FIELDS AND WAVES

4.0 ELECTROMAGNETIC INDUCTION

In Week 2, we studied the electric fields produced by static charges. But there is another way to generate an electric field. Whenever magnetic fields *change* in any way, they generate an *induced electric field*. This induced electric field exerts a force on electric charge, just as does the electrostatic field we have studied earlier. If we have a loop of wire with, for example, a lamp bulb in series in it, the induced electric field can push the electrons around in the wire, and light up the lamp. The changing magnetic field produces an *electromotive force*, or emf in the circuit. The basic law of electromagnetic induction, as formulated by Michael Faraday on the basis of the brilliant experiments he performed at the Royal Institution here in London in the 1820s, relates the induced emf in a closed loop to the rate at which the total flux of the magnetic field through that loop changes. Faraday demonstrated the phenomenon before the Royal Society in November 1831.

We have already seen that stationary charges exert forces on one another; and moving charges produce magnetic fields which exert forces on other moving charges, so that electric currents exert forces on one another. Faraday set out to examine whether an electric current in a wire might produce an electric current in a nearby wire. At first, and to his surprise and disappointment, he failed. A steady current has no effect on a nearby wire at all. However, when he set up apparatus "with 203 feet of copper wire coiled around a large block of wood, and another coil of 203 feet of similar wire, separated electrically by an insulator (twine), and connected one of the coils to a galvanometer to detect current and the other to a battery with 100 pairs of plates, 4 inches square, with double coppers well charged" , there was a surprise. He wrote:

"When the contact was made, there was a sudden and very slight effect at the galvanometer, and there was also a similar slight effect when the contact was broken"

But while the current flowed steadily, he could detect no effect.

4.1 FARADAY'S LAW OF INDUCTION

It was the *change* in the current in the first coil that produced the *induced* flow of current in the second coil. Faraday then went on to replace the galvanometer by a solenoid into which he had inserted an *unmagnetised* iron needle. The induced current was thereby shown to magnetise the needle, and he was thus able to determine the direction of the induced current. Almost at the same time Joseph Henry made the same discovery in the USA. And the Russian physicist HFE Lenz studied the same phenomenon a little later. He formulated a principle that allows us to determine the direction of the induced current. Faraday soon recognised that what was responsible for the induced current was the change in the magnetic field produced by the first coil as the current through it was switched on or off. So he was able to demonstrate that moving a magnet through a loop of wire, or inserting a magnet into a solenoid or withdrawing it from a solenoid, or *vice versa* moving a wire through a magnetic field, can generate an emf.

When a wire is moved between the poles of a magnet, it *cuts across* the lines of the magnetic field. There is an induced emf in the wire, which is proportional to the rate at which the magnetic field lines are crossed. It makes no difference whether the wire moves and the magnet is stationary, or the magnet moves with the wire stationary. The induced emf is in either case associated with the rate at which the wire cuts across the magnetic field.

In a simple *dynamo*, spinning a magnet near a fixed coil generates the emf which then drives the current through the circuit to which it is connected.

The mechanism by which the motion of a conductor through a magnetic field produces an emf is a direct consequence of the force exerted on a moving charge by a magnetic field. For example, if a wire is moved through a magnetic field, the essentially free electrons in it are moved through the field, and so experience a force that (being at right angles to both the direction of motion and to the direction of the field) tends to push them along the wire. This is the induced emf. The direction of the emf is such that the force on the wire associated with the induced current flowing in the presence of the magnetic field *opposes* the motion of the wire. This is one version of Lenz's rule. We can go further, and use this idea to derive a quantitative result.

