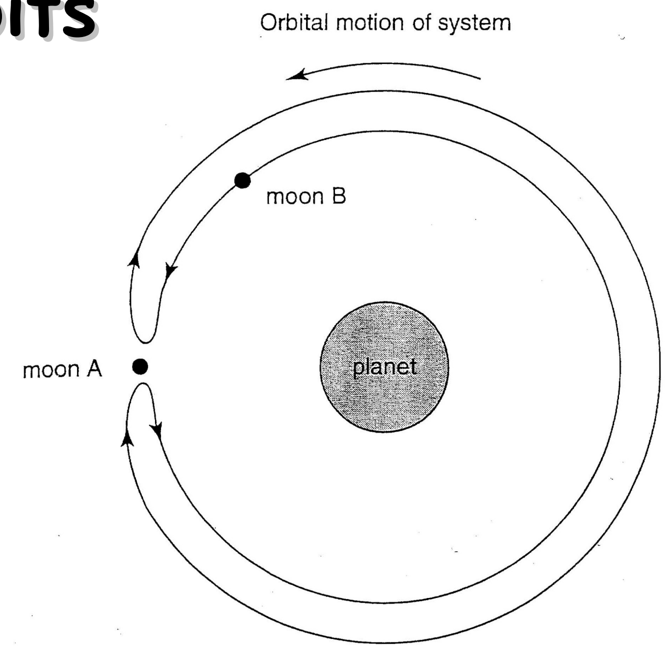
- This powerful equation **does not depend on orbital eccentricity**.
- For instance, if we observe a new object in the SS and know its current velocity and distance, we can determine its orbital semimajor axis and thus have some idea where it came from.

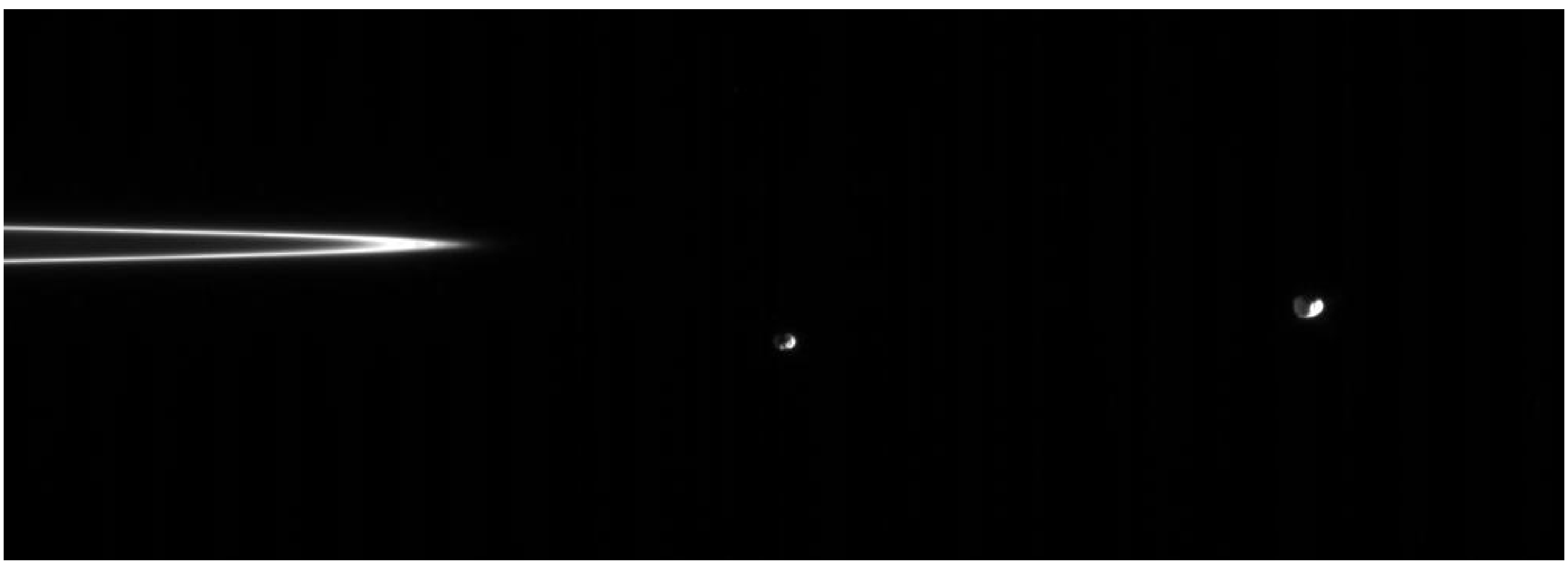
Vis-viva equation

A meteor is observed to be traveling at a velocity of 42 km/s as it hits the Earth's atmosphere. Where did it come from?

