FIELDS AND WAVES

2004

7.0 SOUND

We experience sound when our eardrums are made to vibrate at a frequency somewhere in the range from around 20 Hz to 20,000 Hz. The mechanism which transfers these vibrations from the eardrum to the sensitive nerve fibres in the inner ear is a delicate and wonderful marvel. The ear, as with other of our sensory organs, is remarkably sensitive. The amplitude of the oscillations to which it responds can be so tiny, that by the last stage in the transfer of vibrations to the sensory nerves themselves it is only a few microns. At the threshold of auditory detection these oscillations are not much greater than those caused by the thermal motion of the fluid which at the last stage transmits the sound to the hair-like detectors themselves. Sound is transmitted through air, water, etc., as a mechanical longitudinal wave. That means that that there is a *compression* wave travelling from the source of the sound, an alternation of regions of compression and rarefaction which is generated by the vibration of your vocal chords or the alternation of push and pull of the diaphragm of a loudspeaker. Sound transmission through a solid is rather like the transmission of a wave along a row of loosely coupled railway carriages; again the wave is longitudinal.

7.1 LOUDNESS

The amplitude of the sound wave determines what is perceived as its *loudness*. To quantify this, we need a measure which is related both to the physical properties of sound and to the more subjective perception. It turns out that there is a useful logarithmic relationship relating the sound energy received by the ear to the perceived impression of loudness, and this is used to define the decibel. So first we define the *intensity* (I) of the sound as the sound energy arriving in a unit time at a unit area placed at right angles to the direction of the propagation of the sound. This will be expressed in $\mathrm{J.s^{-1}.m^{-2}} = \mathrm{W.m^{-2}}$. Then we may define the *intensity level* of the sound by relating this to what the human ear is able to respond to. The human ear can detect sounds with an intensity as low as

$$I_0 = 10^{-12}\mathrm{W.m^{-2}}$$

and as high as 1 $\mathrm{W.m^{-2}}$, at which intensity pain results and permanent damage can be done. (So take care at the disco or with your personal stereo!) The intensity level is then defined by reference to the minimum intensity which can be detected through a logarithmic scaling as follows:

$$\mathrm{IL} = \log_{10}(I/I_0).$$

The intensity level is measured in units called bel (B), but it is more usual to use the decibel (dB);

$$10 \text{ dB} = 1 \text{ B}.$$

So a sound with intensity 1 W.m^{-2}, which has an intensity 10^{12} times I_0, has an intensity level IL = 12 B = 120 dB. So far as concerns *perception* of sound, every increase in the intensity level by 10 dB is perceived as the same increase in loudness.

7.2 PITCH AND QUALITY

The ear can respond to frequencies from 20 to 20,000 Hz, and differentiates frequencies in this range by their *pitch*. The middle C note at the centre of the piano keyboard sounds with a frequency of 261.626 Hz. The frequencies to which the ear responds most comfortably range a few octaves on either side of that (an octave below means half the frequency, an octave above is twice the frequency); that should come as no surprise, since it is this range of frequencies which are most often encountered in the environment, and also the range which the human voice can produce. Sound with frequencies above the range of the human ear is called *ultrasound* . Ultrasound is used for example in medical diagnostic imaging, because the wavelength is short enough for it not to be significantly diffracted by structures inside the body. At the intensity levels used, it has no harmful side effects, which is why it is particularly appropriate for pre-natal imaging in pregnancy. Higher intensity ultrasound can sometimes be used to disintegrate gallstones. Ultrasound can be generated by using what is known as the piezoelectric effect – quartz crystals can be made to oscillate by application of an oscillating electric voltage. The reverse effect is then used to detect the sound waves reflected and refracted at interfaces between different structures in their passage through the body: the ultrasound oscillations generate a small oscillating voltage across the faces of a crystal in the detector. Devices such as those which turn one kind of signal into another (here electrical to sound and back again) are called *transducers*.
We can distinguish the difference in sound between a piano and a saxophone, even when both are playing the same pitched note. This difference in *quality* comes about because neither of them is producing a *pure tone* (a sinusoidal wave with just one frequency). They each produce sounds with a superposition of different frequencies, which in the case of musical instruments are predominantly ones related to that of the note being played by simple integer multiples (higher harmonics). It is the different relative intensities of these superposed frequencies which is the main cause for the different qualities of sounds with the same pitch. If the sound has a whole jumble of different frequencies, it is experienced as *noise*.