If a length L of straight conducting wire, part of a closed circuit, is moved at constant speed v at right angles across a uniform magnetic field B, it cuts across magnetic field lines. The rate at which they are cut is BLv (we take B lines of force per unit area, and the area is changing at the rate Lv). The induced emf $\mathcal{E}$ makes a current I flow in the circuit, and this does work (for example in heating a resistor) at the rate $I\mathcal{E}$. Where does this work come from? It is supplied by the work needed to keep the wire moving against the force that the current in the wire moving through the magnetic field produces. Recall the formula

$$F = BIL$$

for the force on a wire of length L carrying a current I in a magnetic field B, and perpendicular to it. The rate at which work has to be done to overcome this force is $BILv$. So, on cancelling out the factor I, we have

$$\mathcal{E} = BLv,$$

which is just the rate at which the lines of magnetic field are cut. This is our first expression for *Faraday's Law of induction.*

4.2 MAGNETIC FLUX

For a more general and complete formulation of this law, we have first to introduce and define the notion of *magnetic flux*. If we imagine a small rectangular area ΔA at right angles to a magnetic field of strength B, the flux through that area is just $\Delta\phi = B.\Delta A$ (The area is taken to be small, so that even if the field is not uniform, it doesn't vary by much across the area ΔA.) If the field is not perpendicular to the area ΔA, but makes an angle θ with it, we have instead

$$\Delta\phi = B.\Delta A \cos\theta$$

For a larger area, of arbitrary shape, the flux of even a non-uniform field can be determined by adding together the contributions from such little "infinitesimal elements of area", using the ideas from calculus. We will not need to do that in this course. But for a uniform field B perpendicular to an area A, the result for the magnetic flux is just

$$\phi = BA.$$

The unit for the magnetic flux is the weber (Wb), defined as T. m^2.

For a coil of n turns enclosing an area A, we will find useful also the idea of the *flux linkage* through the coil. Each turn of the coil contributes an amount ϕ, and the total flux linkage for the coil is

$$\Phi = n\phi = nBA,$$

if B is the magnetic field strength normal to the area A of the coil. Again this idea can be generalised, so as to give the flux linkage through any circuit.

Faraday's law can now be expressed as

$$\mathcal{E} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}.$$

The induced emf in the circuit is equal in magnitude to the rate at which the magnetic flux linkage Φ with the circuit changes, and is in the direction such as to oppose that change (this is Lenz's rule again). The induced current produces a magnetic field and so also a magnetic flux through the circuit: the direction of the induced current is such that this induced magnetic field is in the opposite direction to the field B.

It makes no difference what it is that causes the flux to change. It could be changed by varying the magnitude or direction of the magnetic field, or by moving the circuit or just part of it, so that the area in the magnetic field changes.

4.3 MEASURING THE MAGNETIC FIELD

We have already encountered one method for measuring the magnetic field strength – the Hall probe. But a *ballistic galvanometer* connected to a *search coil* provides an alternative method for measuring the magnetic field. The idea is to detect the charge moved through a circuit by the emf induced in extracting a small coil from the magnetic field to be measured, thereby changing the flux linkage through the coil.

So first of all we need to consider the ballistic galvanometer as a device for measuring the charge moved when a current flows for a short period. Consider a moving coil galvanometer, which is a coil suspended so that it rotates freely in a magnetic field whilst carrying the current. There is a mirror attached to the suspension that allows one to detect and measure the twist in the suspension as the coil rotates. The charge we wish to measure flows through the coil as a current which lasts for only a short time, short that is compared with the period of free oscillations of the coil if it is made to rotate by some applied torque, and then left alone. The effect of the current is then to give the coil a kick, an impulsive twist, which is proportional to the magnitude of the current and its duration, in fact to the charge carried through the coil by the current.

In just the same way that an impulse can set a pendulum swinging, and the amplitude of the swing it induces is proportional to the size of the impulse, so the amplitude of the oscillations of the suspension of the coil is proportional to the size of the impulsive twist delivered by the current as the charge flows through the coil, in turn proportional to the charge which flows through. So by measuring the amplitude – it's best to measure the amplitude of the initial deflection, since as the coil swings back and forth once the current has ceased, the amplitude of the swing is damped out by friction – one obtains a measure of the charge pushed through the coil, and the factor of proportionality (which depends on the stiffness of the support) can be determined by calibrating the instrument with a known current acting for a known time. This then is the basis for constructing the ballistic galvanometer.

