FIELDS AND WAVES

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9.0 OPTICAL INSTRUMENTS In the previous chapter the emphasis was on the wave properties of light. These are of particular importance when considering situations where the wavelength of light cannot be neglected compared with the size of the things being looked at, or the instruments used to observe them. But for many applications, the wave properties like diffraction and interference are not the main concern. The pupil of your eye is a few millimetres across, and so diffraction of the light passing through it is not often a matter of concern. The objective lens of a camera is usually even larger. In such cases where the few hundred nanometres wavelength of light is small enough to allow it the wave properties to neglected, the method of ray tracing and so- called geometrical optics will be enough to develop the theory of lenses and mirrors which will allow us to understand the operation of optical instruments such as telescopes and microscopes. Rays of light are lines which trace out the direction of propagation of the light. They are lines *orthogonal to* (at right angles to) the wave fronts. And in geometrical or ray optics, it is usually sufficient to trace out some few special rays and one or two representative ones in order to determine how an optical instrument functions. In particular, we will often be interested in how rays are brought together by a mirror or a lens so that they all cross at a point, called a focal point, or *focus.*

Drawing diagrams in html (the text preparation language used for these notes) is not easy! So I hope you will be satisfied to refer to the diagrams in the textbook, or those which I will show in the lectures.

9.1 MIRRORS

(See Figures 19.1B and 19.1C) When rays of light are reflected from a mirror, they make an angle with the *normal* to the surface of the mirror which for the *reflected ray* is equal to that made by the *incident ray.*

$$\text{Incident angle } i = \text{Reflected angle } r$$

A simple geometric construction and the use of similar triangles then suffices to show that the rays coming from a point on an *object* in front of a *plane mirror* are reflected in such a way that they appear to diverge from a corresponding point on the *image*, which lies as far behind the mirror as the object is in front of the mirror. So when you "see yourself in the mirror", you see an image of yourself which is "behind the mirror", and which is inverted - inverted that is from front to back. If the mirror is in the $x-y$ plane, the mirror changes z to $-z$.

The same equality ($i = r$) also applies for reflection from curved mirrors (Figures 19.1E and 19.1F). Consider first a *concave spherical mirror*, that is a mirror with a reflective surface which is the *insid* surface of part of a sphere. The curvature of the mirror depends on the radius of that sphere, the *radius of curvature, R.* Parallel rays close to the *axis* (the line joining the *centre of curvature* – the centre of the sphere –to the centre of the mirror) on striking the mirror are reflected to a focus or *focal point.* [In fact strictly speaking, they do not come exactly to a focus because they do not all cross at the same point. They form a *cusp* such as you can see on the surface of a cup of coffee formed by the rays from the sun reflected off the curved inner surface of the cup.] The distance of the focal point F from the centre of the mirror is called the focal length f of the mirror.

The way that a ray diagram can be used to locate where the image of an object is formed on reflection in a concave spherical mirror is illustrated in Figures 19.1G and 19.1H . This is also a characteristic example of how such constructions can be used in other cases too.

It is generally a good idea to identify an optical axis, usually a line of symmetry of the optical system to be studied. In this case, it is the axis of the mirror. What we want to do is to find where the image of some object can be found; the object O is supposed to be close to the axis, and we need to consider a few rays from some representative point of the object and find where they cross after they are reflected from the mirror. It is conventional to suppose that the object is an arrow with its tail on the axis; and we choose to find where the image of its tip is located. So we draw a ray from the tip of the arrow to the centre of the mirror (Ray 1 in Figure 19.1G); since the axis is the normal to the mirror at the centre of the mirror, the reflected ray makes an angle with the axis equal to that of the incident Ray 1. A second ray (Ray 2) is drawn parallel to the axis of the mirror. On reflection, it will pass through the focal point F. This reflected ray will intersect the reflected Ray 1, and this point of intersection is enough to determine where the image I is located. In fact there is another ray (Ray 3) for which it is easy to find the reflected ray. Ray 3 is drawn from the tip of the arrow of the object to pass through the focal point F; since rays parallel to the axis pass through the point F after reflection, rays passing through F will be reflected back parallel to the axis, and so it is easy to draw the reflected Ray 3. It also passes through the point of intersection of the other two reflected rays.

