FIELDS AND WAVES

6.0 WAVE MOTION

Perhaps the most familiar example of waves is provided by the waves on the surface of the sea, or the ripples on the surface of a pond. Wave motion is met in many and varied branches of physics. Waves have much in common with the oscillations which were the subject of the previous section. They have the feature that they oscillate both in space and in time. So if one looks at a cork floating on the surface of a pond, it bobs up and down as a ripple passes under it, showing that a point on the water surface oscillates up and down in time. But also a snapshot of the surface of the water at a given instant of time will show a displacement from the flat surface of an undisturbed pond which varies from point to point in an oscillatory fashion.

We will need to consider waves in one, two or three dimensions. An example of a wave in one dimension is the wave one can see on a vibrating violin or guitar string. The waves on the surface of a pond are an example of waves in two dimensions. And sound waves are an example of waves in three dimensions. In each case there will be a *displacement* from equilibrium which varies from place to place and from time to time. Just as we were able to model many more realistic examples of oscillations in the previous section by considering simple harmonic motion, it is often possible to approximate wave motion by sinusoidal dependence of the displacement on space and time.

Some waves are *transverse*, in that the displacement is perpendicular to the direction in which the wave travels. The ripples on the pond are transverse, as are the vibrations of the violin string. So also are electromagnetic waves which we will meet later in the course. Since in three dimensions there are two different directions at right angles to the direction of propagation of a wave, transverse waves in three dimensions can be *polarized*: the transverse displacement can be in the same direction all along the direction of the wave. Light waves can be polarized.

Other waves are *longitudinal*. In a sound wave the vibrations of the medium transmitting the sound are along the direction of propagation; sound waves are longitudinal. One can excite a longitudinal wave along a spring, an alternation of compression and stretching (or *rarefaction*) which travels along the spring.

6.1 WAVELENGTH AND PERIOD

Waves oscillate in space and in time. So at any point in space, the displacement will oscillate, and one can speak of the time period T , which is the time it takes for the displacement to undergo one complete *cycle of oscillation.* And again one may define the frequency f through

$$f = 1/T.$$

The frequency of a wave motion is measured in hertz (Hz).

A snapshot picture of a wave will show alternations in displacement from point to point, again repeating over and over again, with the separation from one place of maximum displacement to the next giving the *wavelength* λ. If we consider a sinusoidal wave in one dimension (a wave on a stretched string for example), these ideas can be expressed mathematically through an equation for the displacement s which shows how it varies in both space (x) and time (t):

$$s = A\cos(kx - \omega t + \phi),$$

where $k = 2\pi/\lambda$ and $\omega = 2\pi f = 2\pi/T$. As you can see, the *phase*, $kx - \omega t + \phi$, varies both with x and with t, in such a way that the cosine goes through one complete cycle when x changes by λ or when t changes by T. One complete cycle corresponds to a change in phase of $360°$ or 2π radians.

The constant A gives the magnitude of the maximum displacement s from the equilibrium ($s = 0$): A is called the *amplitude* of the wave.

The wave form we have considered can be seen to describe a *travelling* or *progressive* wave. Any point at which the phase takes on a given constant value is found by solving

$$kx - \omega t + \phi = \text{constant},$$

which gives

$$x = ct + \text{constant},$$

where $c = \omega/k = f\lambda$ is the speed with which the point moves. Indeed the whole wave form moves with this speed in the direction of increasing x.

6.2 PLANE WAVES

In three dimensions, the phase of the wave will depend on x, y and z, as well as time t. So we might have something like

$$s = A\cos(\mathbf{k}\cdot\mathbf{r} - \omega t + \phi),$$

where $\mathbf{k}\cdot\mathbf{r} = k_x x + k_y y + k_z z$. There would then be *surfaces* of constant phase

$$\mathbf{k}\cdot\mathbf{r} - \omega t + \phi = \text{constant},$$

which is the equation of a *plane*. This then is a *plane wave*. The crests and troughs of the wave (places of maximum positive or negative displacement) lie on planes which advance with time in the direction specified by the vector $\mathbf{k} = (k_x, k_y, k_z)$. The wavelength is then related to the magnitude $k = \sqrt{(}k_x^2 + k_y^2 + k_z^2)$ of this vector through $k = 2\pi/\lambda$ as before. There are also spherical waves, with spherical surfaces of constant phase, and so on. A wave in two dimensions will have lines of constant phase. When these are straight lines, one has a two-dimensional analogue of a plane wave.

Another name for a surface on which the phase is an integer multiple of 2π is a wave front.

6.3 HUYGENS' PRINCIPLE

A very helpful way of explaining wave properties was introduced by Christian Huygens. One may consider any unobstructed point on a wave front as the source of *wavelets*, spherical (or circular in two dimensions) waves which spread out in the forward direction – the direction in which the wave is moving. The wavelets combine to produce another wave front, and so the wave moves on until it encounters an obstacle. What happens then is discussed in what follows. Another useful concept is that of *rays*. At any point in a region through which a wave passes, one can consider a directed line which points along the direction of the motion of the wave. This is a ray. Rays are always orthogonal to the wave fronts which they intersect.

6.4 REFLECTION AND REFRACTION

Light waves are reflected by a mirror, and other kinds of waves can be reflected by obstacles. For example, spreading ripples on the surface of a swimming pool can be reflected from the containing walls of the pool. It is easiest to consider the situation for waves in two dimensions, if only because it is easier to visualize; but the generalization to three dimensions is straightforward enough. The wave front of the analogue of a plane wave is a straight line. The wave fronts will make an angle i with the reflector; and if the reflector is straight, the reflected wave will also be one with straight line wave fronts which make an angle r with the reflector. The fundamental law of reflection, $i = r$, can then be deduced using Huygens' principle, as can be seen by reference to Fig 16.3C in the textbook.