7.3 WAVE PROPERTIES OF SOUND

Sound waves share many of the properties of waves which we have already described for light waves. In particular, they can be reflected (as with echoes) and refracted, and suffer interference and diffraction. These last are very characteristic wave phenomena, and depend critically on the wavelength. The speed of sound depends on the medium through which the sound waves are passing. For air it is of the around 340 m.s^{-1}. Since the sound we hear has frequencies in the range 20 - 20,000 Hz, the wavelengths involved are from tens of meters down to millimetres. At these wavelengths, diffraction is a significant effect enabling sound to pass round most of the gaps and barriers which stand in the way of straight-line transmission. We can hear round corners, even though we cannot see round them! Interference effects are analogous to those we have mentioned for light, and will discuss in more detail later. But again because the wavelength for sound is so much greater than for light, the effect is easier to demonstrate – it is “scaled up”. Two coherent sources of sound generating the same pure tone can easily be arranged by connecting two loudspeakers in series to a signal generator. They emit spherical waves which interfere where they cross one another, producing an interference pattern reminiscent of that of the ripples on the surface of a pond into which two stones have been dropped. Where crests of the waves (regions of maximum compression) coincide, they reinforce, and so likewise for troughs. But where the crests of the waves from one source coincide with the troughs from the other, they cancel one another. When two wavetrains with frequencies which differ only a little propagate together, they gradually become “out of step” with one another, and then come back into step and so on over and over again. So if at time $t = 0$ they are in phase, as time goes on they get progressively more and more out of phase. The difference in phase between the waves is just $2\pi(f_1 - f_2)t$. Of course, when this phase difference is an integer multiple of 2π, the waves are in fact back in phase with one another. The time between such occurrences is an integer multiple of T, where

$$(f_1 - f_2)T = 1.$$

So the frequency $f_{\rm B}$ with which the waves come back in phase with one another is

$$f_{\rm B} = 1/T = f_1 - f_2.$$

This frequency $f_{\rm B}$ is called the *beat frequency* , because what one hears is a periodic alternation in *intensity* at this frequency as the waves alternately reinforce (when they are in phase with one another) and cancel out (when they are out of phase). This beating of the two tones against one another is used, for example, by piano tuners, who can adjust the frequency of a note on the piano until it no longer beats with that produced by a tuning fork.

7.4 RESONANCE AND ORGAN PIPES

Sound waves can *reverberate* as they are reflected back and forth in an echo chamber. The sound is eventually damped out as it is scattered and absorbed by the walls of the chamber, but persists for an appreciable time. In designing a concert hall, the acoustical engineer will strive to achieve a reverberation time which makes the sound in the hall neither appear “dead” nor “muddy”. The engineer will also make sure that there are no notes which *resonate* in the hall. Resonance, as we have seen with oscillators, occurs when a damped oscillator is driven at an appropriate frequency, close to its natural frequency. It leads to an increase in the build up of the amplitude of the oscillation in response to the driving force. Organ pipes and wind instruments in general utilise resonance. The air column in the pipe is made to resonate in a standing wave by air being blown over a sharp edge near one end of the pipe (or through the mouthpiece of the saxophone for example). The noise produced by this is a mixture of many frequencies, but only the one at the resonance frequencies of the air column in the pipe will build up in amplitude to produce the musical note desired. The resonance frequencies of the air column in a pipe depend on whether the pipe is open at both ends, or closed at one end and open at the other. At a closed end, there is neither compression nor rarefaction, so that the standing wave in the pipe has to have a *node* there. At an open end of a pipe in resonance there will be an *antinode*, a region of maximum displacement. (In fact the antinode is at a distance e, a little outside the pipe.) The distance between a node and any of the antinodes of a standing wave is always an *odd multiple of a quarter wavelength.* So the length L of the pipe has to satisfy