Now we want to use the ballistic galvanometer to measure a magnetic field. This is done by placing a small coil in the magnetic field in the position where the field is to be measured, and perpendicular to the direction in which the field component is required. The coil is in a circuit connected in series with the ballistic galvanometer, and a resistor adjusted to allow the galvanometer coil to swing freely – usually a high setting is needed. The small coil is called a *search coil*. The flux linkage through the search coil is BAn, where A is the known area of the coil, and n is the known number of turns in the coil. B is the quantity to be measured, namely the component of the magnetic field in the vicinity of the search coil, perpendicular to it.

What is done is to remove the search coil from the field *quickly*. The flux linkage through the coil thereby changes rapidly, falling from its initial value Ban to zero. As the flux linkage changes, Faraday's law of induction tells us that there will be an induced emf in the circuit connecting the coil to the galvanometer, and so a current flows during the brief time that the search coil is being removed from the field. And the galvanometer allows us to measure the charge transported around the circuit whilst the current flows.

We now show how to relate that charge to the magnetic field component B

we were trying to measure. Faraday's law

$$\mathcal{E} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}.$$

means that the current flowing in the circuit at any instant is

$$I = \frac{\mathcal{E}}{R},$$

where R is the resistance in the circuit. But the current is the rate at which charge is being transported around the circuit,

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}.$$

This means that in any short interval of time, the amount of charge transported is given by

$$\Delta Q = -\frac{\Delta\Phi}{R},$$

where $\Delta\Phi$ is the change in the flux linkage in the same time interval.

So Q, the *total* amount of charge transported as the search coil is removed is just $-1/R$ times the change in the flux linkage through the search coil, which is minus (its final value minus its initial value) or since its final value is zero,

$$Q = \frac{\Phi_{\text{initial}}}{R}.$$

But we know

$$\Phi_{\text{initial}} = BAn.$$

So we now have

$$BAn = QR.$$

But Q is precisely what is measured by the ballistic galvanometer, as proportional to the angle θ of the initial deflection of the galvanometer mirror,

$$Q = k\theta$$

with the factor of proportionality k, the so-called charge-sensitivity, or charge per unit deflection. So what we now have found is that

$$BAn = kR\theta$$

or on rearranging the equation

$$B = (kR/An)\theta.$$

In practice of course one can calibrate the apparatus to give B directly from the measured value of θ.

4.4 A SPEEDOMETER

Sweeping a wire through a magnetic field generates an emf across the ends of the wire. So spinning a disc in a magnetic field generates an emf between the axis and the perimeter of the disc. This is the basis for the simplest kind of generator of an emf. If you think of the disc as a lot of wires running from the centre to the rim, like the spokes of a bicycle wheel, it is straightforward to use Faraday's law to determine the induced emf. Suppose the magnetic field to be at right angles to the plane of the disc. Then as the disc turns, each of the "spokes" cuts across the magnetic field lines, and as the disc turns through one complete revolution the flux crossed by each spoke is just B times the area πR^2 of the disc. But this happens f times a second if f is the frequency with which the disc turns. So the rate at which the flux is crossed by the turning spoke is $B\pi R^2 f$ and this is the emf between the centre and the rim of the disc.

The direction of the emf can be determined by applying Lenz's rule, or any of the other related ideas we have introduced. Suppose for example that seen from the side from which the disc appears to be rotated clockwise , the magnetic field is directed towards the disc. Then an electron near the top of the disc is moving from left to right in a magnetic field at right angles to it, and so (since it has a negative charge) will experience a force towards the axis around which the disc is rotating. In fact for any position on the disc, the force on an electron is towards the centre, so the axis has a negative potential relative to the rim.

The voltage between the axis and the rim can be measured with a voltmeter. Alternatively, the voltmeter can be used to measure the speed with which the disc is rotating, and that is the way the speedometer on a car operates.

