Wrg-02 Vibrations and Waves in context

An Introduction to Vibrations and Waves

Many things oscillate or vibrate in a regular fashion; waves of vibration can pass through a continuous substance. Many things start to make sense once we realise that they are wave phenomena - in particular, sound and light. We can then understand how reflection and refraction occur, and can go on to discover and explain interference, diffraction, and polarisation. Vibrations and waves crop up in all sorts of surprising places in Nature. But whether we are talking about the wall between you and your noisy neighbour, an oscillator circuit in your hi-fi amplifier, the surface of the Sun, or the wave function of quantum mechanics, the *concepts* and *mathematics* are pretty much the same and most of the phenomena have precise parallels across a wide range of areas of physics. Below is an overview of the main physical phenomena that involve vibrations and waves. We will discuss only a subset of the phenomena in this course.

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VIBRATIONS occur whenever a system has a stable equilibrium position, and when it is removed from this position and released, a force or torque comes into play and pulls the system back to equilibrium. (e.g. mass on a spring, swinging pendulum, plucked guitar string, ship bobbing on water, quartz crystal in a watch, vibrations of atoms in molecules, etc). On returning to the equilibrium position, the system overshoots, finally stopping on the other side. Again it is pulled back towards the equilibrium, which it overshoots. This produces a periodic motion or vibration or oscillation.

Such a system has a **NATURAL FREQUENCY** at which it vibrates when displaced.

For small oscillations it
will vibrate with **SIMPLE HARMONIC MOTION (SHM):**
continue indefinitely with **CONSTANT ENERGY**
or suffer **ENERGY DISSIPATION** caused by **DAMPING.**

Vibrations can **COMBINE** to give phenomena like **INTERFERENCE** and **BEATS**

A system could also be **FORCED** or **DRIVEN** by an external agent acting at a chosen frequency.

- but you only get a large amplitude for the vibration if you use a force with a frequency near the natural frequency of the system. This gives rise to a **RESONANCE** eg opera singer shattering a glass.

The strength and sharpness of a resonance depend on how much damping is involved.

The **ENERGY INPUT** by the driving force replenishes the energy lost through damping.

All these phenomena can be treated in a powerful unified way by using a **COMPLEX REPRESENTATION** which not only simplifies the mathematics involved but also provides a **GEOMENTRICAL REPRESENTATION** of vibrations.
The complex representation is essential in **QUANTUM MECHANICS** where all the phenomena met in this course will be found to occur.

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COUPLED VIBRATIONS occur when two or more oscillators interact through some physical mechanism.
The coupling makes the vibrational energy pass from one oscillator to another periodically with beat frequencies known as **NORMAL MODES** of vibration. With many coupled systems the energy can propagate, thereby producing **RUNNING WAVES IN DISCRETE MEDIA -** the most important example occurs in the quantum mechanical vibrations of atoms in a solid.

The continuum limit corresponds to infinitely many coupled oscillators infinitesimally close to their immediate neighbours, leading to
WAVES, which are travelling disturbances in **CONTINUOUS MEDIA**. The oscillations of each element are passed on to each neighbouring element in turn, creating a travelling wave. Waves can occur whenever a system is disturbed from its equilibrium position and when the disturbance can travel or propagate from one region to another.

Note that vibrations do not propagate and do not move from one region to another.

The wave concept is one of the most important unifying threads running through the entire content of the natural sciences.

Waves are either mechanical in nature, or electromagnetic (light, radio waves, infrared, UV, x-rays and gamma rays, or quantum mechanical explaining the behaviour of atomic and subatomic particles.

The matter doesn't travel, but waves transport **ENERGY** and **INFORMATION**.

Waves can be **TRANSVERSE:** waves on a string, water waves, electromagnetic waves; or **LONGITUDINAL:** pressure waves (sound).

SOUND WAVES are composed of longitudinal vibrations of physical material.
LIGHT WAVES are composed of transverse vibrations of the electromagnetic field.
The **SPEED** of a wave depends on the physical properties of the medium it travels in.

- for waves in solid, speed depends on the stiffness and the density of the material
- for waves in gases, speed depends on pressure and density of the gas
- for light waves, speed depends on the permittivity and permeability of the material.

The same medium can support waves of various different **frequencies** and **wavelengths**, but if you choose one you fix the other. The wave **amplitude** is just an accident of history. How much **energy** a wave carries depends on its amplitude, its frequency, and the stiffness of the medium carrying it.

In some media waves of different wavelength travel at different speeds, and so separate out (**DISPERSION** - e.g. light waves in glass).

Coinciding waves can **INTERFERE**, i.e. add up or cancel out depending on phase

- In open regions, this can lead to **beats**, and **wave-bunches**
- In closed off regions, a fixed pattern of vibration can result (**Standing Waves**)

Two light beams only interfere if they are **COHERENT**, i.e. have constant phase difference.
Coherent beams can be made by splitting a single beam:

- by reflection from THIN **FILM.** (some reflected from top surface, some from bottom)
- by passing light through two **NARROW SLITS**.
- by reflection from half silvered mirror. (some transmitted, some reflected)

In each case, the two beams travel different **PATH LENGTHS**, which introduces a **PHASE DIFFERENCE** when they recombine.