Now let us consider how the location of the image I varies as the distance of the object O from the mirror is changed. If the object is very far away, the rays we have considered are all nearly parallel to the axis, and so the intersection point of the reflected rays must be close to F. In fact as the object recedes to infinite distance from the mirror, the image approaches to F. If the object is distance twice the focal length f from the mirror, the symmetry of the resulting diagram should convince you that the image is at the same distance $2f$ from the mirror. You should also note that the image for object distances greater than $2f$ is *smaller* than the object (a *diminished* image), and that at distance equal to $2f$ it is the same size as the object. In either case it is *inverted.* If the object is *closer* to the mirror than $2f$, the image is further away from the mirror than the object, and is *enlarged* (but still inverted) (See Figure 19.1H)

Things get to be interesting if the object is *closer* to the mirror than f. (See Figure 19.1I - only Rays 1 and 2 have been drawn - you might try to add Ray 3). The reflected rays, considered as lines in a geometrical diagram, continue to intersect, but now *behind* the mirror. The real reflected light rays will of course not meet behind the mirror, but they will *appear* to come from that point of intersection, just as was the case with a plane mirror, where the image lay behind the mirror. The image is now said to be *virtual.* You can cast a *real* image, such as that formed from an object further away from the mirror than f, onto a screen, but you cannot do that with a virtual image! [If the object is a candle, it is not difficult to see its reflected image on a sheet of paper]. The virtual image formed by an object closer than f to a concave mirror is also *enlarged* and *upright* . This is what you see when using a make-up mirror or a shaving mirror.

There is a simple formula which relates the distance u of the object from the mirror to the distance v of its image:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}.$$

The derivation of this formula is given following Figure 19.1J.

Note what happens when u is less than f. It follows from the formula that v has to *negative*, and this is interpreted as meaning that the image is a virtual image, behind the mirror. It also follows from the geometrical construction that the ratio of the height of the image to that of the object (just using similar triangles), which is called the *linear magnification* M is given by:

$$M = v/u.$$

Again there is a sign convention; when M is positive, the image is inverted, and when it is negative, the image is upright.

How about finding the focal distance f? This is clearly related to the radius of curvature R; since rays coming from an object placed at the centre of the spherical surface (so with $u = R$) will strike the surface of the mirror at normal incidence, they will be reflected straight back and so the image will also be at the centre of the sphere (therefore with $v = R$). The formula relating object and image distance then shows that

$$f = R/2.$$

Very similar methods can be applied to the case of *convex* spherical mirrors. Here the reflection takes place at the outside of a spherical surface. (See Figures 19.1K and 19.1L). The same formulas as before apply, but now with the sign convention which sets f negative.

9.2 LENSES

Lenses can bring rays to a focus by refraction. You are reminded of Snell's Law:

$$\sin i/\sin r = n.$$

This can be used to show that a thin convex lens will act to focus a parallel beam of light, parallel to the axis of the lens, bringing the rays to a focal point distance f from the centre of the lens. Such a lens is called a *converging lens.* The lens will have two focal points on its axis, one on either side of the lens. Ray diagrams like that of Figure 19.2H can then be used to derive the *lens formula*:

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f},$$

which is by good fortune the same as the one for mirrors. And again u is the *object distance*, with v the *image distance*, with I positive meaning that the image is on the opposite side of the lens to the object.

If the object is at a distance u greater than f, the image is *real* and *inverted.* And if u is greater than $2f$it will be diminished, but for u less than $2f$ it will be *enlarged.* In either case the linear magnification is again given by the formula:

$$M = v/u.$$

Once more there is an interesting consequence following from taking $u < f$. (See Figure 19.2J). The refracted rays on the far side of the lens from the object *appear* to come from an image at a distance v from the lens as given by the formula, but now with $v < -f$, which is to be interpreted as indicating that the image is *virtual* and further away than the object. It is also *magnified*, and is *upright* as indicated by the fact that M is greater than 1 in magnitude, but is negative. The lens is being used as a *magnifying glass.*

Similar considerations apply once more for a *concave* or *diverging* lens, which will also have focal points on its axis, one on either side of the lens. They are defined as the points from which rays parallel to the axis passing through the lens *appear* to diverge: the focal length f is now negative. But the lens formula still applies, and with the same sign conventions.