4.5 DYNAMOS

Suppose a rectangular coil is rotated between the poles of a magnet. The magnetic flux through the coil changes as the angle between the plane of the coil and the direction of the magnetic field changes. Remember that the flux linkage is given by

$$\Phi = BAn\cos\theta.$$

So there is an induced emf in the coil, and this can be used to drive a circuit fed from the coil by brushes which make contact with the leads from the coil as it rotates.

$$\mathcal{E} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t} = -\frac{\mathrm{d}(BAn\cos\theta)}{\mathrm{d}t}.$$

If the coil is rotated at a steady angular speed, we have

$$\theta = \omega t,$$

and since the area A, number of windings n and the magnetic field strength B are none of them varying, we obtain

$$\mathcal{E} = -\mathrm{d}(BAn\cos\omega t)/\mathrm{d}t = BAn\omega\sin\omega t.$$

The rotating coil generates an a.c. emf, varying sinusoidal with the angular frequency ω and with peak value $BAn\omega$.

If what is wanted is a d.c. emf, we have to arrange for the brushes to make contact with first one end of the coil, then the other, changing every time the coil makes half a revolution. This may be done with what are called *split ring* contacts. The resulting emf is always in one direction, although it is still not constant in time, and so in practice would still need to be "smoothed".

In a power station the electricity is generated in a similar fashion, but there are some important differences. First, the magnet is an electromagnet, supplied with dc current. Secondly, instead of the coil spinning between the poles of a magnet, the magnet is spun between coils. The spinning electromagnet is called the *rotor*, and it will be turned at the steady rate of 50 turns per second (in the UK; 60 in the USA). It rotates between coils which are stationary, and these are called the *stators*. In fact there are three pairs of coils set at 120° to one another, and this means that the output emf from each of the pairs is 120° out of phase with the others. The result is what is called a three phase supply, and this is what is distributed over the main grid transmission lines. Domestic supplies only use one of the three phases, but around the Physics Building you may see three phase power points.

4.6 EDDY CURRENTS and INDUCTION MOTORS

If a magnet is moved close to a conducting plate, the electrons in the plate experience a force, because they are in motion relative to the magnetic field. The same will be true if the plate is moved relative to a fixed magnet. So there are induced in the plate electric currents as the electrons move around. These are called *eddy currents*. These currents then interact with the magnetic field and produce a force on the plate, which (Lenz's rule again!) act to oppose the relative motion of the magnet and the plate. So if the plate were a rotating disc, the force would act to slow it down (This is the principle for the electromagnetic brake. The magnetic field can be provided by an electromagnet; when the electromagnet is switched on the eddy currents and the consequential forces act to brake the turning disc). Similarly if a magnet is moved close to a metal plate, the result is a force which acts to drag the plate along with the moving magnet.

If a metal cylinder, free to turn on its axis, is acted on by two or more ac-supplied electromagnets suitably placed, the cylinder turns. The reason is that the varying magnetic field from say the first electromagnet induces eddy currents in the cylinder, which rise and fall, and so also for the next. But the peak fields are reached out of phase with one another, and the result is that the peak field circles round the cylinder in just the same way as if a magnet were being moved round and round the cylinder. And the result is the same as though this was what was happening, the cylinder is dragged around by the rotating magnetic field.

This is the basis of an *induction motor*. In a three-phase induction motor, there are three pairs of coils (stators) wound from the three phases of the power supply so as to provide a rotating magnetic field. This acts on a rotor, which is a soft iron cylinder (the iron is easily magnetised and so enhances the flux), which is lined with copper rods. Copper is a good conductor, so the eddy currents are strong and so exert greater force on the rotor which spins with the rotating magnetic field.

It is possible to run an induction motor with only one magnet. This is done by "shading" part of the pole face of the magnet with a plate which in effect delays the magnetic field so that the single magnet behave like two magnets with a phase lag between them. A similar effect may be achieved by inserting "shading rings" through the laminations of soft iron which are the core of the stator magnet.

4.7 TRANSFORMERS

If an ac current is passed through a coil wound round an iron bar, it magnetises the bar first one way then the next. The magnetic flux along the bar is therefore changing. So if there is another coil wound round the same bar, there is a changing flux through that second coil, and an emf is generated in it. This is the basis for the action of a *transformer*.