- Phase difference varies with location, producing a pattern of light and dark **FRINGES.**
- Phase difference varies with wavelength, producing **INTERFERENCE COLOURS** (e.g. colours in oil slick).

Multiple interfering beams can be produced with a row of narrow slits
(e.g. **DIFFRACTION GRATING**)

- the fringe pattern then has an isolated spike, position depending on wavelength
- so white light is spread into **SPECTRUM** .

A wide slit, aperture, or edge, acts like many neighbouring narrow slits.
So light encountering obstacles is **DIFFRACTED**.

- can apparently bend round corners
- shadows are not sharp, but have fringe pattern edges

The blurring effect of diffraction by the entrance aperture of an optical instrument limits how sharp an image can be made, i.e. its **RESOLUTION**.

- The bigger the hole the sharper an image is possible.

The following is not included in this course, and is added here for completeness only.

*At boundaries, waves are **REFLECTED** and **TRANSMITTED (refracted)**.*

- reflection is symmetric; angle of transmitted wave depends on relative wave speeds in the two media.
- the proportions transmitted and reflected depend on the "stiffnesses" of the two media
- the reflected wave will be 180^o out of phase if the second medium is "stiffer"

*Pure single light waves are rare; most "natural" light is a random mixture of short wave trains vibrating in different planes. However several effects can **POLARISE** or separate light into orthogonal components:*
- selective absorption by long parallel molecules (as in polaroid sheet)
- double refraction in anisotropic crystals
- reflection from surfaces
- scattering from electrons, atoms, molecules, smoke particles, etc.

The Maths behind Simple Harmonic Motion

We know physically that Simple Harmonic Motion (SHM) arises whenever you pull something away from equilibrium, and the restoring force is proportional to the distance away from equilibrium. As we will check in a moment, if we write down the maths corresponding to this situation, the solution indicates that sinusoidal motion results. The basic equation of SHM that we get from the physics is **differential** in nature, i.e. it involves the *derivative* of a quantity, and we have to *solve* it to get at the behaviour of that quantity itself.

Along the way to starting our study of vibrations, we will spend some time looking at general properties of differential equations. Almost all of advanced physics involves the solution of differential equations.
For example, suppose we know that the velocity of an object (i.e. the rate of change of distance x) is constant, and we want to know how the position of the object, $x(t)$ changes with time -

In this case the equation is $\frac{dx}{dt} = K$ and the solution is $x(t) = x_o + Kt$.

That is x changes linearly with time from wherever it started. $x = x_o$ at $t = 0$ say.

We could solve the equation by rewriting it as $dx = Kdt$; then just integrate both sides,

$$\int dx = \int Kdt \qquad \text{and thus} \qquad x + c_1 = Kt + c_2 \qquad \text{or simply} \qquad x = Kt + C$$

where c_1, c_2 and C are constants.

If we know that $x = x_o$ at $t = 0$, then of course $C = x_o$. Alternatively, we could just guess the right answer from physical intuition, and check whether it's right by substituting the guess into the differential equation and seeing if both sides come out the same, and making whatever adjustment is then obviously needed to make the solution work. (In general, some differential equations can be explicitly solved, by various techniques, but many have to be guessed and checked.)

The simplest differential equations you can think of correspond to the most common kinds of motion in Physics.

	Equation	Solution	Type of motion
(1)	$\frac{dx}{dt} = K$	$x = x_o + Kt$	uniform linear motion
(2)	$\frac{dx}{dt} = \pm Kx$	$x = x_o e^{\pm Kt}$	(+) exponential growth or decay (-)
(3)	$\frac{d^2x}{dt^2} = K$	$x = x_o + v_o t + Kt^2/2$	uniform acceleration
(4a)	$\frac{d^2x}{dt^2} = - Kx$	$x = A\cos\left(K^{\frac{1}{2}}t + \varphi\right)$	regular oscillations

This is the differential equation and solution that we require when studying oscillations**.**
(Check these solutions work by substituting into the differential equation.)

To complete our set of differential equations, we could look at

(4b) $$\frac{d^2x}{dt^2} = + Kx$$

Solution is $x = c_1 \exp\left(K^{\frac{1}{2}}t\right) + c_2 \exp\left(-K^{\frac{1}{2}}t\right)$

This turns out to give a mixture of growth and decay. It doesn't occur often in physics.

The first two equations involve only dx/dt, and so have one constant of integration. The latter two involve the second derivative, and so have two constants of integration. It is always the case with differential equations that the solution consists of two parts - the universal part, which describes the type of physical behaviour, and the constants of integration, which cannot be deduced from the physics, but depends on the individual history of the specific system. It is traditional (and usually convenient) to solve for these in terms of position, velocity, or whatever, at time $t = 0$, as we did above. These are called the "initial conditions". More generally, if we know other values which constrain the solution, these are called "boundary conditions".