Horseshoe orbits

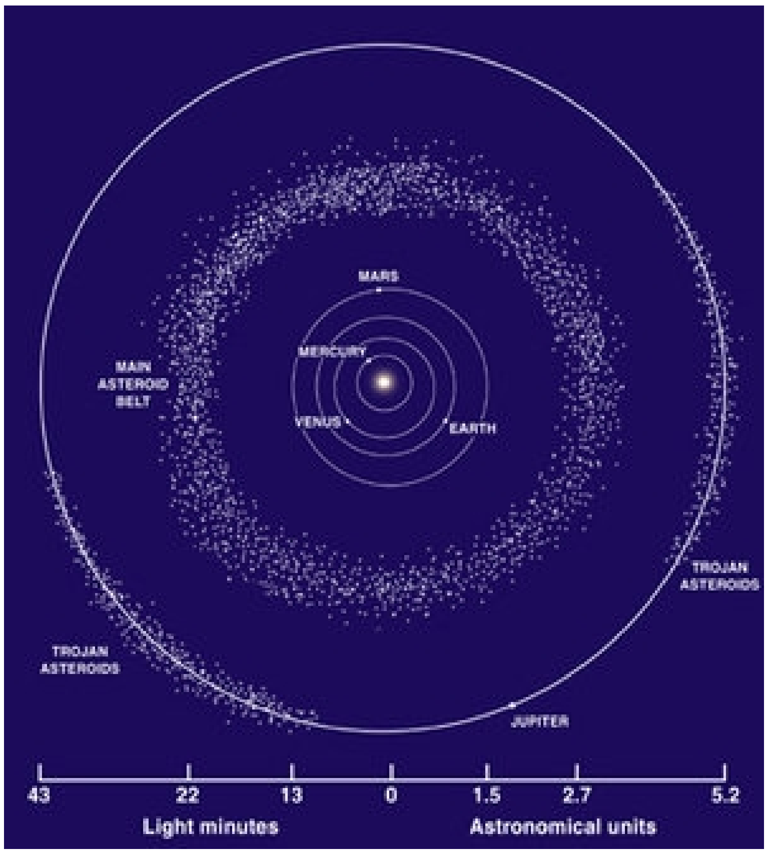
- Two small moons of Saturn, Janus and Epimetheus, only separated by about 50 km.
- As inner (faster moving) moon catches up with slower moon, it is given a gravitational kick into a higher orbit.
- It then moves more slowly and lags behind the other moon.