$$L + e = n\lambda/4 \quad \text{with} n \text{odd.}$$

Correspondingly the resonance frequencies of an air column in a pipe closed at one end and open at the other are given by an *odd* multiple of the *fundamental* frequency $f_0 = c/\lambda_0$ with $\lambda_0 = 4(L+e)$. (c is the speed of *sound* in air). With an organ pipe open at *both* ends, when the air column is resonating, there are antinodes at both ends (again a little outside to be exact). So in this case the length L of the pipe ($+2e$ for the end effects) has to be an integer multiple of *half* a wavelength, giving

$$L + 2e = n\lambda/2 \quad \text{with} n \text{integer,}$$

a fundamental frequency $f_0 = c/\lambda_0$ with $\lambda_0 = 2(L + 2e)$, and *overtones* with frequencies at integer multiples of the fundamental. All wind instruments exploit similar ideas. The quality of the notes generated will depend, amongst others, on such things as whether the pipe or tube in which the air is made to resonate is cylindrical or conical.

7.5 VIBRATING STRINGS, AND STRINGED INSTRUMENTS

A stretched string or wire can be made to vibrate as a standing wave and will show resonance if forced at a suitable frequency. The resonant frequencies are determined by the condition that the displacement at the ends of the string must vanish at all times. The ends of the string then are nodes, and L, the length of the string, is therefore an integer multiple of a half wavelength: $L = n \times \lambda/2$. The resonance frequencies are then given by

$$f = nf_0 \quad \text{with} f_0 = c/(2L).$$

Here c is the speed of waves *on the string.* (Not the speed of sound in air!) This speed can be worked out using mechanics and a little calculus. It is given by

$$c = \sqrt{(T/\mu)},$$

where T is the tension in the string, and μ is its mass per unit length.

7.6 THE DOPPLER EFFECT

Christian Doppler (1803-53) was the first to make note of the effect which is nowadays familiar from the apparently changing pitch of the siren of an ambulance as it speeds past you. This is best understood by considering the succession of wave crests in the sound wave emitted by the siren. If you and the ambulance are both stationary, they will reach you at regular intervals, having travelled from the siren to your ear, at the same frequency f_s with which they were emitted. But now suppose the ambulance is coming towards you at a speed v_s. In the time between successive crests being emitted, the ambulance will have travelled a distance v_s/f_s, and so each crest will have that less distance to travel to reach your ear than the one which preceded it. The distance between successive wave crests is then no longer c/f_s, but instead we have

$$\lambda = (c - v_s)/f_s.$$

The frequency f_o with which they are received is therefore given by

$$f_o/f_s = c/(c - v_s).$$

The pitch of the siren is higher as the ambulance comes towards you. Similarly when the ambulance is moving away from you, the formula becomes

$$f_o/f_s = c/(c + v_s),$$

and the pitch is lower. If the *observer* is moving, but the source is at rest, the argument is a little different. As you move towards the source with speed v_o, the time interval between your passing successive wave crests which are coming towards you at speed c is $\lambda/(c + v_o)$. So we have for the observed frequency

$$f_o = (c + v_o)/\lambda,$$

or

$$f_o/f_s = (c + v_o)/c,$$

when you are moving towards the source, and similarly

$$f_o/f_s = (c - v_o)/c,$$

when you are moving away from the source. There is a similar Doppler shift in the observed frequency of light depending on the relative motion of source and observer. Here one should really use special relativity to get the correct results, but for relative speeds small compared to the speed of light the same formulas as we have just obtained are still a good approximation (but of course c would now be the speed of light, not of sound!). This shift of frequencies is towards the blue end of the spectrum for light coming from a source moving towards the observer, and towards the red end of the spectrum for sources receding from the observer. The observed red shift of spectral frequencies from light emitted by distant galaxies was used by Edwin Hubble to deduce that they were receding at speeds proportional to their distance, one of the cornerstone discoveries in astrophysics and cosmology. It is an essential observation in the arguments leading to the Big Bang cosmology now generally accepted.

Prepared by John M Charap on 03/03/00. Corrected 02/03/04