Opticians measure the *power* of the lenses used to correct vision in *dioptres.* The power of the lens is the reciprocal of its focal length measured in metres. So a lens with a focal length of 2.5 m is said to have power 0.4 dioptres (0.4 D).

9.3 THE CAMERA

The lens or lens system in a camera is designed to focus an image of the objects or scene to be photographed on the film or plate or CCD at its rear. For a typical camera, this might be 50mm from the lens, so that if the camera is to be able to focus objects at a large distance ("at infinity"), the focal length of the lens should be 50mm. For objects closer than that, since (unlike the eye!), it is easier to change the distance from the lens to the film rather than to change the focal length of the lens, the focusing of the camera is indeed achieved by moving the lens. It is possible to make a camera without any lens at all - the pinhole camera. By replacing the lens with a tiny pinhole, an image can be produced on the film for objects at any distance from the camera; the only limitation on the sharpness of the image will come form the diffraction effects which are greater the smaller the aperture of the pinhole. And of course the amount of light admitted to the camera will be less the smaller the aperture. In a camera with a lens, there is a trade-off between the *depth of field* and widening the aperture so as to admit more light. The diameter of the aperture is controlled with an aperture stop, and is measured by the *f*- number . This is simply the ratio between the diameter of the aperture and the focal length of the lens (or lens system). So an aperture number 4 means an aperture diameter of $f/4$, and one of 2.8 means an aperture diameter which is approximately $\sqrt{2}$ times greater. Since the opening area of the increases as the *square* of the aperture width, a lens at f-number 2.8 will admit approximately twice as much light as one at f-number 4 – and so need half the exposure time.

9.4 MICROSCOPES

We have already seen that a single converging lens can function as a magnifying glass. An object placed at a distance u from the lens less than its focal length f will produce as an enlarged upright virtual image on the same side of the lens. And we have also established that the *linear magnification* of the object is

$$\frac{v}{u} = \frac{f}{(u-f)},$$

(which is negative because of the sign convention for a virtual image). But this is not the same thing as the *magnifying power*, which is defined in a way more accurately to reflect the way we perceive the size of an object. This is related to the *angle* it subtends at the eye, and that depends on its distance from the eye as well as its linear size.

When we want to examine something small, we usually try to bring it close to our eyes, but not *too* close, since we are not able to focus on objects closer than about 25 cm. The lens in the eye can *accommodate* to provide sharp images on the retina from that distance out to infinity by changing its focal length- at least for a "normal" eye. Many of us have to use spectacles to assist the lens in the eye to achieve this! But we may suppose that there is a "near point" at which objects can still be brought into focus, and when looking at a small object, it will be best seen when at the near point. It will then subtend an angle α at the eye which is given by

$$\tan\alpha = h/D,$$

Where h is the height of the object, and D is the distance from the eye to the near point (approximately 25 cm.) With the aid of a converging lens used as a magnifying glass, the image of the small object is at a distance v, and its height appears to be v/u times h, so that the angle β which it subtends at the eye is given by

$$\tan\beta = \frac{v}{u} \cdot \frac{h}{v} = \frac{h}{u}.$$

The magnifying power of the lens is then defined as the ratio β/α, or since the angles are small enough that the tangent of the angle is well approximated by the angle itself, the magnifying power is

$$M = \frac{\beta}{\alpha} = \frac{(h/u)}{(h/D)} = \frac{D}{u}.$$

This will vary as we vary the object distance u. There is no point in making u too small, since this will make the image appear too close - the best we can do is to make the image appear at distance D. That means taking $v = -D$ (the minus sign is because the image is virtual), and then u is given by

$$\frac{1}{u} = \frac{1}{f} - \frac{1}{(-D)},$$

and the magnifying power is

$$M = \frac{\beta}{\alpha} = D \cdot \Big(\frac{1}{f} - \frac{1}{(-D)}\Big) = \frac{D}{f} + 1,$$

which is the same as the linear magnification. This is the maximum magnifying power attainable with the magnifying glass. It is sometimes more comfortable to use the magnifying glass with the image at infinity, so that $u = f$; then the magnifying power is D/f and this is the minimum magnifying power of the lens.