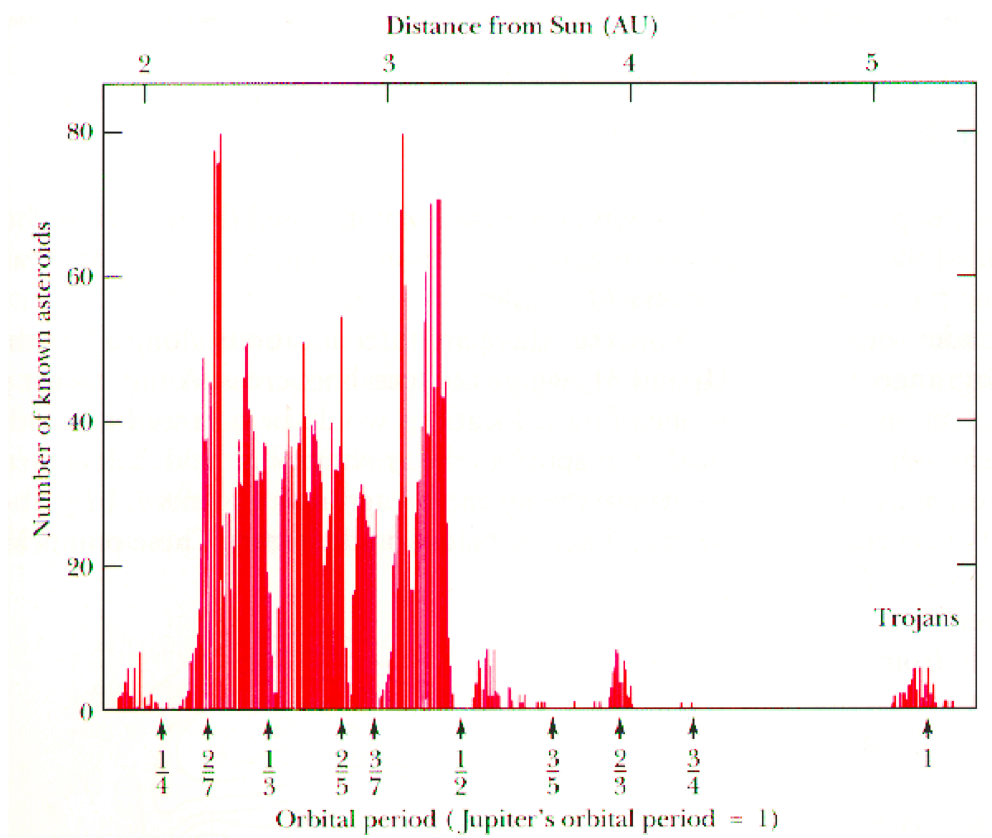




Kirkwood gaps

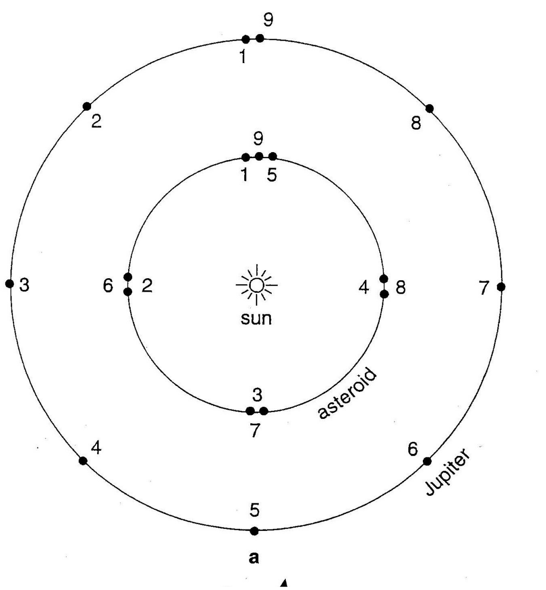
- The distribution of asteroid periods (or semi-major axes) in the main asteroid belt is not smooth, but shows gaps and peaks





Resonances

- If the orbit of a small body around a larger one is a small-integer fraction of the larger body's period, the two bodies are *commensurable*.
- Some resonances (3:2 resonance of Jupiter) actually have a stabilising effect.