If it is arranged so that all of the flux produced by the first coil (the *primary*) passes through the second (the *secondary*), we have a more efficient transformer. If ϕ is the flux (through both coils), the flux linkage with the secondary is $N_s\phi$. So the induced emf in the secondary coil is

$$V_s = N_s \mathrm{d}\phi/\mathrm{d}t.$$

There is also an emf in the primary coil induced by the changing flux, a *back emf*. This is what is opposed to the voltage supply to the primary, from which one deduces

$$V_p = N_p \mathrm{d}\phi/\mathrm{d}t.$$

From this follows the simple relation between the p.d supplied to the primary (V_p) and the induced emf in the secondary (V_s), namely

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

Note also that the power supplied to the primary coils is (assuming no losses to heating of the core, etc.) equal to the power taken from the secondary by the current generated. So for a 100% efficient transformer we have

$$I_s V_s = I_p V_p.$$

Transformers can be either "step-up" or "step-down".

One of the advantages of ac power supply is the ease with which it can with good efficiency be transformed up or down in voltage. High voltage is used for transmission, because it minimises energy loss. To transmit power W at voltage V through cables offering resistance R means sending a current $I = W/V$ through the resistance R, with consequential loss of power

$$I^2 R = W^2 R/V^2,$$

clearly demonstrating the advantage of having V as high as practicable.

4.8 MUTUAL INDUCTANCE

The flux linkage through a secondary coil is proportional to the current in the primary coil, so we may write

$$\Phi_s = M I_p.$$

And since the emf induced in the secondary is given by Faraday's law, we have

$$V_s = -\mathrm{d}\Phi_s/\mathrm{d}t = -M\mathrm{d}I_p/\mathrm{d}t.$$

The constant M is called the *mutual inductance* between the coils. The unit of inductance is the *henry* (H):

$$1\ \mathrm{H} = 1\ \mathrm{V\ A^{-1}\ s} = 1\ \Omega\mathrm{s}.$$

The mutual inductance M also enters in the corresponding relation

$$V_p = -M\mathrm{d}I_s/\mathrm{d}t.$$

It in fact depends only on the geometrical arrangement of the coils.

A varying current in a coil produces a changing flux linkage through that same coil, and so generates a *back emf* in the coil; its direction (Lenz's rule again) is opposite to the voltage driving the varying current. We have for the induced emf

$$V = -\mathrm{d}\Phi/\mathrm{d}t,$$

and the flux linkage Φ is proportional to the current in the coil

$$\Phi = LI,$$

so that

$$V = -L\mathrm{d}I/\mathrm{d}t.$$

The factor of proportionality L is called the self-inductance of the coil. It is again essentially a geometrical factor, again expressed in henry.

When a circuit through a coil is first switched on, the current does not rise instantaneously, but rises gradually from zero. This is best seen by considering a simple circuit of a battery, a coil and a resistor in series, with a switch which is closed at time t. The total emf in the circuit is the battery voltage V_{batt} minus the induced emf in the coil. So we have

$$V_{\mathrm{batt}} - L\mathrm{d}I/\mathrm{d}t = IR.$$

This equation for $I(t)$ may be solved to give a curve for I *vs* t which rises from its initial value $I(0) = 0$ to approach its final value V_{batt}/R. There is likewise an exponential fall in the current when the battery is disconnected. The continuing presence of the back emf even when V_{batt} is removed can lead to sparking at the switch.

4.9 ENERGY STORED IN AN INDUCTOR

There is energy stored in an inductor carrying a current. To determine just how much, calculate the work done in taking the current from an initial value of zero to a final value I. This is obtained from the power supplied, which when the current has reached a value i is $W = iV$, with V equal to the back emf, namely

$$W = iL\mathrm{d}i/\mathrm{d}t.$$

But since W is the rate at which the energy stored in the inductor is increasing, the final energy stored is given by integration, and the result is

$$E = LI^2/2.$$

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