Of course we can do better with a *compound microscope*, which in its simplest form consists of two converging lenses or lens systems. One of them, the *objective* is close to the object, in such a position that the object is just a small distance further away than the focal point ($u > f$), which means that a *real, magnified* image is formed on the other side of the lens. This real image is then viewed through the *eyepiece* lens, which is in effect a magnifying glass which magnifies the real image produced by the objective lens still further. (The eyepiece must of course be beyond the real image produced by the objective).

The magnifying power of the microscope will depend on how the adjustment is made to bring the final virtual image to a suitable distance from the eye. In *normal adjustment* the final image is brought to the near point distance D from the eye. The magnifying power of the microscope is again defined as the ratio between the angle β subtended at the eye by the final image to α, that which the object would subtend if it were placed at the near point of vision. If the height of the object is h, since the angles involved are small, we again have $\alpha = h/D$. What about β? This will be given by $\beta = h_2/D$ at normal adjustment since the image is again at distance D, where h_2 is the height of the virtual image formed by the eyepiece. And this height is the magnified height of the real image formed by the objective; $h_2 = (h_2/h_1) \cdot h_1$ where (h_2/h_1) is the linear magnification of the objective. So the final result for the magnifying power of the microscope is

$$\beta/\alpha = (h_2/h) = (h_2/h_1) \cdot (h_1/h),$$

and the last factor (h_1/h) is just the linear magnification produced by the objective. The magnifying power of the microscope is thus the product of the linear magnification of the objective with that of the eyepiece.

9.5 TELESCOPES

Another class of optical instruments is exemplified by the telescope, and we will consider first the *refracting telescope* which uses lenses. One function of a telescope is to allow one to view *distant* objects, and make them appear larger - which means making them subtend a larger angle at the eye than they would when viewed directly without the aid of the telescope. Again there is an objective and an eyepiece, both of them converging lenses or lens systems. Since the object is distant, it is in effect "at infinity", so the objective forms a real image near the focal point. If the focal point of the eyepiece coincides with that of the objective, the result of viewing this real image produced by the objective is to give again a real final image at infinity. But it will subtend a different angle at the eye than the object would without the telescope. The magnifying power of the telescope is again the ratio of the angles subtended at the eye, with the telescope and without, and is given by

$$M = \beta/\alpha = f_o/f_e,$$

where f_o and f_e are respectively the focal lengths of the objective and the eyepiece.

The distance between the objective and the eyepiece is $f_o + f_e$, since the intermediate image is at the focal point of both lenses. The *resolving power* of the telescope, its ability to bring to distinctly separated focus two closely separated objects, will depend on the extent of diffraction through the objective. As we have seen two point-like objects will have their diffracted images overlapping unless they are separated by an angle greater than $1.22\lambda/W$, so it pays to have as wide an objective diameter as possible. This has the added benefit that it will increase the light gathering power of the telescope.

With all instruments using lenses, the focal lengths of the lenses involved will depend on the refractive index of the glass from which the lens is made, and since this in turn depends on the wavelength of the light (dispersion!), there will be *chromatic aberration*; different colours coming from the same point on the source object will be brought to a focus at different places. This is especially a problem for astronomical observations. Most large modern telescopes minimise this problem by using a mirror rather than a lens for the objective. Reference to Figures 20.52 and 20.53 will explain the two principal methods used to allow the image produced by the objective mirror to be viewed through the eyepiece.

Notes revised by John M Charap on 12/03/04