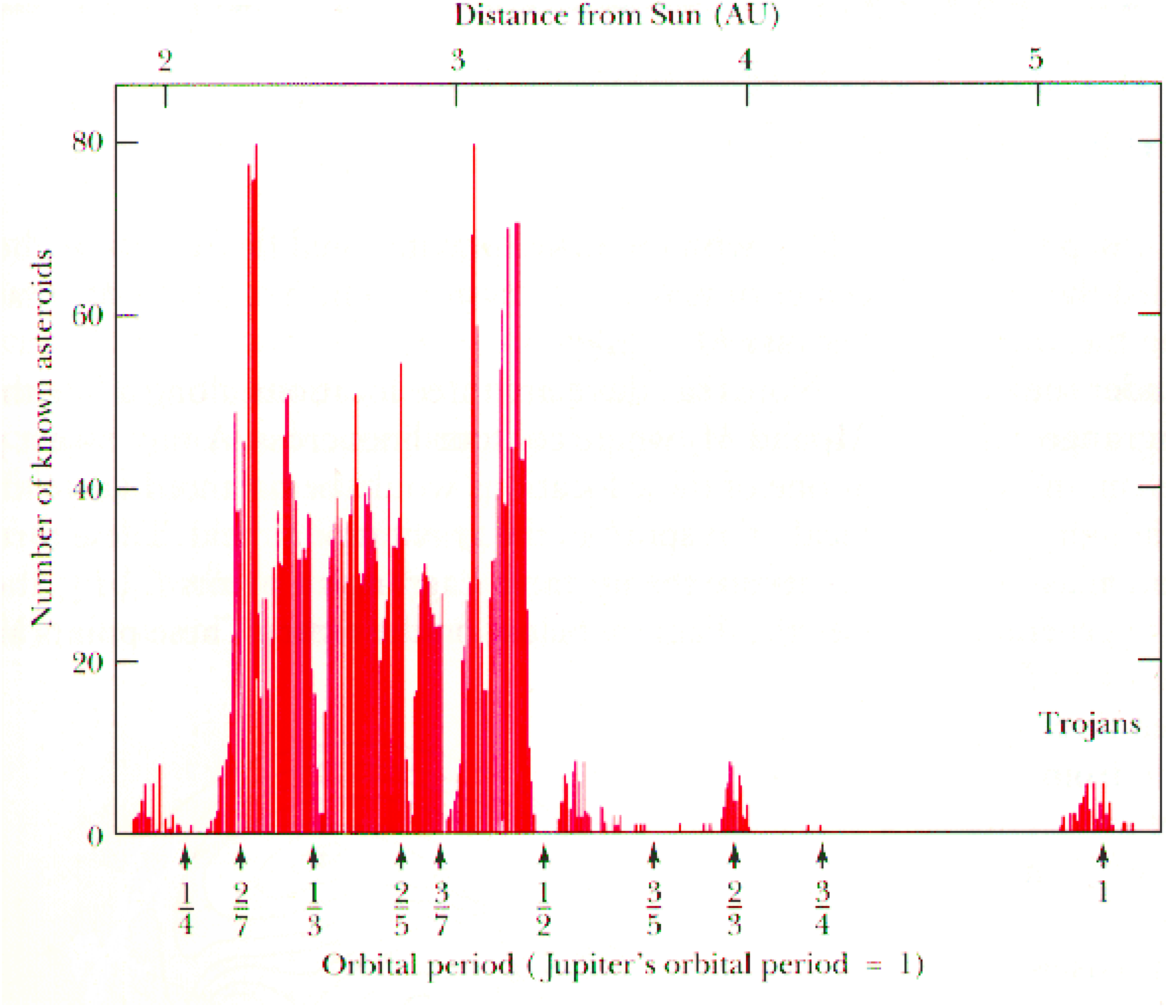


Example:

- An asteroid in a 1:2 resonance with Jupiter completes two revolutions, while Jupiter completes one