Huygens' principle can also be used to deduce what happens when non-planar waves are reflected from a planar reflector (or curved wavefronts in two dimensions from a straight reflector), for example spherical (or circular) waves. The reflected waves appear to come from a point behind the reflector. A point source of waves, from which outgoing spherical waves will emerge, will produce reflected spherical waves which will have as their centre an *image point* behind the mirror or reflector, from which they will appear to emerge. Similar arguments can be used for the discussion of reflection from curved reflectors.

6.5 REFRACTION

When waves pass from one region to another in which the speed of the waves is different, the direction of the waves is changed. This is what is called *refraction*. If a ray in the incident wave makes an angle I with the normal to the boundary between the two regions (again called the incident angle, or angle of incidence) and the continuation of that ray on the other side of the boundary, which will be in a different direction because of the refraction, makes an angle r with the normal (r is the angle of refraction), *Snell's Law* states that

$$\frac{\sin i}{\sin r} = n$$

where n is a constant called the refractive index.

Refraction can also be understood with the aid of Huygens' principle. If the speed of the waves is different in the two regions, since the frequency is the same, it follows from the equation $c = f\lambda$ that the wavelength has to be different, with

$$\frac{\lambda_1}{\lambda_2} = \frac{c_1}{c_2}.$$

What follows from Huygens' principle is an explanation of Snell's Law, which furthermore gives an expression for the refractive index n_{12} for passage from medium 1 in which the speed is c_1 to medium 2 with speed c_2 (See Fig 16.3G in the textbook),

$$n_{12} = \frac{c_1}{c_2}.$$

In the case of light waves, when one speaks of the refractive index of a medium, one means the refractive index from a vacuum (or air) as medium 1, into the medium as medium 2. The refractive index of glass for example is about 1.5, so that the speed of light in glass is about 2/3 the speed of light in the vacuum. The speed of light in water is about 3/4 the speed of light in the vacuum, giving a refractive index of water around 4/3. Then the refractive index from water to glass is around 9/8.

6.6 DISPERSION

Although the speed of light in the vacuum is the same for all wavelengths, its speed in a medium is in general different for different wavelengths. Similarly, the speed of propagation of other kinds of wave may vary with wavelength. Since the amount of bending of the rays, their refraction, depends on the refractive index, which in turn depends on the speed of the waves in the medium. This results in dispersion of the waves as they are refracted on passing from one medium to another, waves of different wavelength being bent through different angles so that a parallel "pencil" of rays which on one side of the boundary are all in the same direction, become on refraction spread out, or dispersed. This is the origin of the colours produced by allowing white light to be refracted by a prism. Because the speed of blue light in glass is less than that of red light, the blue rays are bent more than the red one, and so through the whole spectrum of colours which comprise what we call white light.

6.7 DIFFRACTION

Huygens' principle also allows an understanding of the phenomenon of *diffraction.* This is what happens at a gap in a barrier or at the edge of a barrier. As you may have observed with water waves, the waves spread out on the far side of the barrier, rather than being cut off sharply as they would if forming a "geometric" shadow. The ray picture does not work well at edges! The extent to which the waves spread out requires more than Huygens' principle for a full understanding, but for a simple phenomenological description, it often suffices to note that short waves spread out more than long ones. Here "short" means short compared with the width of the gap. The long waves of an ocean swell spread out into the harbour through the narrow mouth in its protective walls; but because a typical wavelength for light is around 600 nm. they are not diffracted appreciably except by narrow features on the scale of tenths of a millimetre. For gaps or objects larger than that "geometric optics" with rays and sharp shadows is usually adequate, and diffraction can be ignored.

6.8 INTERFERENCE

When two or more wave trains cross, they pass through one another. Each of them contributes to the resultant displacement where they intersect. The resultant displacement is then just the sum of the displacements produced by each of the separate wave trains. This very important and very general property of waves, the *superposition principle* lies at the heart of the most characteristic feature of wave motion - *interference.*

Since the displacements can be either positive or negative, it is possible that the superposition of two waves can show *reinforcement* (where the displacements have the same sign) or *cancellation* (where they are of opposite sign). If the two waves have the same frequency, and if their sources maintain a constant phase difference (in which case the sources are said to be *coherent*), the relative phases of the two wave trains at any given point where they intersect will not change with time. Places where the waves reinforce will not move, nor will places where they cancel. There results an *interference pattern.*

6.9 STANDING WAVES

So far we have concentrated attention on waves which propagate, progressive waves in which the wave fronts move. But one also has *standing waves* (also called stationary waves), in which the wave pattern oscillates with time, but stands still in position. The vibrations of a plucked guitar string are an example of a standing wave. Each point on the string oscillates in time, and the profile of the string at any instant has a sinusoidal shape. But the wave profile does not itself travel along the string. A mathematical expression for such a standing wave is something of the form

$$s = A \cos \omega t \sin kx,$$

where again there is a time period $T = 2\pi/\omega$ and a wavelength $\lambda = 2\pi/k$, but no progression of the wave. There are places where the displacement s vanishes at all times (where $kx =$ an integer multiple of π in the example illustrated by the equation); these are called *nodes.* The distance between neighbouring nodes is half of one wavelength. Halfway between the nodes, there are the *antinodes*; these are points at which the displacement oscillates with maximum amplitude. A standing wave on a stretched string with fixed endpoints has nodes at these fixed points.

Notes prepared by John M Charap

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