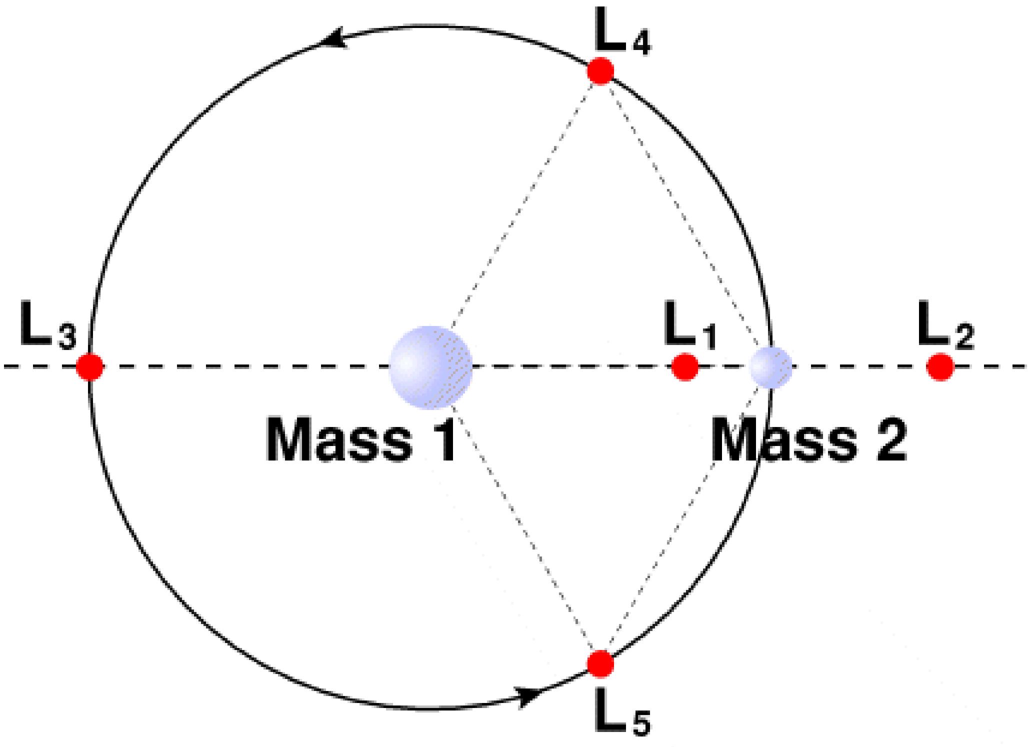
Kirkwood gaps

- Gaps in the distribution of asteroids correspond to resonances with Jupiter



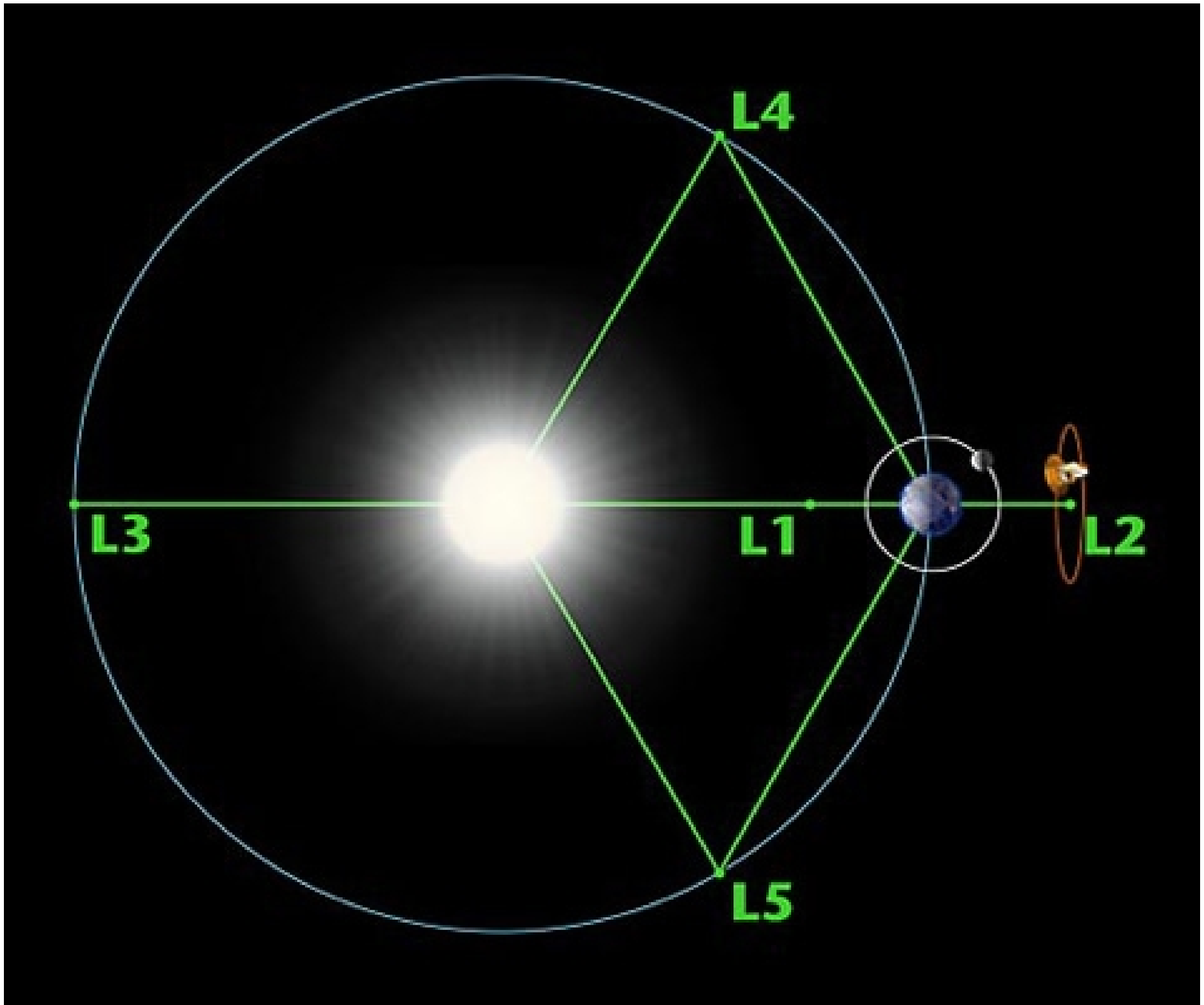
Lagrangian points

- An analytic solution to the 3-body problem is possible for a specific case: with two co-orbiting bodies with nearly circular orbits and a third body with nearly the same revolution period P as the other two.
- There are five points at which the third body can be placed and it will remain fixed relative to the other two bodies. Only L4 and L5 are stable.



Lagrange points

Many satellite missions are being designed to orbit around the Earth-Sun L2 point. This is about 4 times farther away than the Moon (but 1/100 the distance to the Sun)



Trojan asteroids

- Two groups of asteroids, occupying the L_4 and L_5 points of Jupiter.
- Trojans may drift well away from the Lagrangian points, but tend to librate about the L4 and L5
- How did they get there?